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A VIEW ON SOFT αB-CONTINUITY ON SOFT TOPOLOGICAL SPACES

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ABSTRACT

In this paper, the concept of soft αB-open set is introduced. In this connection, the interrelations among the sets are established. Also the interrelation among soft αB-continuity is discussed and counter examples are provided wherever necessary.

Keywords: *Soft αB-open sets, Soft αB-continuous functions.*

AMS Subject Classification: *54A10; 54A20; 54C08.*

1. INTRODUCTION

In 1999, Molodtsov [1] introduced the concept of soft sets with adequate parametrization for dealing with uncertainties. Later Muhammad Shabir and Munazza Naz [2] introduced the concept of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Naime Tozlu and Saziye yüksel [3, 4] introduced the concepts of soft A-sets, soft B-sets and soft C-sets. In this paper, the concept of soft αB-open set is introduced. In this connection, the interrelations among the sets are established. Also the interrelations among soft αB-continuity are discussed and counter examples are provided wherever necessary.

2. PRELIMININARIES

Let U be an initial universe set and E be a set of parameters. Let P(U) denote the power set of U, and let $A \subseteq E$.

Definition 2.1: [1] A pair (F, A) is called a soft set over U, where F is a mapping given by $F: A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For a particular $e \in A$, F(e) may be considered the set of e-approximate elements of the soft set (F, A).

Definition 2.2:. [5] For two soft sets (F, A) and (G, B) over a common universe U,

(i) (F, A) is a soft subset of (G, B), denoted by $(F, A) \sqsubseteq (G, B)$, if $A \subseteq B$ and $\forall e \in A$, $F(e) \subseteq G(e)$.
(ii) (F, A) is said to be a soft superset of (G,B), if (G, B) is a soft subset of (F, A), denoted by $(F, A) \sqsupseteq (G, B)$.

Definition 2.3: [5] Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.4: [5] A soft set (F, A) over U is said to be a null soft set, denoted by Φ, if $\forall e \in A$, $F(e) = \varphi$.

Definition 2.5: [5] A soft set (F, A) over U is said to be an absolute soft set, denoted by $\widetilde{U}$, if $\forall e \in A$, F(e) = U.

Definition 2.6: [5] The union of two soft sets (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$ and for all $\forall e \in C$,

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$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \cup G(e), & \text{if } e \in A \cup B, \end{cases}$$

This relationship is written as (F,A) ⊔ (G,B) = (H,C).

Definition 2.7: [5] The intersection of two soft sets (F,A) and (G,B) over the common universe U is the soft set (H,C), where C = A ∩ B and ∀ e ∈ C, H(e) = F(e) ∩ G(e). This relationship is written as (F,A) ⊓ (G,B) = (H,C).

Definition 2.8: [5] The complement of a soft set (F,A) denoted by $(F,A)'$ and is defined by $(F,A)' = (F',A)$, where F': A →P(U) is a mapping given by $F'(e)$ = U - F(e), ∀ e ∈ A. F' is called the soft complement function of F. Clearly, $(F')'$ is the same as F and $((F,A)')'$ = (F,A).

Definition 2.9: [5] Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if
(i) $\Phi, \tilde{X}$ belong to τ ,
(ii) the union of any number of soft sets in τ belongs to τ ,
(iii) the intersection of any two soft sets in τ belongs to τ .
The triplet (X, τ, E) is called a soft topological space over X. Every member of τ are said to be soft open sets in X. A soft set (F,E) over X is said to be a soft closed set in X, if its relative complement $(F,E)'$ belongs to τ .

Example 2.1: [4] Let X = $\{x_1, x_2, x_3, x_4\}$, E = $\{e_1, e_2\}$ Let $F_1, F_2, .., F_{11}$ be a mapping from E to P(X) defined by,
$(F_1, E) = \{(e_1, \{x_1\}), (e_2, \{x_1\})\}$,
$(F_2, E) = \{(e_1, \{x_2\}), (e_2, \{x_2\})\}$,
$(F_3, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2\})\}$,
$(F_4, E) = \{(e_1, \{x_1, x_2, x_3\}), (e_2, \{x_1, x_3\})\}$
$(F_5, E) = \{(e_1, \{x_1, x_2, x_4\}), (e_2, \{x_1, x_2, x_3\})\}$
$(F_6, E) = \{(e_1, \{x_2\}), (e_2, \Phi)\}$,
$(F_7, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1\})\}$,
$(F_8, E) = \{(e_1, \{x_1, x_2, x_3\}), (e_2, \{x_1, x_2, x_3\})\}$,
$(F_9, E) = \{(e_1, \{X\}), (e_2,\{x_1, x_2, x_3\})\}$
$(F_{10}, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2, x_3\})\}$
$(F_{11}, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_3\})\}$
Then $\tau = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), ..., (F_{11}, E)\}$ is a soft topological space over X.

Definition 2.10: [5] Let (X,τ ,E) be a soft topological space over X and (F,E) be a soft set over X. Then,
(i) the soft closure of (F,E) is the soft set scl(F,E) = ⊓{(G,E)/(G,E) is soft closed and (F,E) ⊑ (G,E)}.
(ii) the soft interior of (F,E) is the soft set sint(F,E) = ⊓{(H,E)/(H,E) is soft open and (H,E) ⊑ (F,E)}.

Definition 2.11: [3] Let (X, τ, E) be a soft topological space. A soft set (F,E) is called soft α-open set in X, if (F,E) ⊑ sint(scl(sint(F,E))).

Definition 2.12: [3] Let (X, τ, E) be a soft topological space. A soft set (F, E) is called
(i) soft t-open set in X, if sint(scl(F,E)) = sint(F,E).
(ii) soft t-closed set in X, if scl(sint(F,E)) = scl(F,E).

Definition 2.13: [3] Let (X,τ,E) be a soft topological space. A soft set (F,E) is called soft B-open(closed) set in X, if (F,A)= (G,E) ⊓ (H,E), where (G,E) is a soft open(closed) set and (H,E) is a soft t-open (closed) set.

Definition 2.14: [3] Let (X,τ,E) be a soft topological space over X and (F,E) be a soft set over X. Then,
(i) the soft B-closure of (F,E) is the soft set sBcl(F,E) = ⊓{(G,E)/(G,E) is soft B-closed and (F,E) ⊑ (G,E)}.
(ii) the soft B-interior of (F,E) is the soft set sBint(F,E) = ⊓{(H,E)/(H,E) is soft B-open and (H,E) ⊑ (F,E)}.

Definition 2.15: [4] Let (X,τ,E) be a soft topological space over X and (F,E) be a soft set over X. Then (F,E) is said to be α*-set if sint(scl(sint(F,E))) = sint(F,E).

Definition 2.16: [4] Let (X,τ ,E) be a soft topological space. A soft set (F,E) is called soft C-open set in X, if (F,A)= (G,E) ⊓ (H,E), where (G,E) is a soft open set and (H,E) is a soft α*- set.

Remark 2.1: [4] Let (X,τ ,E) be a soft topological space. The notion of soft α-open sets is different from the soft C-open sets.

Example 2.2: [4] Let X = {x_1, x_2, x_3, x_4}, E = {e_1, e_2}. Let us take the soft topology τ on X as in example 2.1 and let (G,E) = {(e_1, { x_3, x_4 }), (e_2,{ x_2 })} be a soft set over X. Here (G,E) is a soft α*-set. Also, (G,E) is a soft C-open set since every soft α*-set is a soft C-open set but not a soft α-open set.

Example 2.3: [4] Let X = {x_1, x_2, x_3}, E = {e_1, e_2} and F is a mapping from E to P(X) defined by, (F,E) = {(e_1, {x_1}), (e_2, {x_2})} be a soft set over X. Then τ ={ Φ, X, (F, E)} is soft topology over X. Let (G,E) = {(e_1, { x_1, x_2 }), (e_2, { x_2 })} be a soft set over X. Here (G,E) is a soft α-open set but not a soft C-open set.

Definition 2.17: [3] Let (X, τ_1, E) and (X, τ_2, E) be any two soft topological spaces. A function f from a soft topological space (X,τ_1,E) to another soft topological space (X, τ_2, E) is said to be soft α-continuous function iff the inverse image of every soft α-open(closed) set in (X, τ_2, E) is a soft open(closed) set in (X,τ_1,E).

Definition 2.18: [3] Let (X, τ_1, E) and (X, τ_2, E) be any two soft topological spaces. A function f from a soft topological space (X, τ_1, E) to another soft topological space (X,τ_2,E) is said to be soft B-continuous function iff the inverse image of every soft open(closed) set in (X, τ_2, E) is a soft B-open(closed) set in (X, τ_1, E).

Definition 2.19: [4] Let (X, τ_1, E) and (X, τ_2, E) be any two soft topological spaces. A function f from a soft topological space (X, τ_1, E) to another soft topological space (X,τ_2,E) is said to be soft C-continuous function iff the inverse image of every soft open(closed) set in (X,τ_2,E) is a soft C-open(closed) set in (X, τ_1, E).

Remark 2.2: [4] Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. Then every soft t-open set is a soft α*-open set.

3. SOFT αB-OPEN SET

Definition 3.1: Let (X, τ, E) be a soft topological space. A soft set (F,E) is called soft αB-open(closed) set in X, if (F,A)= (G,E) ⊓ (H,E), where (G,E) is a soft α-open(closed) set and (H,E) is a soft t-open(closed) set.

Definition 3.2: Let (X, τ, E) be a soft topological space over X and (F,E) be a soft set over X. Then,

(i) the soft αB-closure of (F,E) is the soft set sαBcl(F,E) = ⊓ {(G,E)/(G,E) is soft αB-closed and (F,E) ⊑ (G,E)}.
(ii) the soft αB-interior of (F,E) is the soft set sαBint(F,E) = ⊓{(H,E)/(H,E) is soft αB-open and (H,E) ⊑ (F,E)}.

Proposition 3.1:

(i) (i)Finite intersection of soft αB-open sets is a soft αB-open set.
(ii) (ii)Finite union of soft αB-closed sets is a soft αB-closed set.

Proof:
(i) Let (F_i, A_i) be a soft αB-open set. Then (F_i, A_i) = (G_i, B_i) ⊓ (H_i, C_i), where (G_i, B_i) is a soft α-open set and (H_i, C_i) is a soft t-open set. Now,

$$\bigcap_{i=1}^{n}(Fi, Ai) = \bigcap_{i=1}^{n} \{(Gi, Bi) \sqcap (Hi, Ci)\}$$
$$= \bigcap_{i=1}^{n} (Gi, Bi) \sqcap \bigcap_{i=1}^{n}(Hi, Ci)$$

Hence finite intersection of soft αB-open sets is a soft αB-open set.

(ii) Let (F_i, A_i) be a soft αB-closed set. Then (F_i, A_i) = (G_i, B_i) ⊔ (H_i, C_i), where (G_i, B_i) is a soft α-closed set and (H_i, C_i) is a soft t-closed set. Now,

$$\bigcup_{i=1}^{n}(Fi, Ai) = \bigcup_{i=1}^{n} \{(Gi, Bi) \sqcup (Hi, Ci)\}$$
$$= \bigcup_{i=1}^{n} (Gi, Bi) \sqcup \bigcup_{i=1}^{n}(Hi, Ci)$$

Hence finite union of soft αB-closed sets is a soft αB-closed set.

Proposition 3.2: Let (X, τ, E) be a soft topological space over X. For any two soft sets (F, A) and (G, B) the following statements are valid.

(i) sαBcl(F, Φ) = (F, Φ).
(ii) sαBcl(F, $\tilde{X}$) = (F, $\tilde{X}$).
(iii) sαBcl(F, A) ⊒ (F, A).
(iv) If (F, A) ⊑ (G, B), then sαBcl(F, A) ⊑ sαBcl(G, B).
(v) sαBcl(sαBcl(F,A)) = sαBcl(F,A).
(vi) sαBcl((F, A) ⊓ (G, B)) ⊑ (sαBcl(F, A)) ⊓ (sαBcl(G, B)).
(vii) αBcl((F, A) ⊔ (G, B)) = (sαBcl(F, A)) ⊔ (sαBcl(G, B)).

Proposition 3.3: Let (X, τ, E) be a soft topological space over X. For any two soft sets (F,A) and (G,B) the following statements are valid.

(i) sαBint(F, A) ⊑ (F, A).
(ii) If (F, A) ⊑ (G, B), then sαBint(F, A) ⊑ sαBint(G, B).
(iii) $(s\alpha Bint(F,A))' = s\alpha Bcl((F,A)')$.
(iv) $(s\alpha Bcl(F,A))' = s\alpha Bint((F,A)')$.
(v) sαBint(sαBint(F, A)) = sαBint(F, A).
(vi) sαBint((F, A) ⊓ (G, B)) = (sαBint(F, A)) ⊓ (sαBint(G, B)).
(vii) sαBint((F, A) ⊔ (G, B)) ⊒ (sαBint(F, A)) ⊔ (sαBint(G, B)).

Proposition 3.4: Every soft B-open set is a soft αB-open set.

Proof: Let (F, A) be a soft B-open set. Then, (F, A) = (G, B) ⊓ (H, C), where (G, B) is a soft open set and (H, C) is a soft t-open set. Since every soft open set is a soft α-open set, (G, B) is a soft αB-open set. Hence (F, A) is a soft αB-open set.

Remark 3.1: The converse of the proposition 3.4 need not be true.

Example 3.1: Let X be the set of diabetes patients in a hospital and E be the set of parameters. Let X = $\{p_1,p_2,p_3\}$ and E= $\{e_1,e_2,e_3,e_4,e_5,e_6\}$ where e_1,e_2,e_3,e_4,e_5,e_6 be the set of parameters which stands for 'Polyurea',' Fatigue', Polydipsea', 'Polyphagia', 'Weightloss', 'Slow healing of wounds' respectively. Let A= $\{e_1, e_2\}$. Let F be a mapping A to P(X) defined by, (F, A) =$\{(e_1,\{p_1\}),(e_2,\{p_2\})\}$ is a soft set over X. Then, τ = {Φ,$\widetilde{X}$, (F,A)}, is a soft topological space over X. Let (G,A) = $\{(e_1,\{p_1,p_2\}),(e_2,\{p_2\})\}$ and (H,A) = $\{(e_1,\{p_2\}),(e_2,\{\Phi\})\}$ are soft sets over X. Here (G,A) is a soft α-open set but not a soft open set and (F,A)' is a soft t-open set. Then (H,A) is a soft αB-open set but not a soft B-open set.

Proposition 3.5: Every soft αB-open set is a soft αC-open set.

Proof: Let (F,A) be a soft αB-open set. Then, (F,A) = (G,B) ⊓ (H,C), where (G,B) is a soft α-open set and (H,C) is a soft t-open set. Since every soft t-open set is a soft α*- set, (H, C) is a soft α*- set. Hence (F,A) is a soft αB-open set.

Remark: 3.2: The converse of the proposition 3.5 need not be true.

Example: 3.2: Let X be the set of diabetes patients in a hospital and E be the set of parameters. Let X = $\{p_1,p_2,p_3\}$ and E= $\{e_1,e_2,e_3,e_4,e_5,e_6\}$ where e_1,e_2,e_3,e_4,e_5,e_6 be the set of parameters which stands for 'Polyurea','Fatigue', Polydipsea', 'Polyphagia', 'Weightloss', 'Slow healing of wounds' respectively. Let A= $\{e_1,e_2\}$. Let F,G,H,I,J be a mapping A to P(X) defined by,

(F,A) = $\{(e_1, \{p_1\}),(e_2,\{p_1,p_3\})\}$,
(G,A) = $\{(e_1, \{p_1\}),(e_2,\{p_1\})\}$,
(H,A) = $\{(e_1, \{p_2\}),(e_2,\{p_2\})\}$,
(I,A) = $\{(e_1, \{p_1,p_2\}),(e_2,\{p_1,p_2\})\}$ and
(J,A) = $\{(e_1, \{p_1,p_2\}),(e_2,\{\widetilde{X}\})\}$ are a soft sets over X.

Then, τ = {Φ,$\widetilde{X}$, (F,A),(G,A),(H,A),(I,A),(J,A)} is soft topological space over X. Let (K, A) = $\{(e_1,\{p_1\}),(e_2,\{p_2\})\}$ be a soft set over X. Here (K,A) is a soft α*-set and soft α-open set but not a soft t-open set. Therefore (K,A) is a soft αC-open set but not a soft αB-open set.

Remark: 3.3: The interrelations among the sets introduced are given clearly in the following diagram.

soft B-open set
↓
soft αB-open set
↓
soft αC-open set

4. SOFT αB-CONTINUOUS FUNCTION

Definition: 4.1: Let (X, τ_1, E) and (X, τ_2, E) be any two soft topological spaces. A function f from a soft topological space (X, τ_1, E) to another soft topological space (X, τ_2, E) is said to be soft αB-continuous function iff the inverse image of every soft open(closed) set in (X, τ_2, E) is a soft αB-open(closed) set in (X, τ_1, E).

Definition: 4.2: Let (X, τ_1, E) and (X, τ_2, E) be any two soft topological spaces. A function f from a soft topological space (X, τ_1, E) to another soft topological space (X, τ_2, E) is said to be soft αC-continuous function iff the inverse image of every soft open(closed) set in (X, τ_2, E) is a soft αC-open(closed) set in (X, τ_1, E).

Proposition: 4.1: Every soft B-continuous is a soft αB-continuous function.

Proof: Let (X, τ_1, E) and (X, τ_2, E) be any two soft topological spaces. Let f: (X, τ_1, E) → (X, τ_2, E) be a soft B-continuous function. Let (F,A) be a soft open set in (X,τ_2,E). Since f is soft B-continuous function, the inverse image of (F, A) is soft B-open set in (X, τ_1, E). Since every soft B-open set is a soft αB-open set, f^{-1}(F, A) is soft αB-open set in (X, τ_1, E). Hence f is a soft αB-continuous function.

Remark: 4.1: The converse of the proposition 4.1 need not be true.

Example: 4.1: Let τ_1 = {Φ, $\tilde{X}$, (F,A)} and τ_2 = {Φ, $\tilde{X}$, (H,A)}. Let (X, τ_1, E) and (X, τ_2, E) be any two topological spaces over X and Y. Let f: (X, τ_1, E) → (X, τ_2, E) be a mapping. Then f is a soft αB-continuous function in (X, τ_1, E) since in example 3.1, (H,A) is a soft αB-open set but not a soft B-open set in (X, τ_1, E). Therefore f is a soft αB-continuous function but not a soft B-continuous function.

Proposition: 4.2: Every soft αB-continuous is a soft αC-continuous function.

Proof: Let (X, τ_1, E) and (X, τ_2, E) be two soft topological spaces. Let f: (X, τ_1, E) → (X, τ_2, E) be a soft αB-continuous function. Let (F,A) be a soft open set in (X, τ_2, E). Since f is soft αB-continuous function, the inverse image of (F,A) is soft αB-open set in (X, τ_1, E). Since every soft αB-open set is a soft αC-open set, f^{-1}(F,A) is soft αC-open set in (X, τ_1, E). Hence f is a soft αC-continuous function.

Remark: 4.2: The converse of the proposition 4.2 need not be true.

Example: 4.2: Let τ_1 = {Φ,$\tilde{X}$,(F,A),(G,A),(H,A),(I,A),(J,A)} and τ_2 ={Φ,$\tilde{X}$,(K,A)}. Let (X, τ_1, E) and (X, τ_2, E) be two topological spaces over X and Y. Let f:(X, τ_1, E) → (X, τ_2, E) be a mapping. Then f is a soft αC-continuous function in (X, τ_1, E) since in example 3.2, (H,A) is a soft αC-open set but not a soft αB-open set in (X, τ_1, E). Therefore f is a soft αC-continuous function but not a soft αB-continuous function.

Remark: 4.3: By Remark 2.1 and examples 2.2 and 2.3, any soft α-open set need not be soft αC-open set and any soft αC-open set need not be soft α-open set. Hence, soft α-continuous function and soft αC-continuous function are independent as shown by the following Example 4.3 and Example 4.4.

Example: 4.3: Let τ_1 = {Φ,$\tilde{X}$,(F_1,E),(F_2,E),...,(F_{11},E)} defined as in example 2.1 and τ_2 = {Φ,X_e (G,E)} where (G,E) is a defined as in example 2.2. Let (X, τ_1, E) and (X,τ_2,E) be two topological spaces over X and Y. Let f:(X,τ_1,E) → (X,τ_2,E) be a mapping. Then f is a soft αC-continuous function in (X, τ_1, E) since in example 2.2, (G,E) is a soft αC-open set but not a soft α-open set in (X, τ_1, E). Therefore f is a soft αC-continuous function but not a soft α-continuous function.

Example: 4.4: Let τ_1 = {Φ,$\tilde{X}$,(F,E)} and τ_2 = {Φ,$\tilde{X}$,(G,E)} where (F,E) and (G,E) is a defined as in example 2.3. Let (X,τ_1,E) and (X,τ_2,E) be two topological spaces over X and Y. Let f:(X,τ_1,E) → (X,τ_2,E) be a mapping. Then f is a soft α-continuous function in (X,τ_1,E) since in example 2.3,(G,E) is a soft α-open set but not a soft αC-open set in (X,τ_1,E). Therefore f is a soft α-continuous function but not a soft αC-continuous function.

Remark: 4.4: The interrelations among the functions introduced are given clearly in the following diagram.

soft B-continuous function
↓
soft αB-continuous function
↓
soft αC-continuous function

Theorem: 4.1: Let (X, τ_1, E) and (X, τ_2, E) be any two soft topological spaces. If f is any mapping from (X, τ_1, E) to (X, τ_2, E) then the conditions below are equivalent.

(i) The function f is soft αB-continuous function.
(ii) The inverse of every soft α-closed set is a soft αB-closed set.

Proof: The proof follows from the Definition 4.1.

Theorem: 4.2: Let (X, τ_1, E) and (X, τ_2, E) be any two soft topological spaces. If f is any mapping from (X, τ_1, E) to (X, τ_2, E) then the following conditions are equivalent.

(i) f is a soft αB-continuous function.
(ii) For every soft set (F,A) of (X,τ_1,E), $f(s\alpha Bint(F,A)) \sqsupseteq sint(f(F,A))$.
(iii) For every soft set (F,A) of (X,τ_2,E), $f^{-1}(sint(F,A)) \sqsubseteq s\alpha Bint(f^{-1}(F,A))$.

Proof:
(i) ⇒ (ii): Let (F,A) be a soft set in (X, τ_1, E). Then f(F,A) is a soft set in (X, τ_2, E). Since f is a soft αB-continuous function, sint(f(F,A)) is a soft αB-open set in (X, τ_2, E). By hypothesis, $f^{-1}(sint(f(F,A)))$ is a soft αB-open set in (X, τ_1, E). Now,

$$(F,A) \sqsupseteq f^{-1}(sint(f(F,A)))$$
$$\Rightarrow s\alpha Bint(F,A) \sqsupseteq s\alpha Bint(f^{-1}(sint(f(F,A))))$$
$$\sqsupseteq f^{-1}(sint(F,A))$$
$$\Rightarrow f(s\alpha Bint(F,A)) \sqsupseteq sint(f(F,A)).$$

Hence $f(s\alpha Bint(F,A)) \sqsupseteq sint(f(F,A))$.

(ii)⇒ (iii): Let (F,A) be a soft open set in (X, τ_2, E). Now, $f^{-1}(F,A)$ be a soft open set in (X, τ_1, E).

$$By(ii),\ f(s\alpha Bint(f^{-1}(F,A))) \sqsubseteq sint(f(f^{-1}(F,A)))$$
$$\sqsubseteq sint(F,A)$$
$$\Rightarrow s\alpha Bint(f^{-1}(F,A)) \sqsubseteq f^{-1}(sint(F,A)).$$

Hence $s\alpha Bint(f^{-1}(F,A)) \sqsubseteq f^{-1}(sint(F,A))$.

(iii)⇒ (i): Let (F,A) be a soft open set in (X, τ_2, E). By hypothesis,

$$s\alpha Bint(f^{-1}(F,A)) \sqsupseteq f^{-1}(sint(F,A))$$
$$= f^{-1}(F,A)$$
$$But,\ f^{-1}(F,A) \sqsupseteq s\alpha Bint(f^{-1}(F,A))$$
$$\Rightarrow s\alpha Bint(f^{-1}(F,A)) = f^{-1}(F,A)$$

Thus, $f^{-1}(F,A)$ is a soft αB-open set in (X, τ_1, E). Therefore f is a soft αB-continuous function.

Theorem: 4.3: Let (X, τ_1, E) and (X, τ_2, E) be any two soft topological spaces. If f is any mapping from (X, τ_1, E) to (X, τ_2, E) then the following conditions are equivalent.

(i) f is a soft αB-continuous function.
(ii) For every soft set (F,A) of (X,τ_1,E), $f(s\alpha Bcl(F,A)) \sqsubseteq scl(f(F,A))$.
(iii) For every soft set (F,A) of (X,τ_2,E), $s\alpha Bclf^{-1}((F,A)) \sqsubseteq f^{-1}(scl(F,A))$.

Proof:
(i) ⇒ (ii): Let (F,A) be a soft set in (X, τ_1, E). Then f(F,A) is a soft set in (X, τ_2, E). Now, scl(f(F,A)) is a soft closed set in (X,τ_2,E). By hypothesis, $f^{-1}(scl(f(F,A)))$ is a soft αB-closed set in (X,τ_1,E). Hence,

$$(F,A) \sqsubseteq f^{-1}(scl(f(F,A)))$$
$$\Rightarrow s\alpha Bcl(F,A) \sqsubseteq s\alpha Bcl(f^{-1}(scl(f(F,A))))$$
$$= f^{-1}(sclf((F,A)))$$
$$\Rightarrow f(s\alpha Bcl(F,A)) = scl(f(F,A)).$$

Hence $f(s\alpha Bcl(F,A)) = scl(f(F,A))$.

(ii)⇒ (iii): Let (F,A) be a soft closed set in (X,τ_2,E). Now $f^{-1}(F,A)$ be a soft closed set in (X,τ_1,E).

$$By(ii),\ f(s\alpha Bcl(f^{-1}(F,A))) \sqsubseteq scl(f(f^{-1}(F,A)))$$
$$\sqsubseteq scl(F,A)$$
$$\Rightarrow s\alpha Bcl(f^{-1}(F,A)) \sqsubseteq f^{-1}(scl(F,A)).$$

Hence $s\alpha Bcl(f^{-1}(F,A)) \sqsubseteq f^{-1}(scl(F,A))$.

(iii)⇒ (i): Let (F,A) be a soft closed set in (X,τ_2,E). By hypothesis,

$$s\alpha Bcl(f^{-1}(F,A)) \sqsubseteq f^{-1}(scl(F,A))$$
$$= f^{-1}(F,A)$$
$$But, f^{-1}(F,A) \sqsubseteq s\alpha Bcl(f^{-1}(F,A))$$

Therefore, $$s\alpha Bcl(f^{-1}(F,A)) = f^{-1}(scl(F,A))$$
$$= f^{-1}(F,A)$$

Thus $f^{-1}(F,A)$ is a soft αB-closed set in (X, τ_1, E). Therefore f is a soft αB continuous function.

Theorem: 4.4: Let (X, τ_1, E), (X, τ_2, E) and (X, τ_3, E) be any three soft topological spaces. A function f: (X, τ_1, E) → (X, τ_2, E) is a soft αB-continuous function and g: (X, τ_2, E) → (X, τ_3, E) is a soft continuous function. Then $g \circ f$: (X, τ_1, E) → (X, τ_3, E) is a soft αB-continuous function.

Proof: Let (F,A) be a soft open set in (X, τ_3, E). Since g is a soft continuous function, $g^{-1}(F,A)$ is a soft open set in (X, τ_2, E). Also since f is a soft αBcontinuous function, $f^{-1}(g^{-1}(F,A))$ is a soft αB-open set in (X, τ_1, E). Hence $g \circ f$ is a soft αB-continuous function.

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