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ON #pg-CONTINUOUS MAPS AND #pg-COMPACT SPACES

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ABSTRACT

In this paper the concept of #pg-closed set is introduced. Besides studying some properties, the interrelations of #pg-closed set with other related sets are studied. Characterizations and properties of #pg-continuous map is discussed. Equivalently #pg-compact spaces are introduced and some interesting properties are discussed.

Keywords: **pre semi-open set, pw-closed set, #pg-closed set, #pg-continuous map, $T_{\#pg}$-space, #pg-irresolute map, #pg-compact space.*

AMS Subject Classification: *54A05, 54C08.*

1. INTRODUCTION

The notion of pre-open set was introduced by Mashhour *et al.* [12]. Cameron [5], Benchalli [3], and Syed Ali Fathima and Mariasingam [19, 20] introduced and investigated regular semi-open sets, rw-closed sets, #rg-closed sets, #rg-closure, #rg-continuous maps, $T_{\#rg}$-space and #RG-compact spaces. We introduce and study the concept of #pg-closed sets, #pg-continuous maps and #pg-compact spaces.

2. PRELIMINARIES

Throughout this paper X, Y and Z denote the topological spaces (X, T), (Y, S) and (Z, R) respectively on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a topological space X, the closure of A, interior of A, semi-closure of A, semipre-closure of A, the complement of A and #rg-closure of A are denoted by cl(A), int(A), scl(A), spcl(A), X\A and #rg-cl(A) respectively. We recall the following definitions and results.

Definition 2.1: A subset A of a space X is called
(1) a pre-open set [11] if A ⊆ intcl (A) and a pre-closed set if clint (A) ⊆ A.
(2) a semi-open set [11] if A ⊆ clint (A) and a semi-closed set if intcl (A) ⊆ A.
(3) a regular open set [17] if A = intcl (A) and a regular closed set if A = clint (A).
(4) a π-open set [21] if A is a finite union of regular open sets.
(5) a regular semi-open set [5] if there is a regular open U such that U ⊆ A ⊆ cl(U).
(6) a semi-preopen set [1] if A ⊆ cl(int(cl(A))) and a semi-preclosed set if int(cl(int(A))) ⊆ A.

Definition 2.2: A subset A of a space X is called
(1) a generalized closed set (briefly, g-closed) [10] if cl(A) ⊆ U whenever A ⊆ U and U is open in X.
(2) a weakly generalized closed set (briefly, wg-closed) [13] if cl(int(A)) ⊆ U whenever A ⊆ U and U is open in X.
(3) a π-generalized closed set (briefly, πg-closed) [8] if cl(A) ⊆ U whenever A ⊆ U and U is π-open in X.
(4) a weakly closed set (briefly, w-closed) [15] if cl(A) ⊆ U whenever A ⊆ U and U is semi-open in X.
(5) a rw-closed set [3] if cl(A) ⊆ U whenever A ⊆ U and U is regular semi-open in X.
(6) a *g-closed set [20] if cl(A) ⊆ U whenever A ⊆ U and U is w-open in X.

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(7) a generalized semi-closed set (briefly, gs-closed) [2] if scl(A) ⊆ U whenever A ⊆ U and U is open in X.
(8) a generalized semi pre-closed set (briefly, gsp-closed) [7] if spcl(A) ⊆ U whenever A ⊆ U and U is open in X.
(9) a #rg-closed set [18] if cl(A) ⊆ U whenever A ⊆ U and U is rw-open in X.

Definition 2.3: [18] For a subset A of a space X, #rg-cl(A) = ∩ {F : A ⊆ F, F is #rg closed in X} is called the #rg-closure of A.

Definition 2.4: A map f : (X, T) → (Y, S) is called
(1) continuous [4] if $f^{-1}(V)$ is closed set in X for every closed subset V of Y.
(2) π-continuous [8] if $f^{-1}(V)$ is π-closed set in X for every closed subset V of Y.
(3) πg-continuous [8] if $f^{-1}(V)$ is πg-closed set in X for every closed subset V of Y.
(4) wg-continuous [13] if $f^{-1}(V)$ is wg-closed set in X for every closed subset V of Y.
(5) gs-continuous [6] if $f^{-1}(V)$ is gs-closed set in X for every closed subset V of Y.
(6) gsp-continuous [7] if $f^{-1}(V)$ is gsp-closed set in X for every closed subset V of Y.
(7) #rg-continuous[19] if $f^{-1}(V)$ is #rg-closed set in X for every closed subset V of Y.

Definition 2.5: A space X is called $T_{\#rg}$-space [18] if every #rg-closed set in it is closed.

Definition 2.6: [19] Let (X, T) be a topological space and $T_{\#rg}$ = {V ⊆ X : #rg-cl(X\V) = X\V}.

Definition 2.7: [19] A function f : (X, T)→ (Y, S) is called #rg -irresolute if $f^{-1}(V)$ is #rg-closed in (X, T) for every #rg-closed subset V of (Y, S).

Definition 2.8: [16]A family S_n of # rg-open subsets of a topological space (X, T) is said to be # rg-open cover of X, if X ⊆ { S_n : n∈ I }.

Definition 2.9: [9] A topological space (X, T) is said to be compact space if every open cover of X has a finite subcover.

3. #pg-CLOSED SETS AND THEIR BASIC PROPERTIES

In this section, we introduce and study #pg-closed sets.

Definition 3.1: A subset A of a space X is called *pre semi-open (briefly, *ps-open) if there is a pre-open set U such that U ⊆ A ⊆ cl(U).

Definition 3.2: A subset A of a space X is called pre weakly-closed (briefly, pw-closed) if cl(A) ⊆ U whenever A ⊆ U and U is *ps-open in X.

Definition 3.3: A subset A of a space X is called #pre generalized-closed (briefly, #pg-closed) if cl(A) ⊆ U whenever A ⊆ U and U is pw-open in X.

The interrelations among the set introduced and other related sets are exhibited below:

$$\begin{tikzcd}
\text{closed} \arrow[rr] & & \#\text{ pg-closed} \arrow[dll] \arrow[dl] \arrow[d] \arrow[dr] & & \pi\text{-closed} \arrow[ll] \\
\pi\text{g-closed} & \text{gsp-closed} & \text{gs-closed} & \text{wg-closed} &
\end{tikzcd}$$

Proposition 3.1: The union of two #pg-closed subsets of X is also a #pg-closed subset of X.

Proof: Assume that A and B are #pg-closed sets in X. Let A∪B ⊆ U and U be pw-open in X. Then A ⊆ U and B ⊆ U and U is pw-open in X. Since A and B are #pg-closed sets in X, cl(A) ⊆ U and cl(B) ⊆ U.
Hence, cl(A∪B) = cl(A)∪cl(B) ⊆ U. Therefore, A∪B is a #pg-closed set in X.

Proposition 3.2: The intersection of two #pg-closed subsets of X is also a #pg-closed subset of X.

Proposition 3.3: Let A be a #pg-closed set in X. Then cl(A)\A does not contain any non-empty pw-closed set in X.

Proof Let U be a non-empty pw-closed subset of cl(A)\A. Now A ⊆ X\U and X\U is pw-open in X. Since A is #pg-closed, cl(A) ⊆ X\U. Then U ⊆ X\cl(A). This is a contradiction, since by assumption, U ⊆ cl(A).

Proposition 3.4: Let A be a #pg-closed set in X. Then A is closed iff cl(A)\A is pw-closed.

Proposition 3.5: For every point x of a space X, X\{x} is #pg-closed (or) pw-open.

Proposition 3.6: Let A be a #pg-closed subset of (X, T) such that A ⊆ B ⊆ cl(A). Then B is also a #pg-closed subset of (X, T).

Proof: Let B ⊆ U and U be pw-open in (X, T). Since A ⊆ B, A ⊆ U and U is a pw-open set in (X, T). Since A is #pg-closed, cl(A) ⊆ U. Then cl(B) ⊆ cl(cl(A)) = cl(A) ⊆ U. Hence, B is #pg-closed.

The converse of the above Theorem need not be true as seen from the following Example.

Example 3.1: Let X = {a, b, c, d} and T = {ϕ, X, {b}, {c}, {b, c}, {a, b, c}}. Let A = {a, d} and B = {a, b, d} Then A and B are #pg-closed set in (X, T). But A = {a, d} ⊆ B = {a, b, d} ⊈ cl(A) = {c, d}.

Proposition 3.7: If a subset A of a topological space X is both pw-open and #pg-closed. Then A is a closed set.

Proposition 3.8: Let A be pw-open and #pg-closed in X. Suppose that F is closed in X. Then A ⋂ F is a #pg-closed set in X.

Proof: Let A be a pw-open and #pg-closed set in X and let F be a closed set in X. By Proposition 3.5, A is closed and so A ⋂ F is closed. Since every closed set is #pg-closed, A ⋂ F is a #pg-closed set in X. Hence, A ⋂ F is a #pg-closed set in X.

Remark 3.1: If a subset A of a topological space X is
(i) open and g-closed, then A is #pg-closed.
(ii) w-open and *g-closed, then A is #pg-closed.
(iii) π-open and πg-closed, then A is #pg-closed.
(iv) semi-open and w-closed, then A is #pg-closed.
(v) *ps-open and pw-closed, then A is #pg-closed.
(vi) open and wg-closed, then A is #pg-closed.

Definition 3.4: A space X is called a $T_{\#pg}$-space if every #pg-closed set in it is closed.

Proposition 3.9: Every $T_{1/2}$-space is $T_{\#pg}$-space.

4. #pg-CONTINUOUS MAPS

In this section, we introduce and study #pg-continuous maps.

Definition 4.1: For a subset A of a space X, #pg-cl(A) = ⋂{F : A ⊆ F, F is #pg-closed in X} is called the #pg-closure of A.

Definition 4.2: Let (X, T) be a topological space and $T_{\#pg}$ = {V ⊆ X: #pg-cl(X\V) = X\V}.

Definition 4.3: A map f : (X, T) → (Y, S) is called #pg-continuous if $f^{-1}(V)$ is #pg-closed in X for every closed subset V in Y.

Definition 4.4: A map f : (X, T) → (Y, S) is called #pg-irresolute if $f^{-1}(V)$ is #pg-closed in (X, T) for every #pg-closed subset V of (Y, S).

Remark 4.1 Let A and B be subsets of (X, T). Then
1. #pg-cl(ϕ) = ϕ and #pg-cl(X) = X.
2. If A ⊆ B, then #pg-cl(A) ⊆ #pg-cl(B).
3. A ⊆ #pg-cl(A).
4. If A is #pg-closed, then #pg-cl(A) = A.

Proposition 4.1: Suppose $T_{\#pg}$ is a topology. If A is #pg-closed in (X, T), then A is closed in (X, $T_{\#pg}$).

Proposition 4.2: A set A ⊆ X is #pg-open iff F ⊆ int(A) whenever F ⊆ A and F is pw-closed.

Proposition 4.3: Let X be a space in which every singleton set is pw-closed. Then f: (X, T)→ (Y, S) is #pg-continuous iff x ∈ int($f^{-1}(V)$) for every open subset V of Y which contains f(x).

Proposition 4.4: Let f : (X, T) → (Y, S) be a map. Let (X, T) and (Y, S) be any two spaces such that $T_{\#pg}$ is a topology on X. Then the following statements are equivalent:

(i) For every subset A of X, f(#pg-cl(A)) ⊆ cl(f(A)).
(ii) f : (X, $T_{\#pg}$) → (Y, S) is continuous.

Proof:
(i)⇒(ii): Suppose (i) holds. Let A be a closed set in Y. By (i), f(#pg-cl(f^{-1}(A))) ⊆ cl(f(f^{-1}(A))) ⊆ cl(A) = A. So #pg-cl(f^{-1}(A)) ⊆ f^{-1}(A). Also f^{-1}(A) ⊆ #pg-cl(f^{-1}(A)). Hence, #pg-cl(f^{-1}(A)) = f^{-1}(A). This implies $(f^{-1}(A))^c \in T_{\#pg}$. Thus, f^{-1}(A) is closed in (X, $T_{\#pg}$). Hence, f is continuous.
(ii)⇒(i): Suppose (ii) holds. Let A be a subset of X. Then cl(f(A)) is closed in Y. Since f : (X, $T_{\#pg}$) → (Y, S) is continuous, f^{-1}(cl(f(A))) is closed in (X, $T_{\#pg}$). By Definition 4.2, #pg-cl(f^{-1}(cl(f(A))) = f^{-1}(cl(f(A))).
Now A ⊆ f^{-1}(f(A)) ⊆ f^{-1}(cl(f(A))), A ⊆ f^{-1}(cl(f(A))), which implies #pg-cl(A) ⊆ #pg-cl f^{-1}(cl(f(A))) = f^{-1}(cl(f(A))). Therefore, f(#pg-cl(A)) ⊆ cl(f(A)).

The interrelations among the map introduced and other related maps are exhibited below:

$$\begin{tikzcd}
\text{continuous} \arrow[r] & \text{\#pg-continuous} \arrow[dl] \arrow[d] \arrow[dr] \arrow[drr] & & \pi\text{-continuous} \arrow[ll] \\
\pi g\text{-continuous} & \text{gsp-continuous} & \text{gs-continuous} & \text{wg-continuous}
\end{tikzcd}$$

Proposition 4.5: Let f : (X, T) → (Y, S) be a function. Then the following are equivalent:

(i) f is #pg-continuous.
(ii) The inverse mage of each open set in (Y, S) is #pg-open in (X, T).
(iii) The inverse mage of each closed set in (Y, S) is #pg-closed in (X, T).

Proof:
(i)⇒(ii): Suppose (i) holds. Let V be open in Y. Then Y\V is closed in Y. Since f is #pg-continuous, f^{-1}(Y\V) is #pg-closed in X. But f^{-1}(Y\V) = X\ f^{-1}(V) which is #pg-closed in X. Therefore, f^{-1}(V) is #pg-open in X. Hence, the inverse mage of each open set in (Y, S) is #pg-open in (X, T).

(ii)⇒(iii): Suppose (ii) holds. Let V be a closed set in Y. Then Y\V is open in Y. Since the inverse mage of each open set in (Y, S) is #pg-open in (X, T), f^{-1}(Y\V) is #pg-open. But f^{-1}(Y\V) = X\ f^{-1}(V) which is #pg-open. Therefore, f^{-1}(V) is #pg-closed in X. Hence, the inverse mage of each closed set in (Y, S) is #pg-closed in (X, T).

(iii)⇒(i): Suppose (iii) holds. Let V be a closed set in Y. Since, the inverse image of each closed set in (Y, S) is #pg-closed in (X, T), f^{-1}(V) is #pg-closed in X. Hence, f is #pg-continuous.

Proposition 4.6: If a map f : (X, T) → (Y, S) is #pg-continuous, then f(#pg-cl(A)) ⊆ cl(f(A)) for every subset A of X.

5. #pg-COMPACT SPACES

In this section we introduce and study the concept of #pg-compact spaces.
Definition 5.1: A family {S_n : n∈ I} of #pg-open subsets of a topological space (X, T) is said to be #pg-open cover of X, if X ⊆ ∪{S_n : n∈ I}.

Definition 5.2: A topological space (X, T) is said to be #pg-compact space if every #pg-open cover of X has a finite subcover.

Proposition 5.1: Let (X, T) be a #pg-compact space. Then a #pg-closed subset of (X, T) is a #pg-compact set.

Proof: Let (X, T) be a #pg-compact space, A ⊆ X be a #pg-closed set and {S_n : n∈ I}be #pg-open cover of A. Then A ⊆ ∪S_n. Since A^c is a # pg-open set in X, X ⊆ ∪S_n ∪A^c. Now ∪S_n ∪A^c is a #pg-open cover of X and X is a #pg-compact space. Hence, X has finite subcover, such that X ⊆ S_1 ∪ S_2 , . . . , ∪S_n ∪A^c and A ∩ A^c = ϕ. Thus, A ⊆S_1 ∪ S_2 , . . . , ∪S_n. Therefore, A is a #pg-compact set.

Proposition 5.2: If f: (X, T)→(Y, S) is a #pg -continuous map from a #pg-compact space (X, T) onto a topological space (Y, S), then (Y, S) is a compact space.

Proof: Let {A_k : k∈ I} be any open cover of (Y, S). Since f is a #pg-continuous map, {f^{-1} (A_k): k∈ I} is a #pg-open cover of X. By hypothesis, X has a finite sub cover {f^{-1} (A_{k1}), f^{-1} (A_{k2}), . . . , f^{-1}(A_{kn})}. That is, there exists k1, k2, . . . , kn, such that X ⊆ ∪{f^{-1}(A_{ki}): i=1,2,….n}. Since f is onto, Y = f(X) ⊆ ∪{f(f^{-1}(A_{ki}) : i = 1, 2,…, n}, which equals ∪{A_{ki}: i = 1, 2, …, n}. Therefore, Y is compact.

Proposition 5.3: If f : (X, T)→ (Y, S) is a #pg-irresolute map from a #pg-compact space (X, T) onto a topological space (Y, S), then (Y, S) is a compact space.

Proposition 5.4: If f : (X, T)→ (Y, S) is a #pg-irresolute map from a #pg-compact space (X, T) onto a topological space (Y, S), then (Y, S) is a #pg-compact space.

Proof: Let $\{A_k : k\in I\}$ be #pg-open cover of Y. Since f is #pg-irresolute, $\{f^{-1}(A_k) : k\in I\}$ is a #pg-open cover of X. By hypothesis, X has a finite sub cover $\{f^{-1}(A_{k1}), f^{-1}(A_{k2}), \ldots, f^{-1}(A_{kn})\}$. That is, there exist k1, k2, . . . , kn, such that $X \subseteq \cup\{f^{-1}(A_{ki}) : i = 1, 2, \ldots, n\}$. Since f is onto, $Y = f(X) \subseteq \cup\{f(f^{-1}(A_{ki})) : i = 1, 2,\ldots, n\} = \cup\{(A_{ki}) : i = 1, 2,\ldots, n\}$. Therefore, Y is a #pg-compact space.

Proposition 5.5: Let f : (X, T) →(Y, S) be a #pg-irresolute map and G be a subset of X. If G is #pg-compact relative to X, then the image f(G) is #pg-compact relative to Y.

Proof: Let $\{A_k : k\in I\}$ be a collection of #pg-open sets in Y, such that $f(G) \subseteq \cup\{A_k : k\in I\}$. Then $G \subseteq \cup\{f^{-1}(A_k) : k\in I\}$, where $f^{-1}(A_k)$ is a #pg-open set in X for each $k\in I$. Since G is #pg-compact, there exists $\{A_1, A_2,..., A_n\}$ such that $G \subseteq \cup\{f^{-1}(A_k) : k = 1, 2,\ldots, n\}$. Then $f(G) \subseteq \cup\{f(f^{-1}(A_k)) : k = 1,2,\ldots,n\}$. Hence, $f(G) \subseteq \cup\{(A_k) : k = 1, 2,..., n\}$. Therefore, f(G) is a #pg-compact space relative to Y.

6 REFERENCES

1. Andrijvic. D., Semi-pre open sets, Mat. Vesnik, 38(1986), 24-36.
2. Arya. S. P. and Nour. T., Characterizations of s-normal spaces, India J. Pure. Appl. Math., 21(8)(1990), 717-719.
3. Benchalli. S. S. and Wali. R. S., On RW-Closed sets in topological spaces, Bull, Malays. Math. Sci. Soc (2) 30(2) (2007), 99-110.
4. Caldas. M. and Ekici. E., Slightly γ continuous functions, Bol. Soc , Parana Mat (3) 22 (2004) No.2, 63-74.
5. Cameron. D. E., Properties of s-closed spaces, Proc. Amer. Math 29(1987), 376-382.
6. Devi. R, Balachandran. K and Maki. K., Semi generalized homeomorphisms and generalized semi - homeomorphisms in topological spaces. Indian J. Pure. Appl. Math., 26(3):271:284, 1995.
7. Dontchev. J., On generalizing semi-pre-open sets. Mem. Fac. Sci. Kochi Univ. ser. A Math., 16:35:48, 1995.
8. Dontchev. J. and Noiri. T., Quasi-normal spaces and πg-closed sets, Acta Math. Hungar. 89(3) (2000), 211-219.
9. James R. Munkers, The topology, Prentice Hall of India private Limited, New Delhi
10. Levine. N., Generalized closed sets in topology, Rend. Circ. Mat. Palermo 19(1970), 89-96.
11. Levine. N., Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70(1963), 36-41.
12. Mashhour. A. S., Abd El-Monsef. M. E. and El-Deeb. S. N., On precontinuous and weak precontinuous mappings, Proc. Math. Phys. Soc. Egypt, 53 (1982), 47–53.
13. Nagaveni. N., Studies on Generalizations of Homeomorphisms in Topological Spaces, Ph.D. Thesis, Bharathiar University, Coimbatore, 1999.
14. Nijastad. O., Some classes of nearly open sets, Pecific J. Math. 15(1965), 961-970.
15. Pushpalatha. A., Studies on Generalizations of Mappings in Topological Spaces, Ph.D. Thesis, Bharathiar University, Coimbatore, 2000.
16. Dr. Salim Dawood Mohisn and Muntaha Khudair Abbas., On #RG-Compact spaces.
17. Stone. M., Application of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc. 41(1937), 374 - 481.
18. Syed Ali Fathima. S and Mariasingam. M., On #regular generalized closed sets in topological spaces. Int. J. Math. Archive, 2(11):2497:2502, 2011.
19. Syed Ali Fathima. S and Mariasingam. M., On #RG-Continuous and #RG-Irresolute functions (To be appear in Journal of Advanced Studies in Topology).
20. Tong. J., Weak almost continuous mapping and weak nearly compact spaces, Boll. Un. Mat. Ital. 6(1982), 385-391.
21. Zaitsav V., On certain classes of topological spaces and their bicompactifications. Dokl. Akad. Nauk SSSR 178 (1968), 778-779.

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