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IMPROVING THE CONDITION OF TRANSPORTATION IN SALEM CITY USING GAME THEORETICAL APPORACH

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ABSTRACT

We live in a modern era where technology has its hands raised in almost all sectors. Although technological development has been rapid in the past decade, a city does not require only technology for becoming an urban area. In this paper, data is collected from residents of Salem, Tamil Nadu, India about the transportation conditions in their respective areas. Factors such as travel expenses, condition of roads and the frequency of transportation are taken into account. Game Theoretical approach is used to solve the model based on which the factor that needs development to the most regarding transportation is found out. The ultimate aim is to reduce the problems in transportation so that this becomes the first step towards the dream of smart city.

Key Words: *Transportation, matrix of losses and function of risk.*

2010 Mathematics Subject Classification: *90C70, 90C26.*

1. INTRODUCTION

When mathematical models are used to make decisions, certain preliminary activities are need to be done. First, one should determine the parameters which affect the final result. For every parameter one needs to define the value intervals which are of significance for the studied problem.

Transportation in a city is always the most important factor for the people. To serve the needs of increasing population, several plans are implemented in the condition of transportation. But these schemes are not well maintained throughout the entire city. This is leading us towards bigger problems like increased dependence upon private sector for transport, travel fares, conditions of roads, increase in number of accidents on roads etc. Among these, the biggest issue that needs a solution today is the poor state of public transport.

The road conditions are severely damaged due to many factors such as laying pipelines, underground cables etc. Taking the development of condition of transportation as the central aim, in this paper a game theoretical approach is used based on [1], for which data has been collected from residents of Salem about the transportation condition that needs to be improved in their localities, to find out which factor has to be developed to the most ultimately so that it becomes a sophisticated area.

Game theory is the study of behaviour in conflict situations. Game theory also includes conflict situations that have almost no relation to games. In order to use the data in game models they have to be properly processed, if the problem has the property of a conflict situation, game theory can be used [5].

Game Theoretical approach is used to solve the model based on which the factor that needs development to the most regarding transportation is found out. The ultimate aim is to reduce the problems in transportation so that this becomes the first step towards the dream of smart city.

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1.1 PRELIMINARIES

Definition 1.1: [6] If X is a collection of objects denoted generally x, then a fuzzy set $\tilde{A}$ in X is defined as a set of ordered pairs $\tilde{A} = \{\frac{x,\mu_{\tilde{A}}(x)}{x} \in X\}$. Where $\mu_{\tilde{A}}(x)$ is called the membership function (or grade membership value of x) for the fuzzy set $\tilde{A}$. The membership function which maps each element of X to a membership value between 0 and 1.

Definition 1.2: [2] A fuzzy number $\tilde{A} = (a^L, a^U, \alpha_1, \alpha_2)$ is called a trapezoidal fuzzy number if its membership function meets the following mapping:

$$f(x) = \begin{cases} \frac{x-(a^L-\alpha_1)}{\alpha_1} & (a^L-\alpha_1) \le x \le a^L \\ 1 & a^L \le x \le a^U \\ \frac{(a^U+\alpha_2)-x}{\alpha_2} & a^U \le x \le (a^u+\alpha_2) \\ 0 & others \end{cases}$$

2. IMPLEMENTATION OF MATRIX GAMES FOR IMPROVING THE TRANSPORTATION

In this section, the working method of the problem in consideration is given in detail.

Initially, the general problems the residents face in terms of transportation are taken into account. Factors such as conditions of roads, maintenance of roads during adverse conditions, the preference of people towards public/private transport, the expenses of their travel etc are taken into consideration. The aim is to find out the factors that are to be developed in the city. In a city with more than hundreds of areas, different areas require different factors to be developed. So, in order to propose a consolidated outcome, here game theoretical model is used.

The structure of the game is taken as (A, B, L), where A denotes the group of people who have specified the developmental factors, B is the collection of those parameters and L is the payoff matrix of the game. Based on [3], the group of people are represented as $A = \{a_1, a_2, \ldots, a_i\}$where a_1 is the group of people who have the notion that all the factors are best in their area, a_2 is the group of people who have the view that all the parameters specified are moderate(somewhat lesser quality than mentioned by a_1) in their area and so on. The collection $B = \{b_1, b_2, \ldots, b_j\}$ represents the factors of maintenance, expenses and so on.

If the factors that needs development are not improved then there is a chance that people feel a little mutilated towards the introduction of new schemes on these factors. Without improving the existing flaws, if these factors are given some additional improvisation, the ultimate outcome will be a loss. The matrix of loss for these factors upon the group of residents is given below:

Table-1: Matrix of Losses

A \ B	b_1	…	b_j
a_1	$L(a_1, b_1)$	…	…
…	…	…	…
a_i	…	…	$L(a_i, b_j)$

Now, a collection of questions are given to the residents and based on their preferences they are grouped as $C = \{c_1, c_2, \ldots, c_k\}$, where c_1 denotes the residents who prefer the factor b_1 as good, c_2 denotes the residents who prefer the factor b_2 as good, and also special emphasis is given to c_k which denotes residents who do not have any opinion upon the development. The distribution of probabilities for $\{c_1, c_2, \ldots, c_k\}$ based on $\{a_1, a_2, \ldots, a_i\}$ is given by:

$$P\{c_i | a_j\}, i = 1,2, \ldots., k \text{ and } j = 1,2, \ldots., n$$

We now define a mapping $S : C \rightarrow B$ based on [4] which gives all the possible combinations of the resident's views on the transportation factors. The total number of combinations for the factors $B = \{b_1, b_2, \ldots, b_j\}$ based on the collection $C = \{c_1, c_2, \ldots, c_k\}$ will be j^k which are given in the collection $S = \{s_1, s_2, \ldots, s_k,$ where $m = j^k\}$

Table-2: Selected sample of correspondent factors

C \ S	s_1	s_2	s_3	s_4	s_5	…
c_1	b_1	b_1	b_1	b_1	b_2	…
c_2	b_2	b_2	b_2	b_1	b_2	…
…	…	…	…	…	…	…
c_4	b_1	b_2	b_3	b_1	b_2	…

From the matrix of loss of Table 1 and the probability distribution we calculate the functions of risk as follows:

$$R(a_i, s_j) = P(c_i|a_j)L(a_i, s_j(X))$$

If there is no saddle point in the matrix of risk, we use linear programming problem method to solve the optimal solution.

3. ESTABLISHMENT OF THE PROPOSED MODEL FOR THE DATA COLLECTED FROM PEOPLE

Initially the data was collected from the residents about the condition of transportation in their area. Based on their opinions it is found that the factors such as conditions of roads, preference for public/private transport, expenses they bear for travel, frequency of the transportation were mostly having oscillated views among various residents. Hence, the game theoretical model is applied to these strategies to find out which factor among these must be improved still.
The collected data is formulated in the following manner:

Let the set of decisions be $B = \{b_1, b_2, b_3\}$, where, b_1 corresponds to the condition of roads in the city, , b_2 is the expenses of travel, b_3 is the frequency of the transportation in the city. B is the collection of all these parameters.

Let the people who prefer each of these factors be grouped as $A = \{a_1, a_2, a_3\}$ where, a_1 is the group of people who have best views upon most of the strategies, a_2 is the group of people who have good views upon all the strategies and a_3 is the group of people who have poor opinion upon all the strategies. We now calculate the matrix of loss. For example, if we consider the condition of roads in a Kitchipalayam area which is in a bad condition, the condition of that road must be developed first. Without developing the existing condition of roads, if new routes are being laid in the same area, it is doubtful whether that will be welcomed by the residents, which is taken as the loss. Similarly, the loss for other factors such as expenses, frequency of transportation are considered. This matrix of loss $L(A, B)$ is presented in the following table:

Table-1: Matrix of Losses

A \ B	b_1	b_2	b_3
a_1	5	10	0
a_2	0	5	5.
a_3	5	10	10

Now, the people who gave their views on the transportation factors are grouped as $C = \{c_1, c_2, c_3, c_4\}$ where, c_1 denotes that the people of the city prefer the factor b_1 as good, c_2 denotes that the people of the city prefer the factor b_2 as good, c_3 denotes that the people of the city prefer the factor b_3 as good, and finally, c_4 denotes the people who have no preference for any of the factors.

Let the probability distribution of the results $\{c_1, c_2, c_3, c_4\}$ depending on the initial data $\{a_1, a_2, a_3\}$ be:
$P(c_1|a_1)$ = 0.2; $P(c_1|a_2)$ = 0.7; $P(c_1|a_3)$ = 0.09; $P(c_2|a_1)$ = 0.07; $P(c_1|a_2)$ = 0.1; $P(c_2|a_3)$ = 0.4; $P(c_3|a_1)$ = 0.7; $P(c_3|a_2)$ = 0.1; $P(c_3|a_3)$ = 0.4; $P(c_4|a_1)$ = 0.03; $P(c_4|a_2)$ = 0.1; $P(c_4|a_3)$ = 0.01

To find the optimal strategy, we now introduce a function which maps C to B, so that each and every view of the residents'are analysed. Since there are 3^4 = 81 combinations, instead of analysing all these we consider the trivial combinations of their views for the factors. Let them be denoted as $S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$ these strategies are the combination of the factors that are taken into consideration and which are represented in a matrix as follows:

Table-2: Selected sample of correspondent factors

C \ S	s_1	s_2	s_3	s_4	s_5	s_6
c_1	b_1	b_1	b_1	b_1	b_2	b_3
c_2	b_2	b_2	b_2	b_1	b_2	b_3
c_3	b_3	b_3	b_3	b_1	b_2	b_3
c_4	b_1	b_2	b_3	b_1	b_2	b_3

Now, the formula for the function of risk is given by
$R(a_i, s_j) = P(c_i|a_j)L(a_i, s_j(X))$
$R(a_1, s_1)$ = 5.(0.2) + 10.(0.07) + 0.(0.7) + 5.(0.03) = 1.85
$R(a_2, s_1)$ = 0.(0.7) + 5.(0.1) + 5.(0.1) + 0.(0.1) = 1
$R(a_3, s_1)$ = 5.(0.09) + 10.(0.4) + 10. (0.4) + 5.(0.01) = 8.5
and similarly other functions of risk can be calculated to obtain the matrix.

The function of risk $R(A,S)$ for the selected s_i is given

$$\begin{array}{lc}
A & s_1 \quad s_2 \quad s_3 \quad s_4 \quad s_5 \quad s_6 \quad Rowmin \\
\begin{array}{l} a_1 \\ a_2 \\ a_3 \end{array} & \begin{pmatrix} 1.85 & 2 & 1.7 & 5 & 10 & 0 \\ 1 & 1.5 & 1.5 & 0 & 5 & 5 \\ 8.5 & 8.55 & 8.55 & 4.5 & 9 & 9 \end{pmatrix} \begin{array}{c} 0 \\ 0 \\ 4.5 \end{array} \\
ColumnMax & 8.5 \quad 8.55 \quad 8.55 \quad 5 \quad 10 \quad 9
\end{array}$$

Since, there is no saddle point, we now proceed to solve this by fuzzy linear programming problem method [4]. Now, the probabilities for the strategies are introduced as $p_1 = P(s_1)$, $p_2 = P(s_2)$, $p_3 = P(s_3)$, $p_4 = P(s_4)$, $p_5 = P(s_5)$, $p_6 = P(s_6)$.

The fuzzy linear programming problem for the strategies based on the probabilities is given by,
Min x = (9,10,13,14)x_1 + (11,13,14,15) x_2 + (10,12,13,14) x_3 + (7,8,10,11) x_4 + (22,23,25,26) x_5 + (11,12,13,15) x_6
subject to the constraints,
1.85 p_1 + 2 p_2 + 1.7 p_3 + 5 p_4 + 10 $p_5 \leq x$
p_1 + 5 p_2 + 1.5 p_3 + 5 p_5 + 6 $p_6 \leq x$
8.5 p_1 + 8.55 p_2 + 8.55 p_3 + 4.5 p_4 + 9 p_5 +9 $p_6 \leq x$
p_1 + p_2 + p_3 + p_4 + p_5 + p_6 = 1
$p_1 \geq 0, p_2 \geq 0, p_3 \geq 0, p_4 \geq 0, p_5 \geq 0, p_6 \geq 0$

Since $x > 0$, the above three inequalities are divided by x and a new variable $y_i = \frac{p_i}{x}$ is introduced.

Hence, we obtain the new fuzzy linear programming problem
Max z = (9,10,13,14)y_1 + (11,13,14,15) y_2 + (10,12,13,14) y_3 + (7,8,10,11) y_4 + (22,23,25,26) y_5 + (11,12,13,15) y_6
subject to the constraints,
1.85 y_1 + 2 y_2 + 1.7 y_3 + 5 y_4 + 10 $y_5 \leq 1$
y_1 + 5 y_2 + 1.5 y_3 + 5 y_5 + 6 $y_6 \leq 1$
8.5 y_1 + 8.55 y_2 + 8.55 y_3 + 4.5 y_4 + 9 y_5 +9 $y_6 \leq 1$
$y_1 \geq 0, y_2 \geq 0, y_3 \geq 0, y_4 \geq 0, y_5 \geq 0, y_6 \geq 0$

Using the fuzzy LPP method, the following solution is obtained.
y_1 = 0, y_2 = 0, y_3 = 0, y_4 = 0, y_5 = 0.1, y_6 = 0.01
z_1 = 0.11 and hence the value of the game x = 9.09

Now, $y_i = \frac{p_i}{x}$, p_1 = 0, p_2 = 0, p_3 = 0, p_4 = 0, p_5 = 0.1, p_6 = 0.01

Thus, probability distribution is
$P\{s_5\}$ = 0.90, $P\{s_6\}$ = 0.09

This means that the final strategy must be determined based on the strategies s_5 and s_6. Now, from Table 2, we can see that s_5 and s_6 differ in all the entries. Hence the final decision is to choose between b_2 and b_3. Now the probability of s_5 is greater than the probability of s_6. Hence, the strategy b_2 is in the best condition. Thus, the conditions of roads must be improved and the frequency of transportation must be increased in and around the city of Salem, so that the residents welcome all other new plans.

4. CONCLUSION

Game theory plays a major role in decision making process. The strategies are fixed in such a way that each and every conditions are taken into account. In this paper, as a part of aiming to change Salem as a smart city the factors that needs development regarding transportation are taken into account. Based on the residents' views the strategies are framed and the final solution is given. This solution is provided for the situation when there is a dilemma between the selection of factors that requires development.

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