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ON FUZZY UPPER AND LOWER CONTRA e^* (δs and δp)-CONTINUOUS MULTIFUNCTIONS

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ABSTRACT

In this paper, we introduce the concepts of fuzzy upper and fuzzy lower contra e^* (resp. δ-semi and δ-pre)-continuous multifunction on fuzzy topological spaces in $\hat{S}$ostak sense. Several characterizations and properties of these fuzzy upper (resp. fuzzy lower) contra e^*(resp. δ-semi and δ-pre)-continuous multifunctions are presented and their mutual relationships are established in L-fuzzy topological spaces. Later, composition and union between these multifunctions have been studied.

Keywords and phrases: *fuzzy upper contra e^*(resp. δ-semi and δ-pre)-continuous multifunction, fuzzy lower contra e^*(resp. δ-semi and δ-pre)-continuous multifunction.*

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1. INTRODUCTION AND PRELIMINARIES

Kubiak [9] and $\hat{S}$ostak [12] introduced the notion of (L-) fuzzy topological space as a generalization of L-topological spaces (originally called (L-) fuzzy topological spaces by Chang [5] and Goguen [6]. It is the grade of openness of an L-fuzzy set. Berge [4] introduced the concept multimapping $F: X \multimap Y$ where X and Y are topological spaces. After Chang introduced the concept of fuzzy topology [5], continuity of multifunctions in fuzzy topological spaces have been defined and studied by many authors from different view points [3]. Tsiporkova *et.al*, [15, 16] introduced the continuity of fuzzy multivalued mappings in the Chang's fuzzy topology [5]. Later, Abbas *et.al*, [1], [2] introduced the concepts of fuzzy upper and lower semi-continuous multifunctions, fuzzy upper and lower β-continuous multifunctions in L-fuzzy topological spaces. Hebeshi., [7] introduced the concepts of fuzzy upper and lower α-continuous multifunctions in L-fuzzy topological spaces. Recently, Vadivel *et.al*, [17] and Prabhu *et.al*, [18] introduced r-fe^*o sets and fuzzy e^*-continuity in a smooth topological space. Sujatha *et.al* [14] introduced fuzzy upper and lower contra e-continuous multifunctions on fuzzy topological spaces in $\hat{S}$ostak sense. In this paper, we introduce the concepts of fuzzy upper and fuzzy lower contra e^*(resp. δ-semi and δ-pre)-continuous multifunction on fuzzy topological spaces in $\hat{S}$ostak sense. Several characterizations and properties of these multifunctions are presented and their mutual relationships are established in L-fuzzy topological spaces. Later, composition and union between these multifunctions have been studied.
Throughout this paper, nonempty sets will be denoted by X, Y etc., $L = [0, 1]$ and $L_0 = (0, 1]$. The family of all fuzzy sets in X is denoted by L^X. The complement of an L-fuzzy set λ is denoted by λ^c. This symbol $\multimap$ for a

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multifunction. For $\alpha \in L$, $\overline{\alpha}(x) = \alpha$ for all $x \in X$. A fuzzy point x_t for $t \in L_0$ is an element of L^X such that $x_t(y) = \begin{cases} t & \text{if } y = x \\ 0 & \text{if } y \neq x. \end{cases}$ The family of all fuzzy points in X is denoted by $Pt(X)$. A fuzzy point $x_t \in \lambda$ iff $t \leq \lambda(x)$. All other notations are standard notations of L-fuzzy set theory.

Let $F: X \multimap Y$, then F is called a fuzzy multifunction (FM, for short) [1] if and only if $F(x) \in L^Y$ for each $x \in X$. The degree of membership of y in $F(x)$ is denoted by $F(x)(y) = G_F(x, y)$ for any $(x, y) \in X \times Y$. The domain of F, denoted by $domain(F)$ and the range of F, denoted by $rng(F)$, for any $x \in X$ and $y \in Y$, are defined by: $dom(F)(x) = \bigvee_{y \in Y} G_F(x, y)$ and $rng(F)(y) = \bigvee_{x \in X} G_F(x, y)$. Let $F: X \multimap Y$ be a FM. Then F is called: (i) Normalized iff for each $x \in X$, there exixts $y_0 \in Y$ such that $G_F(x, y_0) = \overline{1}$. (ii) A crisp iff $G_F(x, y) = \overline{1}$ for each $x \in X$ and $y \in Y$. Let $F: X \multimap Y$ be a FM. Then (i) The image of $\lambda \in L^X$ is an L-fuzzy set $F(\lambda) \in L^Y$ defined by $F(\lambda)(y) = \bigvee_{x \in X}[G_F(x, y) \wedge \lambda(x)]$. (ii) The lower inverse of $\mu \in L^Y$ is an L-fuzzy set $F^l(\mu) \in L^X$ defined by $F^l(\mu)(x) = \bigvee_{y \in Y}[G_F(x, y) \wedge \mu(y)]$. (iii) The upper inverse of $\mu \in L^Y$ is an L-fuzzy set $F^u(\mu) \in L^X$ defined by $F^u(\mu)(x) = \bigwedge_{y \in Y}[G_F^c(x, y) \vee \mu(y)]$.

An L-fuzzy topological space (L-fts, in short) [9,12] is a pair (X, τ), where X is a nonempty set and $\tau: L^X \to L$ is a mapping satisfying the following properties. (i) $\tau(\overline{0}) = \tau(\overline{1}) = 1$, (ii) $\tau(\mu_1 \wedge \mu_2) \geq \tau(\mu_1) \wedge \tau(\mu_2)$, for any $\mu_1, \mu_2 \in I^X$. (iii) $\tau(\bigvee_{i \in \Gamma} \mu_i) \geq \bigwedge_{i \in \Gamma} \tau(\mu_i)$, for any $\{\mu_i\}_{i \in \Gamma} \subset I^X$. Then τ is called an L-fuzzy topology on X. For every $\lambda \in L^X$, $\tau(\lambda)$ is called the degree of openness of the L-fuzzy set λ. A mapping $f: (X, \tau) \to (Y, \eta)$ is said to be continuous with respect to L-fuzzy topologies τ and η iff $\tau(f^{-1}(\mu)) \geq \eta(\mu)$ for each $\mu \in L^Y$. Let (X, τ) be a an L-fts. Then for each $\lambda \in L^X$, $r \in L_0$, we define L-fuzzy operators C_τ and $I_\tau: L^X \times L_0 \to L^X$ as follows:
$C_\tau(\lambda, r) = \bigwedge\{\mu \in L^X: \lambda \leq \mu, \tau(\overline{1} - \mu) \geq r\}$. $I_\tau(\lambda, r) = \bigvee\{\mu \in L^X: \lambda \geq \mu, \tau(\mu) \geq r\}$.

Let (X, τ) be a fts. For $\lambda, \mu \in I^X$ and $r \in I_0$, λ is called r-fuzzy regular open [8] (for short, r-fro) (resp. r-fuzzy regular closed (for short, r-frc)) if $\lambda = I_\tau(C_\tau(\lambda, r), r)$ (resp. $\lambda = C_\tau(I_\tau(\lambda, r), r)$). Let (X, τ) be a fts. Then for each $\mu \in I^X$, $x_t \in P_t(X)$ and $r \in I_0$, (i) μ is called r-open Q_τ-neighbourhood of x_t if $x_t q \mu$ with $\tau(\mu) \geq r$. (ii) μ is called r-open R_τ-neighbourhood of x_t if $x_t q \mu$ with $\mu = I_\tau(C_\tau(\mu, r), r)$. We denoted $Q_\tau(x_t, r) = \{\mu \in I^X: x_t q \mu, \tau(\mu) \geq r\}$, $R_\tau(x_t, r) = \{\mu \in I^X: x_t q \mu, \mu = I_\tau(C_\tau(\mu, r), r)\}$. Let (X, τ) be a fts. Then for each $\lambda \in I^X$, $x_t \in P_t(X)$ and $r \in I_0$, (i) x_t is called r-τ cluster point of λ if for every $\mu \in Q_\tau(x_t, r)$, we have $\mu q \lambda$. (ii) x_t is called r-δ cluster point of λ if for every $\mu \in R_\tau(x_t, r)$, we have $\mu q \lambda$. (iii) An δ-closure operator is a mapping $D_\tau: I^X \times I \to I^X$ defined as follows: $\delta C_\tau(\lambda, r)$ or $D_\tau(\lambda, r) = \bigvee\{x_t \in P_t(X): x_t$ is r-δ-cluster point of $\lambda\}$. Equivalently, $\delta C_\tau(\lambda, r) = \bigwedge\{\mu \in I^X: \mu \geq \lambda, \mu \text{ is a } r-\text{frc set}\}$ and $\delta I_\tau(\lambda, r) = \bigvee\{\mu \in I^X: \mu \leq \lambda, \mu \text{ is a } r-\text{fro set}\}$. Let (X, τ) be a fuzzy topological space. For $\lambda \in I^X$ and $r \in I_0$, λ is called r-fuzzy δ-closed iff $\lambda = \delta C_\tau(\lambda, r)$ or $D_\tau(\lambda, r)$.

Let (X, τ) be a fuzzy topological space. For $\lambda, \mu \in I^X$ and $r \in I_0$,λ is called an (i) r-fuzzy δ-semiopen [13] (resp. r-fuzzy δ-semiclosed) set if $\lambda \leq C_\tau(\delta\text{-}I_\tau(\lambda, r), r)$ (resp. $I_\tau(\delta\text{-}C_\tau(\lambda, r), r) \leq \lambda$). (ii) r-fuzzy δ-preopen [13] (resp. r-fuzzy δ-preclosed) set if $\lambda \leq I_\tau(\delta\text{-}C_\tau(\lambda, r), r)$ (resp. $C_\tau(\delta\text{-}I_\tau(\lambda, r), r) \leq \lambda$). (iii) r-fuzzy α-open [11] (resp.r-fuzzy α-closed) set if $\lambda \leq I_\tau(C_\tau(I_\tau(\lambda, r), r), r)$ (resp. $C_\tau(I_\tau(C_\tau(\lambda, r), r), r) \leq \lambda$). (iv) r-fuzzy β-open [11] (resp.r-fuzzy β-closed) set if $\lambda \leq C_\tau(I_\tau(C_\tau(\lambda, r), r), r)$ (resp. $I_\tau(C_\tau(I_\tau(\lambda, r), r), r) \leq \lambda$). (v) r-fuzzy e-open [13] (resp. r-fuzzy e-closed) set if $\lambda \leq C_\tau(\delta\text{-}I_\tau(\lambda, r), r) \vee I_\tau(\delta\text{-}C_\tau(\lambda, r), r)$ (resp. $C_\tau(\delta\text{-}I_\tau(\lambda, r), r) \wedge I_\tau(\delta\text{-}C_\tau(\lambda, r), r) \leq \lambda$). (vi) r-fuzzy e^*-open [17] (resp. r-fuzzy e^*-closed) set if $\lambda \leq C_\tau(I_\tau(\delta\text{-}C_\tau(\lambda, r), r), r)$ (resp. $I_\tau(C_\tau(\delta\text{-}I_\tau(\lambda, r), r), r) \leq \lambda$). [17] Let (X, τ) be a fuzzy topological space. $\lambda, \mu \in I^X$ and $r \in I_0$, $e^* I_\tau(\lambda, r) = \bigvee\{\mu \in I^X: \mu \leq \lambda, \mu$ is a $r\text{-}fe^*o$ set $\}$ is called the r-fuzzy e^*-interior of λ. $e^* C_\tau(\lambda, r) = \bigwedge\{\mu \in I^X: \mu \geq \lambda, \mu$ is a $r\text{-}fe^*c$ set $\}$ is called the r-fuzzy e^*-closure of λ.

Let $F: X \multimap Y$ be a FM between two L-fts's (X, τ), (Y, η) and $r \in L_0$. Then F is called: (i) Fuzzy upper semi (or Fuzzy upper) (in short, FUS (or FU)) (resp. $FU\alpha$, FUe and $FU\beta$)-continuous at a L-fuzzy point $x_t \in dom(F)$ iff $x_t \in F^u(\mu)$ for each $\mu \in L^Y$ and $\eta(\mu) \geq r$, there exists $\lambda in L^X$, $\tau(\lambda) \geq r$ (resp. r-fαo, r-feo and r-fβo set) and $x_t \in \lambda$ such that $\lambda \wedge dom(F) \leq F^u(\mu)$. F is FU (resp. $FU\alpha$, FUe and $FU\beta$)-continuous iff it is FU(resp. $FU\alpha$, FUe and $FU\beta$)-continuous at every $x_t \in dom(F)$. (ii) Fuzzy lower semi (or Fuzzy lower) (in short, FLS (or FL)) (resp. $FL\alpha$, FLe and $FL\beta$)-continuous at a L-fuzzy point $x_t \in dom(F)$ iff $x_t \in F^l(\mu)$ for each $\mu \in L^Y$ and $\eta(\mu) \geq r$, there exists $\lambda \in L^X$, $\tau(\lambda) \geq r$ (resp. r-fαo, r-feo and r-fβo set) and $x_t \in \lambda$ such that $\lambda \leq F^l(\mu)$. F is FL (resp. $FL\alpha$, FLe and $FL\beta$)-continuous iff it is FL (resp. $FL\alpha$, FLe and $FL\beta$)-continuous at every $x_t \in dom(F)$. (iii) Fuzzy [1] (resp. $FU\alpha$ [7], FUe [19] and $FU\beta$ [2])-continuous if it is FU (resp. $FU\alpha$, FUe and $FU\beta$)-continuous and FL (resp. $FL\alpha$, FLe and $FL\beta$)-continuous.

Let (X,τ) and (Y,η) be a fts's. The fuzzy sets of the form $\lambda\times\mu$ with $\tau(\lambda)\geq r$ and $\eta(\mu)\geq r$ form a basis for the product fuzzy topology [3,20] $\tau\times\eta$ on $X\times Y$, where for any $(x,y)\in X\times Y$, $(\lambda\times\mu)(x,y)=min\{\lambda(x),\mu(y)\}$. [3,10] Let $F:X\multimap Y$ be a FM between two fts's (X,τ) and (Y,η). The graph fuzzy multifunction $G_f:X\to X\times Y$ of F is defined as $G_f(x)=x_1\times F(x)$, for every $x\in X$. [14] Let $F:X\multimap Y$ be a FM between two L-fts's (X,τ), (Y,η) and $r\in L_0$. Then F is called: (i) Fuzzy upper contra e-continuous ($FUCe$-continuous, in short) at any L-fuzzy point $x_t\in dom(\square)$ iff $x_t\in F^u(\mu)$ for each $\mu\in L^Y$ and $\eta(\mu^c)\geq r$, there exists r-feo set $\lambda\in L^X$ and $x_t\in\lambda$ such that $\lambda\wedge dom(F)\leq F^u(\mu)$. (ii) Fuzzy lower contra e-continuous ($FLCe$-continuous, in short) at any L-fuzzy point $x_t\in dom(F)$ iff $x_t\in F^l(\mu)$ for each $\mu\in L^Y$ and $\eta(\mu^c)\geq r$, there exists r-feo set $\lambda\in L^X$ and $x_t\in\lambda$ such that $\lambda\leq F^l(\mu)$. (iii) Fuzzy upper contra e-continuous (resp. Fuzzy lower contra e-continuous) iff it is $FUCe$-continuous (resp. $FLCe$-continuous) at every $x_\square\in dom(F)$.

2. FUZZY UPPER AND LOWER CONTRA e^* (resp. δ-semi and δ-pre)-CONTINUOUS MULTIFUNCTIONS

Definition 2.1: *Let $F:X\multimap Y$ be a FM between two L-fts's (X,τ), (Y,η) and $r\in L_0$. Then F is called:*

1. *Fuzzy upper contra e^* (resp. δ-semi and δ-pre) (in short, $FUCe^*$ (resp. FUCδS and $FUC\delta P$))-continuous at any L-fuzzy point $x_t\in dom(F)$ iff $x_t\in F^u(\mu)$ for each $\mu\in L^Y$ and $\eta(\mu^c)eqr$ there exists r-fe^*o (resp. r-fδso and r-fδpo) set, $\lambda\in L^X$ and $x_t\in\lambda$ such that $\lambda\wedge dom(F)\leq F^u(\mu)$.*
2. *Fuzzy lower contra e^* (resp. δ-semi and δ-pre) (in short, $FLCe^*$ (resp. FLCδS and $FLC\delta P$))-continuous at any L-fuzzy point $x_t\in dom(F)$ iff $x_t\in F^l(\mu)$ for each $\mu\in L^Y$ and $\eta(\mu^c)geqr$ there exists r-fe^*o (resp. r-fδso and r-fδpo) set, $\lambda\in L^X$ and $x_t\in\lambda$ such that $\lambda\leq F^l(\mu)$.*
3. *$FUCe^*$ (resp. FUCδS, FUCδP, FLCe^*, FLCδS and FLCδP)-continuous iff it is $FUCe^*$ (resp. FUCδS, FUCδP, FLCe^*, FLCδS and FLCδP)-continuous at every $x_t\in dom(F)$.*

Proposition 2.1: *If F is normalized, then F is $FUCe^*$ (resp. FUCδS and FUCδP)-continuous at an L-fuzzy point $x_t\in dom(F)$ iff $x_t\in F^u(\mu)$ for each $\mu\in L^Y$ and $\eta(\mu^c)\geq r$ there exists $\lambda\in L^X$, λ is r-fe^*o (resp. r-fδso and r-fδpo) set and $x_t\in\lambda$ such that $\lambda\leq F^u(\mu)$.*

Theorem 2.1: *Let $F:X\multimap Y$ be a FM between two L-fts's (X,τ), (Y,η) and $\mu\in L^Y$, then the following are equivalent: (i) F is FLe^*-continuous. (ii) $F^l(\mu)$ is r-fe^*o set, for any $\eta(\mu)\geq r$. (iii) $F^u(\mu)$ is r-fe^*c set, for any $\eta(\overline{1}-\mu)\geq r$. (iv) $e^*C_\tau(F^u(\mu),r)\leq F^u(C_\eta(\mu,r))$, for any $\mu\in L^Y$. (v) $I_\tau(C_\tau(\delta I_\tau(F^u(\mu),r),r),r)\leq F^u(C_\eta(\mu,r))$, for any $\mu\in L^Y$.*

Proof:

(i) ⇒ (ii): Let $x_t\in dom(F)$, $\mu\in L^Y$, $\eta(\mu)\geq r$ and $x_t\in F^l(\mu)$ then, there exist $\lambda\in L^X$, λ is r-fe*o set and $x_t\in\lambda$ such that $\lambda\leq F^l(\mu)$ and hence $x_t\in e^*I_\tau(F^l(\mu),r)$. Therefore, we obtain $F^l(\mu)\leq e^*I_\tau(F^l(\mu),r)$. Thus $F^l(\mu)$ is r-fe*o (resp. r-fδso and r-fδpo) set.

(ii) ⇒ (iii): Let $\mu\in L^Y$ and $\eta(\overline{1}-\mu)\geq r$ hence by (ii), $F^l(\overline{1}-\mu)=\overline{1}-F^u(\mu)$ is r-fe*o. Then $F^u(\mu)$ is r-fe*c.

(iii) ⇒ (iv): Let $\mu\in L^Y$ hence by (iii), $F^u(C_\eta(\mu,r))$ is r-fe*c. Then we obtain $e^*C_\tau(F^u(\mu),r)\leq F^u(C_\eta(\mu,r))$.

(iv) ⇒ (v): Let $\mu\in L^Y$ hence by (iv), we obtain $I_\tau(C_\tau(\delta I_\tau(F^u(\mu),r),r),r)\leq e^*C_\tau(F^u(\mu),r)\leq F^u(C_\eta(\mu,r))$.

(v) ⇒ (ii): Let $\mu\in L^Y$, $\eta(\mu)\geq r$, hence by (v), we have

$$\begin{aligned}\overline{1}-F^l(\mu)&=F^u(\overline{1}-\mu)\\&\geq I_\tau(C_\tau(\delta I_\tau(F^u(\overline{1}-\mu),r),r),r)\\&=I_\tau(C_\tau(\delta I_\tau(\overline{1}-F^l(\mu),r),r),r)\\&=\overline{1}-[C_\tau(I_\tau(\delta C_\tau(F^l(\mu),r),r),r)]\\F^l(\mu)&\leq C_\tau(I_\tau(\delta C_\tau(F^l(\mu),r),r),r).\end{aligned}$$

Hence, $F^l(\mu)$ is r-fe*o.

(ii) ⇒ (i): Let $x_t\in dom(F)$, $\mu\in L^Y$, $\eta(\mu)\geq r$, with $x_t\in F^l(\mu)$ we have by (ii), $F^l(\mu)$ is r-fe^*o set. Let $F^l(\mu)=\lambda$(say), then there exists $\lambda\in L^X$, λ is r-fe*o set and $x_t\in\lambda$ such that $\lambda\leq F^l(\mu)$. Thus F is FLe^*-continuous.

Theorem 2.2: *Let $F:X\multimap Y$ be a FM and normalized between two L-fts's (X,τ), (Y,η) and $\mu\in L^Y$, then the following are equivalent: (i) F is FUe^*-continuous. (ii) $F^u(\mu)$ is r-fe^*o set, for any $\eta(\mu)\geq r$. (iii) $F^l(\mu)$ is r-fe^*c set, for any $\eta(\overline{1}-\mu)\geq r$. (iv) $e^*C_\tau(F^l(\mu),r)\leq F^l(C_\eta(\mu,r))$, for any $\mu\in L^Y$. (v) $I_\tau(C_\tau(\delta I_\tau(F^l(\mu),r),r),r)\leq F^l(C_\eta(\mu,r))$, for any $\mu\in L^Y$.*

Proof: This can be proved in a similar way as Theorem 2.1.

Corollary 2.1: *Let $F:X\multimap Y$ be a FM between two fts's (X,τ), (Y,η) and $\mu\in L^Y$. Then we have the following:*
(i) If F is normalized, then F is FUe^*-continuous. at x_t iff $x_t\in$ r-fe^*o set of $F^u(\mu)$, for each $\eta(\mu)\geq r$ and $x_t\in F^u(\mu)$. (ii) F is FLe^*-continuous at x_t iff $x_t\in$ r-fe^*o set of $F^l(\mu)$, for each $\eta(\mu)\geq r$ and $x_t\in F^l(\mu)$.

Remark 2.1: *From the above definitions, it is clear that every (FUCδS, FUCα and FUCδP)(resp. FLCδS, FLCα and FLCδP)-continuous is FUCe-continuous. Also, it is clear that every FUCe(resp. FLCe)-continuous is FUCβ (resp. FLCβ)-continuous and FUCe* (resp. FLCe*)-continuous. Also, every FUCβ (resp. FLCβ)-continuous is FUCe* (resp. FLCe*)-continuous. The converses need not be true in general and it is clear that the following implications are true.*
where (FUC-conts, FUCδS-conts, FUCα-conts, FUCδP-conts, FUCe-conts, FUCβ-conts, FUCe^*-conts)(resp. FLC-conts, FLCδS-conts, FLCα-conts and FLCδP-conts, FLCe-conts, FLCβ-conts, FLCe^*-conts) areabbreviated by fuzzy upper (resp. fuzzy lower) contra continuous, fuzzy upper (resp. fuzzy lower) contra δ-semicontinuous, fuzzy upper (resp. fuzzy lower) contra α-continuous, fuzzy upper (resp. fuzzy lower) contra δ-precontinuous,fuzzy upper (resp. fuzzy lower) contra e-continuous, fuzzy upper (resp. fuzzy lower) contra β-continuous and fuzzy upper (resp. fuzzy lower) contra e^*-continuous mappings respectively.

From the following examples, we see that the converses of these implications are not true.

Example 1: *Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2, y_3\}$ and $F: X \multimap Y$ be a FM defined by $G_F(x_1, y_1) = 0.8$, $G_F(x_1, y_2) = 0.9$, $G_F(x_1, y_3) = 0.8$, $G_F(x_2, y_1) = \overline{1}$, $G_F(x_2, y_2) = 0.7$,and $G_F(x_2, y_3) = 0.9$. Let λ_1 and λ_2 be a fuzzy subset of X be defined as $\lambda_1(x_1) = 0.3$, $\lambda_1(x_2) = 0.1$; $\lambda_2(x_1) = 0.1$, $\lambda_2(x_2) = 0.2$ and μ be a fuzzy subset of Y defined as $\mu(y_1) = 0.7$, $\mu(y_2) = 0.9$, $\mu(y_3) = 0.8$. We assume that $\overline{1} = 1$ and $\overline{0} = 0$. Define L-fuzzy topologies $\tau: L^X \to L$ and $\eta: L^Y \to L$ as follows:*

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{2}, & \text{if } \lambda = \lambda_1, \\ 0, & \text{otherwise,} \end{cases} \qquad \eta(\mu) = \begin{cases} 1, & \text{if } \mu = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{2}, & \text{if } \mu = \mu, \\ 0, & \text{otherwise.} \end{cases}$$

are fuzzy topologies on X and Y. For $r = \frac{1}{2}$, then F is $FUC\beta$-continuous but not $FUCe$-continuous because for any closed set μ in (Y, η), $F^u(\mu) = \lambda_2$ is not $\frac{1}{2}$-feo set in (X, τ).

Example 2: *Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2, y_3\}$ and $F: X \multimap Y$ be a FM defined by $G_F(x_1, y_1) = 0.2$, $G_F(x_1, y_2) = 1$, $G_F(x_1, y_3) = \overline{0}$, $G_F(x_2, y_1) = 0.5$, $G_F(x_2, y_2) = ne0$, and $G_F(x_2, y_3) = 0.3$. Let λ_1 and λ_2 be a fuzzy subset of X be defined as $\lambda_1(x_1) = 0.4$, $\lambda_1(x_2) = 0.3$; $\lambda_2(x_1) = 0.2$, $\lambda_2(x_2) = 0.4$ and μ be a fuzzy subset of Y defined as $mu(y_1) = 0.6$, $\mu(y_2) = 0.9$, $\mu(y_3) = 0$. We assume that $\overline{1} = 1$ and $\overline{0} = 0$. Define L-fuzzy topologies $\tau: L^X \to L$ and $\eta: L^Y \to L$ as follows:*

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{2}, & \text{if } \lambda = \lambda_1, \\ 0, & \text{otherwise,} \end{cases} \qquad \eta(\mu) = \begin{cases} 1, & \text{if } \mu = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{2}, & \text{if } \mu = \mu, \\ 0, & \text{otherwise.} \end{cases}$$

are fuzzy topologies on X and Y. For $r=\frac{1}{2}$, then F is $FLC\beta$-continuous but not $FLCe$-continuous because for any closed set μ in (Y, η), $F^l(\mu) = \lambda_2$ is not $\frac{1}{2}$-feo set in X.

Example 3: *Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2, y_3\}$ and $F: X \multimap Y$ be a FM defined by $G_F(x_1, y_1) = 0.8$, $G_F(x_1, y_2) = 0.9$, $G_F(x_1, y_3) = 0.8$, $G_F(x_2, y_1) = \overline{1}$, $G_F(x_2, y_2) = 0.7$,and $G_F(x_2, y_3) = 0.9$. Let λ_1 and λ_2 be a fuzzy subset of X be defined as $\lambda_1(x_1) = 0.5$, $\lambda_1(x_2) = 0.1$; $\lambda_2(x_1) = 0.1$, $\lambda_2(x_2) = 0.2$ and μ be a fuzzy subset of Y defined as $\mu(y_1) = 0.7$, $\mu(y_2) = 0.9$, $\mu(y_3) = 0.8$. We assume that $\overline{1} = 1$ and $\overline{0} = 0$. Define L-fuzzy topologies $\tau: L^X \to L$ and $\eta: L^Y \to L$ as follows:*

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{2}, & \text{if } \lambda = \lambda_1, \\ 0, & \text{otherwise,} \end{cases} \qquad \eta(\mu) = \begin{cases} 1, & \text{if } \mu = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{2}, & \text{if } \mu = \mu, \\ 0, & \text{otherwise.} \end{cases}$$

are fuzzy topologies on X and Y. For $r = \frac{1}{2}$, then F is $FUCe^*$-continuous but not $FUCe$-continuous because for any closed set μ in (Y, η), $F^u(\mu) = \lambda_2$ is not $\frac{1}{2}$-feo set in (X, τ).

Example 4: *Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2, y_3\}$ and $F: X \multimap Y$ be a FM defined by $G_F(x_1, y_1) = 0.2$, $G_F(x_1, y_2) = \overline{1}$, $G_F(x_1, y_3) = \overline{0}$, $G_F(x_2, y_1) = 0.5$, $G_F(x_2, y_2) = \overline{0}$, and $G_F(x_2, y_3) = 0.3$. Let λ_1 and λ_2 be a fuzzy subset of X be defined as $\lambda_1(x_1) = 0.5$, $\lambda_1(x_2) = 0.3$; $\lambda_2(x_1) = 0.2$, $\lambda_2(x_2) = 0.4$ and μ be a fuzzy subset of Y defined as $\mu(y_1) = 0.6$, $\mu(y_2) = 0.9$, $\mu(y_3) = \overline{0}$. We assume that $\overline{1} = 1$ and $\overline{0} = 0$. Define L-fuzzy topologies $\tau: L^X \to L$ and $\eta: L^Y \to L$ as follows:*

$$\tau(\lambda)=\begin{cases}1, & \text{if } \lambda=\overline{0} \text{ or } \overline{1},\\ \frac{1}{2}, & \text{if } \lambda=\lambda_1,\\ 0, & \text{otherwise,}\end{cases}\qquad \eta(\mu)=\begin{cases}1, & \text{if } \mu=\overline{0} \text{ or } \overline{1},\\ \frac{1}{2}, & \text{if } \mu=\mu,\\ 0, & \text{otherwise.}\end{cases}$$

are fuzzy topologies on X and Y. For $r=\frac{1}{2}$, then F is $FLCe^*$-continuous but not $FLCe$-continuous because for any closed set μ in Y, $F^l(\mu)=\lambda_2$ is not $\frac{1}{2}$-feo set in X.

Example 5: *Let $X=\{x_1, x_2\}$, $Y=\{y_1, y_2, y_3\}$ and $F: X \multimap Y$ be a FM defined by $G_F(x_1, y_1)=0.8$, $G_F(x_1, y_2)=0.9$, $G_F(x_1, y_3)=0.8$, $G_F(x_2, y_1)=\overline{1}$, $G_F(x_2, y_2)=0.7$, and $G_F(x_2, y_3)=0.9$. Let λ_1 and λ_2 be a fuzzy subset of X be defined as $\lambda_1(x_1)=0.7$, $\lambda_1(x_2)=0.7$; $\lambda_2(x_1)=0.2$, $\lambda_2(x_2)=0.1$ and μ be a fuzzy subset of Y defined as $\mu(y_1)=0.6$, $\mu(y_2)=0.7$, $\mu(y_3)=0.9$. We assume that $\overline{1}=1$ and $\overline{0}=0$. Define L-fuzzy topologies $\tau: L^X \to L$ and $\eta: L^Y \to L$ as follows:*

$$\tau(\lambda)=\begin{cases}1, & \text{if } \lambda=\overline{0} \text{ or } \overline{1},\\ \frac{1}{2}, & \text{if } \lambda=\lambda_1,\\ 0, & \text{otherwise,}\end{cases}\qquad \eta(\mu)=\begin{cases}1, & \text{if } \mu=\overline{0} \text{ or } \overline{1},\\ \frac{1}{2}, & \text{if } \mu=\mu,\\ 0, & \text{otherwise.}\end{cases}$$

are fuzzy topologies on X and Y. For $r=\frac{1}{2}$, then F is $FUCe^*$-continuous but not open in (X, τ).

Example 6: *Let $X=\{x_1, x_2\}$, $Y=\{y_1, y_2, y_3\}$ and $F: X \multimap Y$ be a FM defined by $G_F(x_1, y_1)=0.2$, $G_F(x_1, y_2)=\overline{1}$, $G_F(x_1, y_3)=\overline{0}$, $G_F(x_2, y_1)=0.5$, $G_F(x_2, y_2)=\overline{0}$, and $G_F(x_2, y_3)=0.3$. Let λ_1 and λ_2 be a fuzzy subset of X be defined as $\lambda_1(x_1)=0.6$, $\lambda_1(x_2)=0.6$; $\lambda_2(x_1)=0.2$, $\lambda_2(x_2)=0.4$ and μ be a fuzzy subset of Y defined as $\mu(y_1)=0.6$, $\mu(y_2)=0.9$, $\mu(y_3)=0.7$. We assume that $\overline{1}=1$ and $\overline{0}=0$. Define L-fuzzy topologies $\tau: L^X \to L$ and $\eta: L^Y \to L$ as follows:*

$$\tau(\lambda)=\begin{cases}1, & \text{if } \lambda=\overline{0} \text{ or } \overline{1},\\ \frac{1}{2}, & \text{if } \lambda=\lambda_1,\\ 0, & \text{otherwise,}\end{cases}\qquad \eta(\mu)=\begin{cases}1, & \text{if } \mu=\overline{0} \text{ or } \overline{1},\\ \frac{1}{2}, & \text{if } \mu=\mu,\\ 0, & \text{otherwise.}\end{cases}$$

are fuzzy topologies on X and Y. For $r=\frac{1}{2}$, then F is $FLCe^*$-continuous but not $FLC\beta$-continuous because for any closed set μ in (Y, η), $F^l(\mu)=\lambda_2$ is not $\frac{1}{2}$-fuzzy beta open set in X.

Example 7: *Let $X=\{x_1, x_2\}$, $Y=\{y_1, y_2, y_3\}$ and $F: X \multimap Y$ be a FM defined by $G_F(x_1, y_1)=0.8$, $G_F(x_1, y_2)=0.9$, $G_F(x_1, y_3)=0.8$, $G_F(x_2, y_1)=\overline{1}$, $G_F(x_2, y_2)=0.7$, and $G_F(x_2, y_3)=0.9$. Let λ_1 and λ_2 be a fuzzy subset of X be defined as $\lambda_1(x_1)=0.3$, $\lambda_1(x_2)=0.1$; $\lambda_2(x_1)=0.7$, $\lambda_2(x_2)=0.7$ and μ be a fuzzy subset of Y defined as $\mu(y_1)=0.3$, $\mu(y_2)=0.1$, $\mu(y_3)=0.2$. We assume that $\overline{1}=1$ and $\overline{0}=0$. Define L-fuzzy topologies $\tau: L^X \to L$ and $\eta: L^Y \to L$ as follows:*

$$\tau(\lambda)=\begin{cases}1, & \text{if } \lambda=\overline{0} \text{ or } \overline{1},\\ \frac{1}{2}, & \text{if } \lambda=\lambda_1,\\ 0, & \text{otherwise,}\end{cases}\qquad \eta(\mu)=\begin{cases}1, & \text{if } \mu=\overline{0} \text{ or } \overline{1},\\ \frac{1}{2}, & \text{if } \mu=\mu,\\ 0, & \text{otherwise.}\end{cases}$$

are fuzzy topologies on X and Y. For $r=\frac{1}{2}$, then F is $FUCe$-continuous but not $FUC\alpha$-continuous because for any closed set μ in (Y, η), $F^u(\mu)=\lambda_2$ is not $\frac{1}{2}$-fuzzy alpha open set in (X, τ).

Example 8: *Let $X=\{x_1, x_2\}$, $Y=\{y_1, y_2, y_3\}$ and $F: X \multimap Y$ be a FM defined by $G_F(x_1, y_1)=0.2$, $G_F(x_1, y_2)=\overline{1}$, $G_F(x_1, y_3)=\overline{0}$, $G_F(x_2, y_1)=0.5$, $G_F(x_2, y_2)=\overline{0}$, and $G_F(x_2, y_3)=0.3$. Let λ_1 and λ_2 be a fuzzy subset of X be defined as $\lambda_1(x_1)=0.4$, $\lambda_1(x_2)=0.3$; $\lambda_2(x_1)=0.9$, $\lambda_2(x_2)=0.5$ and μ be a fuzzy subset of Y defined as $\mu(y_1)=0.4$, $\mu(y_2)=0.1$, $\mu(y_3)=\overline{1}$. We assume that $\overline{1}=1$ and $\overline{0}=0$. Define L-fuzzy topologies $\tau: L^X \to L$ and $\eta: L^Y \to L$ as follows:*

$$\tau(\lambda)=\begin{cases}1, & \text{if } \lambda=\overline{0} \text{ or } \overline{1},\\ \frac{1}{2}, & \text{if } \lambda=\lambda_1,\\ 0, & \text{otherwise,}\end{cases}\qquad \eta(\mu)=\begin{cases}1, & \text{if } \mu=\overline{0} \text{ or } \overline{1},\\ \frac{1}{2}, & \text{if } \mu=\mu,\\ 0, & \text{otherwise.}\end{cases}$$

are fuzzy topologies on X and Y. For $r=\frac{1}{2}$, then F is $FLCe$-continuous but not $FLC\alpha$-continuous because for any closed set μ in (Y, η), $F^l(\mu)=\lambda_2$ is not $\frac{1}{2}$-fuzzy alpha open set in X.

Example 9: *Let* $X = \{x_1, x_2\}$, $Y = \{y_1, y_2, y_3\}$ *and* $F: X \multimap Y$ *be a FM defined by* $G_F(x_1, y_1) = 0.8$, $G_F(x_1, y_2) = 0.9$, $G_F(x_1, y_3) = 0.8$, $G_F(x_2, y_1) = \overline{1}$, $G_F(x_2, y_2) = 0.7$,*and* $G_F(x_2, y_3) = 0.9$. *Let* λ_1 *and* λ_2 *be a fuzzy subset of* X *be defined as* $\lambda_1(x_1) = 0.3$, $\lambda_1(x_2) = 0.1$; $\lambda_2(x_1) = 0.7$, $\lambda_2(x_2) = 0.7$ *and* μ *be a fuzzy subset of* Y *defined as* $\mu(y_1) = 0.3$, $\mu(\square_2) = 0.1$, $\mu(y_3) = 0.2$. *We assume that* $\overline{1} = 1$ *and* $\overline{0} = 0$. *Define* L*-fuzzy topologies* $\tau: L^X \to L$ *and* $\eta: L^Y \to L$ *as follows:*

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{2}, & \text{if } \lambda = \lambda_1, \\ 0, & \text{otherwise,} \end{cases} \qquad \eta(\mu) = \begin{cases} 1, & \text{if } \mu = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{2}, & \text{if } \mu = \mu, \\ 0, & \text{otherwise.} \end{cases}$$

are fuzzy topologies on X *and* Y. *For* $r = \frac{1}{2}$, *then*

(i) F *is* $FUCe$*-continuous but not* $FUC\delta P$*-continuous because for any closed set* μ *in* (Y, η), $F^u(\mu) = \lambda_2$ *is not* $\frac{1}{2}$*-fuzzy* δ*-pre open in* (X, τ).

(ii) F *is* $FUC\delta S$*-continuous but not* FUC*-continuous because for any closed set* μ *in* (Y, η), $F^u(\mu) = \lambda_2$ *is not* $\frac{1}{2}$*-fuzzy open in* (X, τ).

Example 10: *Let* $X = \{x_1, x_2\}$, $Y = \{y_1, y_2, y_3\}$ *and* $F: X \multimap Y$ *be a FM defined by* $G_F(x_1, y_1) = 0.2$, $G_F(x_1, y_2) = \overline{1}$, $G_F(x_1, y_3) = \overline{0}$, $G_F(x_2, y_1) = 0.5$, $G_F(x_2, y_2) = \overline{0}$, *and* $G_F(x_2, y_3) = 0.3$. *Let* λ_1 *and* λ_2 *be a fuzzy subset of* X *be defined as* $\lambda_1(x_1) = 0.1$, $\lambda_1(x_2) = 0.3$; $\lambda_2(x_1) = 0.9$, $\lambda_2(x_2) = 0.5$ *and* μ *be a fuzzy subset of* Y *defined as* $\mu(y_1) = 0.6$, $\mu(y_2) = 0.9$, $\mu(y_3) = 0$. *We assume that* $\overline{1} = 1$ *and* $\overline{0} = 0$. *Define* L*-fuzzy topologies* $\square: L^X \to L$ *and* $\eta: L^Y \to L$ *as follows:*

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{2}, & \text{if } \lambda = \lambda_1, \\ 0, & \text{otherwise,} \end{cases} \qquad \eta(\mu) = \begin{cases} 1, & \text{if } \mu = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{2}, & \text{if } \mu = \mu, \\ 0, & \text{otherwise.} \end{cases}$$

are fuzzy topologies on X *and* Y. *For* $r=\frac{1}{2}$, *then*

(i) F *is* $FLCe$*-continuous but not* $FLC\delta P$*-continuous because for any closed set* μ *in* Y, $F^l(\mu) = \lambda_2$ *is not* $\frac{1}{2}$*-fuzzy* δ*-pre open set in* X.

(ii) F *is* $FLC\delta S$*-continuous but not* FLC*-continuous because for any closed set* μ *in* Y, $F^l(\mu) = \lambda_2$ *is not* $\frac{1}{2}$*-fuzzy open set in* X.

Example 11: *Let* $X = \{x_1, x_2\}$, $Y = \{y_1, y_2, y_3\}$ *and* $F: X \multimap Y$ *be a FM defined by* $G_F(x_1, y_1) = 0.8$, $G_F(x_1, y_2) = 0.9$, $G_F(x_1, y_3) = 0.8$, $G_F(x_2, y_1) = \overline{1}$, $G_F(x_2, y_2) = 0.7$,*and* $G_F(x_2, y_3) = 0.9$. *Let* λ_1 *and* λ_2 *be a fuzzy subset of* X *be defined as* $\lambda_1(x_1) = 0.6$, $\lambda_1(x_2) = 0.8$; $\lambda_2(x_1) = 0.7$, $\lambda_2(x_2) = 0.7$ *and* μ *be a fuzzy subset of* Y *defined as* $\mu(y_1) = 0.3$, $\mu(y_2) = 0.1$, $\mu(y_3) = 0.2$. *We assume that* $\overline{1} = 1$ *and* $\overline{0} = 0$. *Define* L*-fuzzy topologies* $\tau: L^X \to L$ *and* $\eta: L^Y \to L$ *as follows:*

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{2}, & \text{if } \lambda = \square_1, \\ 0, & \text{otherwise,} \end{cases} \qquad \eta(\mu) = \begin{cases} 1, & \text{if } \mu = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{2}, & \text{if } \mu = \mu, \\ 0, & \text{otherwise.} \end{cases}$$

are fuzzy topologies on X *and* Y. *For* $r = \frac{1}{2}$, *then*

(i) F *is* $FUCe$*-continuous but not* $FUC\delta S$*-continuous because for any closed set* μ *in* Y, $F^u(\mu) = \lambda_2$ *is not* $\frac{1}{2}$*-fuzzy* δ*-semi open in* X.

(ii) F *is* $FUC\delta P$*-continuous but not* FUC*-continuous because for any closed set* μ *in* Y, $F^u(\mu) = \lambda_2$ *is not* $\frac{1}{2}$*-fuzzy open in* X.

(iii) F *is* $FUC\alpha$*-continuous but not* FUC*-continuous because for any closed set* μ *in* Y, $F^u(\mu) = \lambda_2$ *is not* $\frac{1}{2}$*-fuzzy open in* X.

Example 12: *Let* $X = \{x_1, x_2\}$, $Y = \{y_1, y_2, y_3\}$ *and* $F: X \multimap Y$ *be a FM defined by* $G_F(x_1, y_1) = 0.2$, $G_F(x_1, y_2) = \overline{1}$, $G_F(x_1, y_3) = \overline{0}$, $G_F(x_2, y_1) = 0.5$, $G_F(x_2, y_2) = \overline{0}$, *and* $G_F(x_2, y_3) = 0.3$. *Let* λ_1 *and* λ_2 *be a fuzzy subset of* X *be defined as* $\lambda_1(x_1) = 0.4$, $\lambda_1(x_2) = 0.3$; $\lambda_2(x_1) = 0.9$, $\lambda_2(x_2) = 0.5$ *and* μ *be a fuzzy subset of* Y *defined as* $\mu(y_1) = 0.4$, $\mu(y_2) = 0.1$, $\mu(y_3) = 1$. *We assume that* $\overline{1} = 1$ *and* $\overline{0} = 0$. *Define* L*-fuzzy topologies* $\tau: L^X \to L$ *and* $\eta: L^Y \to L$ *as follows:*

$$\tau(\lambda)=\begin{cases}1, & \text{if } \lambda=\overline{0} \text{ or } \overline{1},\\ \frac{1}{2}, & \text{if } \lambda=\lambda_1,\\ 0, & \text{otherwise,}\end{cases}\qquad \eta(\mu)=\begin{cases}1, & \text{if } \mu=\overline{0} \text{ or } \overline{1},\\ \frac{1}{2}, & \text{if } \mu=\mu,\\ 0, & \text{otherwise.}\end{cases}$$

are fuzzy topologies on X and Y. For $r=\frac{1}{2}$, then (i) F is $FLCe$-continuous but not $FLC\delta S$-continuous because for any closed set μ in Y, $F^l(\mu)=\lambda_2$ is not $\frac{1}{2}$-fuzzy δ-semi open set in X. (ii) F is $FLC\delta P$-continuous but not FLC-continuous because for any closed set μ in Y, $F^l(\mu)=\lambda_2$ is not $\frac{1}{2}$-fuzzy open set in X.

Example 13: *Let $X=\{x_1, x_2\}$, $Y=\{y_1, y_2, y_3\}$ and $F: X \multimap Y$ be a FM defined by $G_F(x_1, y_1)=0.2$, $G_F(x_1, y_2)=\overline{1}$, $G_F(x_1, y_3)=\overline{0}$, $G_F(x_2, y_1)=0.5$, $G_F(x_2, y_2)=\overline{0}$, and $G_F(x_2, y_3)=0.3$. Let λ_1 and λ_2 be a fuzzy subset of X be defined as $\lambda_1(x_1)=0.7$, $\lambda_1(x_2)=0.5$; $\lambda_2(x_1)=0.9$, $\lambda_2(x_2)=0.5$ and μ be a fuzzy subset of Y defined as $\mu(y_1)=0.4$, $\mu(y_2)=0.1$, $\mu(y_3)=1$. We assume that $\overline{1}=1$ and $\overline{0}=0$. Define L-fuzzy topologies $\tau: L^X \to L$ and $\eta: L^Y \to L$ as follows:*

$$\tau(\lambda)=\begin{cases}1, & \text{if } \lambda=\overline{0} \text{ or } \overline{1},\\ \frac{1}{2}, & \text{if } \lambda=\lambda_1,\\ 0, & \text{otherwise,}\end{cases}\qquad \eta(\mu)=\begin{cases}1, & \text{if } \mu=\overline{0} \text{ or } \overline{1},\\ \frac{1}{2}, & \text{if } \mu=\mu,\\ 0, & \text{otherwise.}\end{cases}$$

are fuzzy topologies on X and Y. For $r=\frac{1}{2}$, then F is $FLC\alpha$-continuous but not FLC-continuous because for any closed set μ in Y, $F^l(\mu)=\lambda_2$ is not $\frac{1}{2}$-fuzzy open set in X.

Theorem 2.3: *Let $\{F_i\}_{i\in\Gamma}$ be a family of FLe^*(resp. FLδS and FLδP)-continuous between two fts's (X, τ) and (Y, η). Then $\bigcup_{i\in\Gamma} F_i$ is FLe^*(resp. FLδS and FLδP)-continuous.*

Proof: Let $\mu \in L^Y$, then $(\bigcup_{i\in\Gamma} F_i)^l(\mu)=\bigvee_{i\in\Gamma}(F_i^l(\mu))$ by, Theorem 2.3 (ii) in [14]. Since $\{F_i\}_{i\in\Gamma}$ is a family of FLe^*(resp. FLδS and FLδP)-continuous between two fts's (X, τ) and (Y, η), then $F_i^l(\mu)$ is r-fe*o (resp. r-fδso and r-fδpo), for any $\eta(\mu)\geq r$. Then we have $(\bigcup_{i\in\Gamma} F_i)^l(\mu)=\bigvee_{i\in\Gamma}(F_i^l(\mu))$ is r-fe*o (resp. r-fδso and r-fδpo) set for any $\eta(\mu)\geq r$. Hence $\bigcup_{i\in\Gamma} F_i$ is FLe^*(resp. FLδS and FLδP)-continuous.

Theorem 2.4: *Let $\{F_i\}_{i\in\Gamma}$ be a family of normalized FUe^*(resp. FUδS and $FU\delta P$)-continuous between two fts's (X, τ) and (Y, η). Then $F_1 \cup F_2$ is FUe^*(resp. FUδS and $FU\delta P$)-continuous.*

Proof: Let $\mu \in L^Y$, then $(F_1 \cup F_2)^u(\mu)=F_1^u(\mu)\wedge F_2^u(\mu)$ by, Theorem 2.3(iii) in [14]. Since $\{F_i\}_{i\in\Gamma}$ is a family of normalized FUe^*(resp. FUδS and $FU\delta P$)-continuous between two fts's (X, τ) and $(Y, \square)$, then $(F_i^u(\mu))$ if r-fe*o (resp. r-fδso and r-fδpo), for any $\eta(\mu)\geq r$ for each $i\in\{1,2\}$. Then for each $\mu \in L^Y$, we have $(F_1 \cup F_2)^u(\mu)=F_1^u(\mu)\wedge F_2^u(\mu)$ is r-fe*o (resp. r-fδso and r-fδpo) set for any $\eta(\mu)\geq r$. Hence $F_1 \cup F_2$ is $FU\square^*$(resp. FUδS and $FU\delta P$)-continuous.

Definition 2.2: *A fuzzy set λ in a fts (X, τ) is called r-fuzzy e^*(resp. δ semi and δ pre)-compact iff every family in $\{\mu : \mu$ is r-fe^*o (resp. r-fδso and r-fδpo), $\mu \in L^X$ and $r \in L\}$ covering λ has a finite subcover.*

Definition 2.3: *Let $F: X \multimap Y$ be a FM between two fts's (X,τ), (Y,η) and $r \in L_0$. Then F is called fuzzy e^*(resp. δ semi and δ pre)-compact valued iff $F(x_t)$ is r-fuzzy e^*-compact for each $x_t \in dom(F)$.*

Theorem 2.5: *Let $F: X \multimap Y$ be a crisp FUe-continuous and fuzzy e^*(resp. δ semi and δ pre)-compact valued between two fts's (X, τ) and (Y, η). Then the direct image of a r-fuzzy e^*-compact in X under F is also r-fuzzy e^*(resp. δ semi and δ pre)-compact.*

Proof: Let λ be r-fuzzy e^*-compact set in X and $\{\gamma_i: \gamma_i$ is r-fe*o set in Y, $i\in\Gamma\}$ be a family of covering of $F(\lambda)$. i.e. $F(\lambda)\leq \bigvee_{i\in\Gamma}\gamma_i$.Since $\lambda=\bigvee_{x_t\in\lambda} x_t$, we have $F(\lambda)=F(\bigvee_{x_t\in\lambda} x_t)=\bigvee_{x_t\in\lambda} F(x_t)\leq \bigvee_{i\in\Gamma}\gamma_i$. It follows that for each $\square_t \in \lambda$, $F(x_t)$łeq $\bigvee_{i\in\Gamma}\gamma_i$. Since F is r-fuzzy e^*-compact valued, then there exists finite subset Γ_{x_t} of Γ such that $F(x_t)\leq \bigvee_{n\in\Gamma_{x_t}}\gamma_n=\gamma_{x_t}$. By Theorem 2.1 (viii) in [14], we have $x_t\leq F^u(F(x_t))\leq F^u(\gamma_{x_t})$ and $\lambda=\bigvee_{x_t\in\lambda} x_t=\bigvee_{x_t\in\lambda} F^{\square}(\gamma_{x_t})$. Since, $\eta(\gamma_{x_t})\geq r$, then from Theorem 2.2., we have $F^u(\gamma_{x_t})$ is r -f e^* o-set. Hence $\{F^u(\gamma_{x_t}): F^u(\gamma_{x_t})$ is r-fe*o-set, $x_t\in\lambda\}$ is a family covering the set λ. Since λ is r-fuzzy e^*-compact, then there exists finite index set N such that $\lambda\leq \bigvee_{n\in N} F^u(\gamma_{x_{t_n}})$. From Theorem 2.1(vii) in [14], we have $F(\lambda)\leq F(\bigvee_{n\in N} F^u(\gamma_{x_{t_n}}))=\bigvee_{n\in N} F(F^u(\gamma_{x_{t_n}}))\leq \bigvee_{n\in N}\gamma_{x_{t_n}}$. Then $F(\lambda)$ is r-fuzzy e^*-compact. The proof of the others are similar.

Theorem 2.6: *Let $F: X \multimap Y$ and $H: Y \multimap Z$ be two FM's and let (X, τ), (Y, η) and (Z, δ) be three fts's. Then we have the following: (i) If F and H are normalized, FUe^* (resp. δ semi and δ pre)-continuous, then $H \circ F$ is FUe^* (resp. δ semi and δ pre)-continuous. (ii) If F and H are FLe^* (resp. δ semi and δ pre)-continuous, then $H \circ F$ is FLe^* (resp. δ semi and δ pre)-continuous.*

Proof: (i) Let F and H are normalized, FUe^*-continuous and $\nu \in L^Z$. Then from Theorem 2.2 in [14], we have $(H \circ F)^u(\nu) = F^u(H^u(\nu))$ is fe*o with $\nu(H^u(\nu)) \geq \delta(\nu)$. Thus $H \circ F$ is FUe^*-continuous. (ii) Similar of (i). The proof of the others are similar.

Theorem 2.7: *Let $F: X \multimap Y$ and $H: Y \multimap Z$ be two FM's and let (X, τ), (Y, η) and (Z, δ) be three L-fts's. If F is FLe^* (resp. δ semi and δ pre)-continuous and H is FL-continuous, then $H \circ F$ is FLe^*(resp. δ semi and δ pre)-continuous.*

Proof: Let $\nu \in L^Z$, $\delta(\nu) \geq r$. Since H is FL-continuous, then by Theorem 2.5 in [14], $H^l(\nu)$ is r-fuzzy open set in Y. Also, F is FLe^*-irresolute implies $F^l(H^l(\nu))$ is fe*o set in X. Hence, we have $(H \circ F)^l(\nu) = F^l(H^l(\nu))$ is r-fe*o. Thus $H \circ F$ is FLe^*-continuous. The proof of the others are similar.

Theorem 2.8: *Let $F: X \multimap Y$ and $H: Y \multimap Z$ be two FM's and let (X, τ), (Y, η) and (Z, δ) be three L-fts's. If F and H are normalized, F is FUe^* (resp. δ semi and δ pre)-continuous and H is FU-continuous, then $H \circ F$ is FUe^* (resp. δ semi and δ pre)-continuous.*

Theorem 2.9: *Let $F: X \multimap Y$ be a FM between two fts's (X, τ) and (Y, η). If G_f is FLe^* (resp. δ semi and δ pre)-continuous, then F is FLe^* (resp. δ semi and δ pre)-continuous.*

Proof: For the fuzzy sets $\rho \in L^X$, $\tau(\rho) \geq r$, $\nu \in L^Y$ and $\eta(\nu) \geq r$, we take, $(\rho \times \nu)(x, y) = \begin{cases} 0, & \text{if } x \notin \rho, \\ \nu(y), & \text{if } x \in \rho. \end{cases}$ Let $x_t \in dom(F)$, $\mu \in L^Y$ and $\eta(\mu) \geq r$ with $x_t \in F^l(\mu)$, then we have $x_t \in G_f^l(X \times \mu)$ and $\eta(X \times \mu) \geq r$. Since G_f is FLe^*-continuous, it follows that there exists $\lambda \in L^X$, λ is fe*o and $x_t \in \lambda$ such that $\lambda \leq G_f^l(X \times \mu)$. From here, we obtain that $\lambda \leq F^l(\mu)$. Thus F is FLe^*-continuous. The proof of the others are similar.

Theorem 2.10: *Let $F: X \multimap Y$ be a FM between two fts's (X, τ) and (Y, η). If G_f is FUe^* (resp. δ semi and δ pre)-continuous, then F is FUe^* (resp. δ semi and δ pre)-continuous.*

Theorem 2.11: *Let (X, τ) and (X_i, τ_i) be L-fts's $(i \in I)$. If a FM $F: X \multimap \Pi_{i \in I} X_i$ is FLe-continuous (where $\Pi_{i \in I} X_i$ is the product space), then $P_i \circ F$ is FLe^* (resp. δ semi and δ pre)-continuous for each $i \in I$, where $P_i: \Pi_{i \in I} X_i \multimap X_i$ is the projection multifunction which is defined by $P_i(x_i) = \{x_i\}$ for each $i \in I$.*

Proof: Let $\mu_{i_0} \in L^{X_{i_0}}$ and $\tau_i(\mu_{i_0}) \geq r$. Then $(P_{i_0} \circ F)^l(\mu_{i_0}) = F^l(P_{i_0}^l(\mu_{i_0})) = F^l(\mu_{i_0} \times \Pi_{i \neq i_0} X_i)$. Since F is FLe^*-continuous and $\tau_i(\mu_{i_0} \times \Pi_{i \neq i_0} X_i) \geq r$, it follows that $F^l(\mu_{i_0} \times \Pi_{i \neq i_0} X_i)$ is fe^*o set. Then $P_i \circ F$ is an FLe^*-continuous. The proof of the others are similar.

Theorem 2.12: *Let (X, τ) and (X_i, τ_i) be L-fts's $(i \in I)$. If a FM $F: X \multimap \Pi_{i \in I} X_i$ is FUe^* (resp. δ semi and δ pre)-continuous (where $\Pi_{i \in I} X_i$ is the product space), then $P_i \circ F$ is FUe^* (resp. δ semi and δ pre)-continuous for each $i \in I$, where $P_i: \Pi_{i \in I} X_i \multimap X_i$ is the projection multifunction which is defined by $P_i(x_i) = \{x_i\}$ for each $i \in I$.*

Theorem 2.13: *Let (X_i, τ_i) and (Y_i, η_i) be L-fts's and $F_i: X_i \multimap Y_i$ be a FM for each $i \in I$. Suppose that $F: \Pi_{i \in I} X_i \multimap \Pi_{i \in I} Y_i$ is defined by $F(x_i) = \Pi_{i \in I} F_i(x_i)$. If F is FLe^* (resp. δ semi and δ pre)-continuous, then F_i is FLe^* (resp. δ semi and δ pre)-continuous for each $i \in I$.*

Proof: Let $\mu_i \in L^{Y_i}$ and $\eta_i(\mu_i) \geq r$. Then $\eta_i(\mu_i \times \Pi_{i \neq j} Y_j) \geq r$. Since F is FLe^*-continuous, it follows that $F^l(\mu_i \times \Pi_{i \neq j} Y_j) = F^l(\mu_i) \times \Pi_{i \neq j} X_j$ is fe*o. Consequently, we obtain that $F^l(\mu_i)$ is r-fe*o for each $i \in I$. Thus, F_i is FLe^*-continuous. The proof of the others are similar.

Theorem 2.14: *Let (X_i, τ_i) and (Y_i, η_i) be L-fts's and $F_i: X_i \multimap Y_i$ be a FM for each $i \in I$. Suppose that $F: \Pi_{i \in I} X_i \multimap \Pi_{i \in I} Y_i$ is defined by $F(x_i) = \Pi_{i \in I} F_i(x_i)$. If F is FUe^* (resp. δ semi and δ pre)-continuous, then F_i is FUe^* (resp. δ semi and δ pre)-continuous for each $i \in I$.*

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