Volume 9, No. 3, March - 2018 (Special Issue)
International Journal of Mathematical Archive-9(3), 2018, 194-198
IJMA Available online through www.ijma.info ISSN 2229 – 5046

ON INTUITIONISTIC FUZZY SOFT PRE-CONTINUOUS FUNCTIONS

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ABSTRACT

The purpose of this paper is to introduce the concepts of an intuitionistic fuzzy soft pre-open set, intuitionistic fuzzy soft pre-interior, intuitionistic fuzzy soft pre-closure, intuitionistic fuzzy soft pre-continuous function are introduced and studied. Some interesting properties are also discussed.

Keywords: *intuitionistic fuzzy soft pre-open set, intuitionistic fuzzy soft pre-closed set, intuitionistic fuzzy soft pre-interior, intuitionistic fuzzy soft pre-closure, intuitionistic fuzzy soft pre-continuous function are introduced and interesting properties are established.*

2010 Mathematics Subject Classification: *54A40-03E72.*

1. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [7] and later Atanassov [1] generalized the idea to intuitionistic fuzzy sets. On the otherhand, Coker [2] introduced the notions of an intuitionistic fuzzy topological spaces, intuitionistic fuzzy continuity, intuitionistic fuzzy compactness and some other related concepts. Roy, A.R and P.K.Maji [5] was studied the definition of fuzzy soft sets. Maji P.K., R. Biswas and A.R.Roy [3] was introduced the definition of an intuitionistic fuzzy soft sets. NeclaTuranli and A. HaydarEs [4] was introduced and studied the concept of an intuitionistic fuzzysoft topological spaces. In this paper, the concepts of an intuitionistic fuzzy soft pre-open set, intuitionistic fuzzy soft pre-closed set, intuitionistic fuzzy soft pre-interior, intuitionistic fuzzy soft pre-closure, intuitionistic fuzzy soft pre-continuous function.

2. PRELIMINARIES

Definition 2.1: Let Ube an initial universe set and E be the set of parameters. Let IF^U denotes the collection of all intuitionistic fuzzy subsets of U. Let (F,A)⊆ E. A pair (F, E) is called an *intuitionistic fuzzy soft set* over U where φ_ψ is a mapping given byφ_ψ : (F,A)→ IF^U.

Definition 2.2: The relative complement of an intuitionistic fuzzy soft set (F, A) over U is denoted by $(F, A)^r$ and is defined by (F^r, A), where for each e ∈ (F,A), $\mu_{F^r(e)}= \lambda_{F(e)}$ and$\gamma_{F^r(e)}= \mu_{F(e)}$, that is $F^r(e) = (\lambda_{F(e)}, \mu_{F(e)})$

Clearly, $((F, A)^r)^r = (F, A)$.

An intuitionistic fuzzy soft set (F, A) over U is said to be a *relative null intuitionistic fuzzy softset* (with respect to the parameter A), denoted by $\tilde{\phi}_A$, if F(e) = 1_U for all e ∈ A. An intuitionistic fuzzy soft set (F, A) over U is said to be a relative whole intuitionistic fuzzy soft set (with respect to the parameter A), denoted by $\tilde{E}_A$, if F(e) = 1^U for all e ∈ (F,A) .

Denote by 1_x and 1^x the intuitionistic fuzzy sets of X defined by $\mu_{1_X}(x) = 0, \lambda_{1_X}(x) = 1$ and $\mu_{1^X}(x) = 1$, $\lambda_{1^X}(x) = 0$, respectively for all x ∈X.

CONFERENCE PAPER
National Conference dated 26th Feb. 2018, on "New Trends in Mathematical Modelling" (NTMM - 2018), Organized by Sri Sarada College for Women, Salem, Tamil Nadu, India.

Definition 2.3: An intuitionistic fuzzy soft topology τ on (U, E) is a family of intuitionistic fuzzy soft sets over (U, E) satisfying the following properties:

(i) $\tilde{\phi}_E, \tilde{E} \in \tau$,

(ii) If (F, A),(G, B) $\in \tau$, then (F, A)∩(G, B) $\in \tau$,

(iii) If $(F_\alpha, A) \in \tau$ for all $\alpha \in \wedge$, an index set, then $\cup_{\alpha \in \wedge}(F_\alpha, A) \in \tau$.

In this case the (U, E, τ) is called an *intuitionistic fuzzy soft topological space* (IFSTS for short) and each IFSS in τ is known as an intuitionistic fuzzy soft open set(IFSOS for short) in (U, E). An intuitionistic fuzzy soft set is called τ-closed iff its complement is τ-open.

Definition 2.4: Let (U,τ, E) and (U′, τ', E′) be IFSTS's and let $\emptyset_\varphi$: U→U′ and $\emptyset_\varphi$: E → E′ be two mappings. Then a mapping (ϕ, φ): IFSTS (U, τ, E)→ IFSTS (U′, τ', E′)is said to be *intuitionistic fuzzy softcontinuous* iff the preimage of each IFSOS in (U′, τ', E′) is an IFSOS in (U, τ, E).

3. INTUITIONIST FUZZY SOFT PRE –OPEN

In this section the concepts of intuitionistic fuzzy soft topological spaces, intuitionistic fuzzy soft pre-open sets, intuitionistic fuzzy soft pre-interior, intuitionistic fuzzy soft pre-closure are introduced an some of the properties are discussed.

Definition 3.1: Let (U,τ, E) be an intuitionistic fuzzy soft topological space. Let (F,A) be an intuitionistic fuzzy soft set in an intuitionistic fuzzy soft topological space (U,τ,E). Then (F,A) is said to be an *intuitionistic fuzzy soft pre- open set.*

The complement of an intuitionistic fuzzy soft pre- open set is said to be an *intuitionistic fuzzy soft pre- closed set.*

Definition 3.2: Let (U, τ, E) be an intuitionistic fuzzy soft topolpgical space. Let (F,A) be an intuitionistic fuzzy soft set in an intuitionistic fuzzy soft topological space (U, τ, E). The *intuitionistic fuzzy soft pre- closure* of (F,A) is denoted and defined by $FS\mathbb{p} - cl\ (F, A) = \cap \{(G, B): (G, B)$is an $IFS\mathbb{p}$ closed set and $(F, A) \subseteq (G, B)\}$.

Definition 3.3: Let (U,τ,E) be an intuitionistic fuzzy soft topological space. Let (F,A) be an intuitionistic fuzzy soft set in an intuitionistic fuzzy soft topological space (U,τ,E). The *intuitionistic fuzzysoft pre- interior* of (F,A) is denoted and defined by $IFS\mathbb{p}\ int\ (F, A) = \cup\{(G, B): (G, B)$ is an $IFS\mathbb{p}$ open set and $(G, B) \subseteq (F, A)\}$.

4. INTUITIONISTIC FUZZY SOFT PRE-CONTINUOUS

In this section the concepts of intuitionistic fuzzy soft pre-continuous, intuitionistic fuzzy soft pre-neighbourhood, intuitionistic fuzzy soft pre-quasineighbourhood are introduced and some of the properties are discussed.

Definition 4.1: Let (U, τ, E) be an intuitionistic fuzzy soft topological space. Let (F,A) be an intuitionistic fuzzy soft set in a intuitionistic fuzzy soft topological space (U,τ,E). Then (F,A) is said to be an *intuitionistic fuzzy soft pre-neighbourhood* of an intuitionistic fuzzy soft point e_G if there exists an intuitionistic fuzzy soft pre- open set (G,B) in an intuitionistic fuzzy soft topological space (U,τ,E) such that $e_G \in (G, B)$, $(G, B) \subseteq (F, A)$. It is denoted by $IF\mathbb{p}nbd$.

Definition 4.2: Let (U, τ, E) be an intuitionistic fuzzy soft topological space. Let (F,A) be an intuitionistic fuzzy soft set in an intuitionistic fuzzy soft topological space (U,τ,E). Then (F,A) is said to be an *intuitionistic fuzzy soft pre-quasi neighbourhood* of an intuitionistic fuzzy point e_G if there exists an intuitionistic fuzzy soft pre-open set (G,B) in an intuitionistic fuzzy soft topological space (U,τ,E) such that $e_G\ q(G, B)$, $(G, B) \subseteq (F, A)$. It is denoted by $IF\mathbb{p}qnbd$.

Definition 4.3: Let (U, τ, E) and (Y, σ, K) be any two intuitionistic fuzzy soft topological spaces.
Let φ_ψ: (U, τ, E) → (Y, σ, K) is a fuzzy soft mapping. Then φ_ψ is said to be an *intuitionistic fuzzysoft pre-continuous function.* If for each intuitionistic fuzzy soft point e_G in $X$and $(G, B) \epsilon N_\sigma \varphi_\psi (e_G)$, there exists $(F, A) \epsilon N_\tau^{IFS\mathbb{p}} q(e_G)$.

Notation 4.4:
- Intuitionistic fuzzy soft topological space is denoted by (IFSTS).
- Intuitionistic fuzzy soft set is denoted by (IFSS).
- Intuitionistic fuzzy soft pre-open set is denoted by (IFS𝕡OS).
- Intuitionistic fuzzy soft pre-closed set is denoted by (IFS𝕡CS).
- Intuitionistic fuzzy soft pre-neighbourhood is denoted by (IFS𝕡nbd).
- Intuitionistic fuzzy soft pre quasi neighbourhood is denoted by (IFS𝕡qnbd).

Remark 4.5:

(i)$IF\mathbb{p}cl(F,A) = (F,A)$if and only if (F,A) is an intuitionistic fuzzy pre closed set.
(ii)$IF\mathbb{p}int(F,A) \subseteq (F,A) \subseteq IF\mathbb{p}\, cl(F,A)$.
(iii)$IF\mathbb{p}\, int(1_{\sim}) = (1_{\sim})$
(iv)$IF\mathbb{p}\, int\ (0_{\sim}) = (0_{\sim})$
(v)$IF\mathbb{p}cl(1_{\sim}) = (1_{\sim})$

Proposition 4.6: Let (U,τ,E) and (Y,σ,K) be any two intuitionistic fuzzy soft topological spaces. Let $\varphi_\psi: (U,\tau,E) \to (Y,\sigma,K)$ be an intuitionistic fuzzy soft mapping. Then the following are equivalent.

i)$\varphi_\psi$is an $IFS\mathbb{p}$ continuous function.
ii) $(\varphi_\psi)^{-1}\,(F,A)$ is an $IFS\mathbb{p}$ open set in an $IFSTS\ (U,\tau,E)$, for each IFS open set (F,A) is an $IFSTS\ (Y,\sigma,K)$.
iii) $(\varphi_\psi)^{-1}\,(G,B)$ is an IFS$\mathbb{p}$ closed set in an $IFSTS\ (U,\tau,E)$ for each IFS closed set (G,B) in an $IFSTS\ (Y,\sigma,K)$.
iv) $IFS\mathbb{p}\ cl\ ((\varphi_\psi)^{-1}\,(F,A)\ \subseteq (\varphi_\psi)^{-1}\,(IFS\ cl\ (F,A))$ for each $IFSS\ (F,A)$ in an $IFSTS(Y,\sigma,K)$
v) $(\varphi_\psi)^{-1}(IFS\ int\ (F,A)) \subseteq IFS\mathbb{p}\ int\ ((\varphi_\psi)^{-1}(F,A)$for each $IFSS\ (F,A)$ in an $IFSTS\ (Y,\sigma,K)$.

Proof:

(i)⟹(ii): Let (F,A) be a IFSOS in an IFSTS (Y,σ,K) and e_G be an intuitionistic fuzzy soft Point in an IFSTS (U,τ,E) such that $e_G q(\varphi_\psi)^{-1}(F,A)$ since φ_ψ is an intuitionistic fuzzy soft pre- continuous function there exists

$$(G,B)\epsilon N_\tau^{IFS\mathbb{p}} q(e_G) \text{ Such that } \varphi_\psi(G,B) \subseteq (F,A)$$

Then,

$$e_G\ \epsilon(G,B) \tag{1}$$
$$(G,B) \subseteq (\varphi_\psi)^{-1}\left(\varphi_\psi(G,B)\right) \tag{2}$$

From (1) & (2) it follows that

$$\epsilon(G,B) \subseteq (\varphi_\psi)^{-1}\left(\varphi_\psi(G,B)\right) \subseteq (\varphi_\psi)^{-1}(F,A)$$
$$\Rightarrow (\varphi_\psi)^{-1}(F,A) \text{is an IFS pre -open set in an IFSTS } (U,\tau,E)$$

(ii)⟹(i): this can be proved by taking complement of (i)
(iii)⟹(iv): let (F,A) be an intuitionistic fuzzy soft set in an intuitionistic fuzzy soft topological space (Y,σ,K).

Since $(F,A) \subseteq IFS\ cl\ (F,A)$, $(\varphi_\psi)^{-1}(F,A) \subseteq (\varphi_\psi)^{-1}(IFS\ cl\ (F,A))$

By (iii)$(\varphi_\psi)^{-1}(IFS\ cl\ (F,A))$ is an intuitionistic fuzzy soft pre-closed set in an intuitionistic fuzzy soft topological space (U,τ,E).
Thus IFS$\mathbb{p}$ $cl\left((\varphi_\psi)^{-1}(F,A)\right) \subseteq (\varphi_\psi)^{-1}(IFS\ cl\ (F,A))$
(iv)⟹(v): Using (iv)

$$\text{IFS}\mathbb{p}cl\left((\varphi_\psi)^{-1}(F,A)\right) \subseteq (\varphi_\psi)^{-1}(IFS\ cl\ (F,A))$$

Then

$$\overline{IFS\mathbb{p}\ cl(\varphi_\psi^{-1}(F,A))} \supseteq (\varphi_\psi)^{-1}\overline{(IFS\ cl\ (F,A))}$$
$$\text{IFS}\mathbb{p}\ int\left(\overline{(\varphi_\psi)^{-1}(F,A)}\right) \supseteq (\varphi_\psi)^{-1}(IFS\ int\ \overline{(F,A)})$$
$$\text{IFS}\mathbb{p}\ int\left((\varphi_\psi)^{-1}\overline{(F,A)}\right) \supseteq (\varphi_\psi)^{-1}(IFS\ int\ \overline{(F,A)})$$
$$(\varphi_\psi)^{-1}(IFS\ int\ \overline{(F,A)}) \subseteq \text{IFS}\mathbb{p}\ int\ \left((\varphi_\psi)^{-1}(F,A)\right)$$

Put $\overline{(F,A)}$= (F,A).

$$(\varphi_\psi)^{-1}(IFS\ int\ (F,A) \subseteq \text{IFS}\mathbb{p}\ int\left((\varphi_\psi)^{-1}(F,A)\right)$$

(V) ⟹(i): Let (F,A) be an intuitionistic fuzzy soft open set in an intuitionistic fuzzy soft topological space (Y,σ,K). Then IFS$int(F,A) = (F,A)$. Using (v)

$$(\varphi_\psi)^{-1}(IFS\ int\ (F,A)) \subseteq \text{IFS}\mathbb{p}\ int\left((\varphi_\psi)^{-1}(F,A)\right)$$
$$(\varphi_\psi)^{-1}(F,A) \subseteq \text{IFS}\mathbb{p}\ int\left((\varphi_\psi)^{-1}(F,A)\right)$$

But IFS$\mathbb{p}$ $int\left((\varphi_\psi)^{-1}(F,A)\right) \subseteq (\varphi_\psi)^{-1}(F,A)$

That is, $(\varphi_\psi)^{-1}(F,A)$ is an intuitionistic fuzzy soft pre - open set in an intuitionistic fuzzy soft topological space (U,τ,E). Let e_G be an intuitionistic fuzzy point in $(\varphi_\psi)^{-1}(F,A)$.
Then, $e_G\, q(\varphi_\psi)^{-1}(F,A)$.This implies that $\varphi_\psi(e_G)q\left((\varphi_\psi)^{-1}(F,A)\right)$

But $\varphi_\psi\left((\varphi_\psi)^{-1}(F,A)\right) \subseteq (F,A)$. Thus for any intuitionistic fuzzy soft point e_G and $(F,A)\epsilon\, N_\tau\varphi_\psi(e_G)$, there exists $(G,B) = (\varphi_\psi)^{-1}(F,A)\epsilon N_\tau^{IFS\mathbb{p}}q(e_G)$ such that
$(\varphi_\psi)^{-1}\left(\varphi_\psi(F,A)\right) \subseteq (F,A)$. there fore $\varphi_\psi(G,B) \subseteq (F,A)$.
Thus φ_ψ is an intuitionistic fuzzy soft pre -continuous function.

Proposition 4.6: Let (U,τ,E) and(Y,σ,K) be any two intuitionistic fuzzy soft topological spaces.
Let $\varphi_\psi: (U,\tau,E) \to (Y,\sigma,K)$ be an intuitionistic fuzzy soft bijection function. Then φ_ψ is an intuitionistic fuzzy soft pre-continuous function if and only if

$$IFS\ int\ \left(\varphi_\psi(F,A)\right) \subseteq \varphi_\psi(IFS\mathbb{p}int(F,A))$$

For each IFSS (F,A) of an IFSTS (U,τ,E).

Proof: Assume that φ_ψ is IFS pre-continuous function and let (F,A) be an IFSS in an IFSTS (U,τ,E).
Hence $((\varphi_\psi)^{-1}(IFS\ int\ (\varphi_\psi(F,A))$ is an intuitionistic fuzzy soft pre- open set in an IFSTS(U,τ,E).
From proposition (v) of (1)

$$(\varphi_\psi)^{-1}\left(IFS\ int\ \varphi_\psi\ (F,A)\right) \subseteq \text{ IFS}\mathbb{p}\ int\left((\varphi_\psi)^{-1}\varphi_\psi(F,A)\right)$$

$(\varphi_\psi)^{-1}\left(IFS\ int\ \varphi_\psi\ (F,A)\right) \subseteq$ IFS$\mathbb{p}$ $int(F,A)$ (Since φ_ψ is an intuitionistic fuzzy soft injective function)
$\varphi_\psi((\varphi_\psi)^{-1}\left(IFS\ int\ \varphi_\psi\ (F,A)\right) \subseteq \varphi_\psi($ IFS$\mathbb{p}$ $int(F,A))$
$\left(IFS\ int\ \varphi_\psi\ (F,A)\right) \subseteq \varphi_\psi$ (IFS$\mathbb{p}$ $int\left((F,A)\right)$ (Since φ_ψ is an intuitionistic fuzzy soft surjective function).

Conversely,
Assume that $\left(IFS\ int\ \varphi_\psi\ (F,A)\right) \subseteq \varphi_\psi($ IFS$\mathbb{p}$ $int(F,A)$), for each intuitionistic fuzzy set (F,A) in an IFSTS (U,τ,E).Let (G,B) be an IFSOS in an IFSTS (Y,σ,K).
Then (G,B) = IFS int (G,B).Since φ_ψ is an intuitionistic fuzzy soft surjective function,

(G,B) = IFS int (G,B)
= IFS int $(\varphi_\psi((\varphi_\psi)^{-1}(G,B))$
$\subseteq \varphi_\psi($ IFS$\mathbb{p}$ $int(\varphi_\psi)^{-1}(G,B))$

$$(\varphi_\psi)^{-1}(G,B) \subseteq (\varphi_\psi)^{-1}\varphi_\psi\left(IFS\ int(\varphi_\psi)^{-1}\ (G,B)\right)$$

Since φ_ψ is an intuitionistic fuzzy soft injective function.

$$(\varphi_\psi)^{-1}(G,B) \subseteq \left(IFS\mathbb{p}\ int(\varphi_\psi)^{-1}\ (G,B)\right) \quad (1)$$

But,

$$\left(IFS\mathbb{p}\ int(\varphi_\psi)^{-1}\ (G,B)\right) \subseteq (\varphi_\psi)^{-1}(G,B) \quad (2)$$

From (1) and (2) it follows that $(\varphi_\psi)^{-1}(G,B) = \left(IFS\mathbb{p}\ int(\varphi_\psi)^{-1}\ (G,B)\right)$. That is, $(\varphi_\psi)^{-1}(G,B)$is an intuitionistic fuzzy soft pre-open set in an intuitionistic fuzzy soft topological space (U,τ,E).Thusφ_ψ is an intuitionistic fuzzy soft pre – continuous function.

Proposition 4.7: Let (U,τ,E) and (Y,σ,K) be any two intuitionistic fuzzy soft topological spaces.
Let $\varphi_\psi: (U,\tau,E) \to (Y,\sigma,K)$ be an intuitionistic fuzzy soft bijection function. Then φ_ψ is an intuitionistic fuzzy soft pre-continuous function if and only if

$$\varphi_\psi\left(IFS\mathbb{p}cl(F,A)\right) \subseteq IFcl\left(\varphi_\psi(F,A)\right).$$

For each IFSS (F,A) of an IFSTS (U,τ,E).

Proof: Simply to proposition (4.6).

CONCLUSION

It is known that various types of functions play a significant role in the theory of classical point set topology and engineering, economics etc. A great number of papers dealing with such functions have appeared and a good many of them have been extended to the fuzzy topological spaces, soft topological spaces and fuzzy soft topological spaces, intuitionistic fuzzy soft topological space by workers. The purpose of the present paper is to define pre-continuous in intuitionistic fuzzy soft topological spaces.

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Source of support: National Conference on "New Trends in Mathematical Modelling" (NTMM - 2018), Organized by Sri Sarada College for Women, Salem, Tamil Nadu, India.

