

APPLICATION OF FUZZY MATRICES IN TAPIOCA CULTIVATION

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INTRODUCTION

The concept of fuzzy set was first introduced by Zadeh [20] in 1965. Fuzzy set theory is an extension of classical set theory. The concept of fuzzy relation on a set was defined by Zadeh [20, 21]. In the last thirty years Bell [1], Dubois and Prade [2], Kerre [5], Lowen [8], Meenakshi *et al.*[9], [10], [11], Roesenfeld [14], Zimmermann [22] and others have extended the ideas of fuzzy set theory to topology, algebra, Hilbert spaces, graphs, games theory, logic and computing, etc. The basic concept of fuzzy matrices introduced by Vasantha Kandasamy W.B., Florentin Smarandache and Ilanthenral K. [17, 18] are studied. They gave the basic notions of matrices, the properties of fuzzy matrices and graphical presentation. This paper describes a simple (yet powerful) methodology for decision making based on fuzzy sets. The paper proposes a multi level fuzzy evaluation function which will totally order a number of tapioca growing district alternatives in a zone. It demonstrates how to carry out production in a profitable manner and allocation of land for selecting suitable cropping options in agriculture. The domain is interesting because decision makers base their choices on a wide range of crops for example, cash crops, food crops, non- food crops, oil seeds, fruits and vegetables etc. this paper attempts to study, with the application of fuzzy sets which district is best suited for maximum tapioca production which is a much luring crop. For the purpose of study, time series agriculture data from published sources has been taken for six districts from north western zone during the period of ten years from 2006-07 to 2015-16.

2010 AMS Classification: 03E72.

2. PRELIMINARIES

Definition 2.1 [12]: Let \mathcal{M}_{mn} denote the set of all $m \times n$ matrices over \mathcal{J} . If $m = n$, in short, we write \mathcal{J}^n . Elements of \mathcal{M}_{mn} are called as membership value matrices, binary fuzzy relation matrices (or) in short, fuzzy matrices. Boolean matrices over the Boolean algebra $\{0, 1\}$ are special types of fuzzy matrices.

Definition 2.2 [12]: Let $A = (a_{ij}) \in \mathcal{M}_{mn}$. Then the element a_{ij} is called the (i, j) entry of A . Let A_i^* (A^*_j) denote the i th row (j th column) of A .

Definition 2.3 [12]: The $m \times n$ zero matrix O is the matrix all of whose entries are zero. The $n \times n$ identity matrix I is the matrix (δ_{ij}) such that $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$. The $n \times m$ universal matrix J is the matrix all of whose entries are 1.

Definition 2.4 [17]: In certain fuzzy matrices we include $[1, -1]$ to be the fuzzy interval. So any element $a_{ij} \in [1, -1]$ can be positive or negative. If a_{ij} is positive then $0 < a_{ij} \leq 1$, if a_{ij} is negative then $-1 < a_{ij} \leq 0$; $a_{ij} = 0$ can also occur. So $[0, 1]$ or $[-1, 1]$ will be known as fuzzy interval. Thus if $A = (a_{ij})$ is a matrix and if in particular $a_{ij} \in [1, -1]$ we call A to be fuzzy matrix.

Example 2.1:

$$A = \begin{bmatrix} .3 & .1 & .4 & 1 \\ .2 & 1 & .7 & 0 \\ .9 & .8 & .5 & .8 \end{bmatrix}$$

Thus A is a fuzzy matrix

Definition 2.5 [18]: Let $X = X_1 \cup X_2 \cup \dots \cup X_M$ ($M \geq 2$) where each X_i is a $1 \times s$ fuzzy row vector/matrix then we define X to be a special fuzzy row vector / matrix ($i = 1, 2, \dots, M$). If in particular $X = X_1 \cup X_2 \cup \dots \cup X_M$ ($M \geq 2$) where each X_i is a $1 \times s_i$ fuzzy row vector/matrix where for atleast one $s_i \neq s_j$ with $i \neq j$, $1 \leq i, j \leq M$ then we define X to be a special fuzzy mixed row vector / matrix.

Definition 2.6 [18]: Let $Y = Y_1 \cup Y_2 \cup \dots \cup Y_m$ ($m \geq 2$) we have each Y_i to be a $t \times 1$ fuzzy column vector / matrix then we define Y to be a special fuzzy column vector / matrix. If in particular in $Y = Y_1 \cup Y_2 \cup \dots \cup Y_m$ ($m \geq 2$) we have each Y_i to be $t_i \times 1$ fuzzy column vector where atleast for one or some $t_i \neq t_j$ for $i \neq j$, $1 \leq i, j \leq m$ then we define Y to be a special fuzzy mixed column vector/matrix.

3. THE METHOD OF APPLICATION OF CETD MATRIX

In this section on the collected data a simple fuzzy matrix model with an effective technique is discussed. At first the collected raw data is transformed into a fuzzy matrix model. In the first stage, the raw data given in the matrix representation is converted into a time dependent matrix. Secondly, after obtaining the time dependent matrix, using the techniques of average and standard deviation the time dependent data matrix is converted into an Average Time Dependent Data Matrix (ATD Matrix) by taking districts along the rows and the years along the columns. ATD Matrix is obtained by dividing each entry of the raw data matrix by the number of years. This matrix represents a data which is totally uniform. At the third stage, using the average μ_j of each j th column and σ_j the Standard Deviation of each j th column, a parameter α has been chosen from the interval $[0, 1]$ and then the Refined Time Dependent Matrix (RTD Matrix) a_{ij} is formed using the formula:

$$\begin{aligned} \text{If } a_{ij} &\leq (\mu_j - \alpha * \sigma_j) && \text{then } e_{ij} = -1 \text{ else} \\ \text{If } a_{ij} &\in (\mu_j - \alpha * \sigma_j, \mu_j + \alpha * \sigma_j) && \text{then } e_{ij} = 0 \text{ else} \\ \text{If } a_{ij} &\geq (\mu_j + \alpha * \sigma_j) && \text{then } e_{ij} = 1 \end{aligned}$$

Where a_{ij} 's are the entries of the ATD Matrix.

The ATD Matrix is thus converted into the Refined Time Dependent Data (RTD) Matrix. This fuzzy matrix has the entries $-1, 0$ and 1 . Now, the row sum of this matrix gives the maximum production of districts which is prone to good performance of tapioca production. We can also combine these matrices by varying $\alpha \in [0, 1]$ so that the Combined Effective Time Dependent Data (CETD) Matrix is obtained. The row sum is found out for the CETD matrix and conclusions are derived based on the row sums. All these are represented by graphs and graphs play a vital role in exhibiting the data by the simplest means that can be even understood by a layman.

4. IDENTIFICATION OF MAXIMUM PRODUCTION OF TAPIOCA PRODUCTION IN NORTH WESTERN ZONE USING 5x8 MATRICES

Using the secondary data we have taken the following ten attributes (P_1, P_2, \dots, P_{10}) to our study.

- P_1 . 2006-07
- P_2 . 2007-08
- P_3 . 2008-09
- P_4 . 2009-10
- P_5 . 2010-11
- P_6 . 2011-12
- P_7 . 2012-13
- P_8 . 2013-14
- P_9 . 2014-15
- P_{10} . 2015-16

These attributes are taken as the columns of the initial raw data matrix. The tapioca production in districts Dharmapuri, Krishnagiri, Salem, Namakkal, Perambalur and Ariyalur are taken as the row of the matrix.

The tapioca cultivation in India accounts to 207.59 mn hectares in terms of area and 4372.68 mn tones of production. Yield is a good indicator which shows the fertility of area, intensity of cropping, climatic factors, environmental factors, profit, price and so on. In terms of yield rate Tamil Nadu state tops the list and the yield rate of tapioca in Tamil Nadu is double the time greater than the National average. Hence it is better understood that Tamil Nadu has good potential for tapioca cultivation. But certain limiting factors warrant certain zones which are familiar and best suited for this crop. Since the researcher has the curiosity to know the districts which are famous in tapioca production, North Western Zone was taken for the study purpose. North Western Zone comprises of Dharmapuri, Krishnagiri, Salem, Namakkal, Perambalur and Ariyalur districts. Fuzzy matrix was applied to know the top districts in the study zone.

Table-1: Initial Raw Data Matrix of Tapioca Production in North Western Zone of order 6x10

District	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}
Dharmapuri	932824	1215185	747565	661723	904040	588414	537900	409393	381274	359358
Krishnagiri	78662	32305	52482	44567	35109	49753	25475	9089	26016	12099
Salem	1080501	1178658	839930	749227	596363	865062	410175	317727	427563	424243
Namakkal	1236457	1210020	1237518	1007100	569893	981818	583911	586088	486393	461471
Perambalur	107892	156923	71850	48570	44986	63688	90231	89958	124123	62780
Ariyalur	0	0	18138	8858	7950	10282	5913	5655	9826	9162

Table-2: The ATD Matrix of Tapioca Production in North Western Zone of order 6×10

District	P ₁	P ₂	P ₃	P ₄	P ₅	P ₆	P ₇	P ₈	P ₉	P ₁₀
Dharmapuri	93282.4	121518.5	74756.5	66172.3	90404.0	58841.4	53790.0	40939.3	38127.4	35935.8
Krishnagiri	7866.2	3230.5	5248.2	4456.7	3510.9	4975.3	2547.5	908.9	2601.6	1209.9
Salem	108050.1	117865.8	83993.0	74922.7	59636.3	86506.2	41017.5	31772.7	42756.3	42424.3
Namakkal	123645.7	121002.0	123751.8	100710.0	56989.3	98181.8	58391.1	58608.8	48639.3	46147.1
Perambalur	10789.2	15692.3	7185.0	4857.0	4498.6	6368.8	9023.1	8995.8	12412.3	6278.0
Ariyalur	0	0	1813.8	885.8	795.0	1028.2	591.3	565.5	982.6	916.2

Table-3: The Average and Standard deviation of the above ATD Matrix

Average(μ)	68726.72	75861.82	49458.05	42000.76	172472.35	42650.28	27560.08	23631.83	24253.25	22151.88
Standard Deviation (σ)	49449.29	54373.12	47194.16	39988.64	330497.81	40285.21	24208.22	2180.44	19493.84	19656.38

Now by using the formula defined in section 3, we get the RTD Matrix for different parameters

The RTD Matrix for $\alpha = 0.1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}$$

The row sum matrix

$$\begin{bmatrix} 8 \\ -10 \\ 8 \\ 8 \\ -10 \\ -10 \end{bmatrix}$$

The RTD Matrix for $\alpha = 0.3$

$$\begin{bmatrix} 1 & -1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}$$

The row sum matrix

$$\begin{bmatrix} 7 \\ -9 \\ 6 \\ 6 \\ -8 \\ -10 \end{bmatrix}$$

The RTD Matrix for $\alpha = 0.5$

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}$$

The row sum matrix

$$\begin{bmatrix} 6 \\ -10 \\ 8 \\ 9 \\ -10 \\ -10 \end{bmatrix}$$

The RTD Matrix for $\alpha = 0.7$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & 0 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & 0 & -1 & -1 & 0 & 0 & -1 \\ -1 & -1 & -1 & -1 & 0 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}$$

The row sum matrix

$$\begin{bmatrix} 5 \\ -9 \\ 7 \\ 9 \\ -7 \\ -9 \end{bmatrix}$$

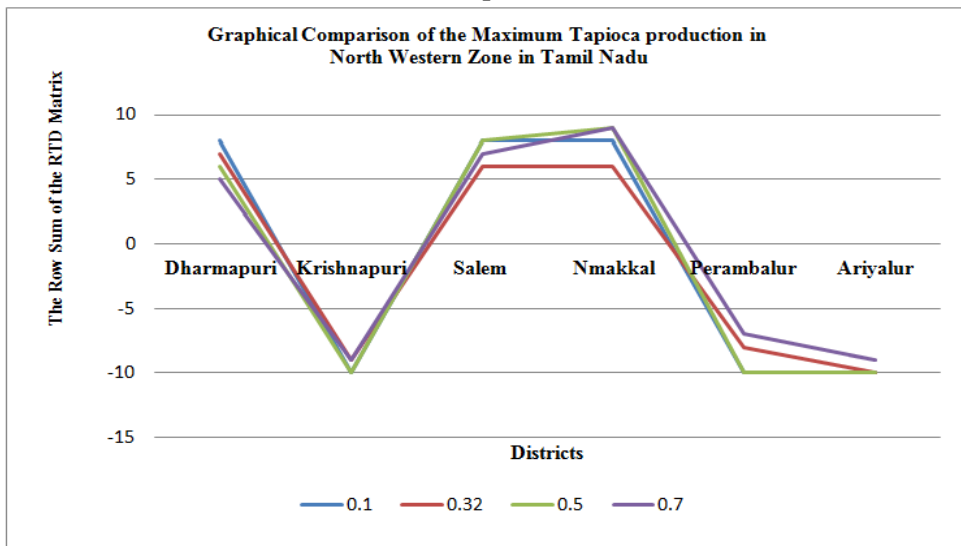
Now we combine these matrices by varying $\alpha \in [0, 1]$ and we get the Combined Effective Time Dependent Data (CETD) Matrix which is given as below.

CETD Matrix	The row sum
$\begin{bmatrix} 2 & 2 & 3 & 2 & -1 & 2 & 4 & 4 & 4 & 4 \\ -3 & -4 & -4 & -4 & -3 & -4 & -4 & -4 & -4 & -4 \\ 4 & 2 & 4 & 4 & -2 & 4 & 3 & 2 & 4 & 4 \\ 4 & 2 & 4 & 4 & -2 & 4 & 4 & 4 & 4 & 4 \\ -2 & -4 & -4 & -4 & -3 & -4 & -4 & -3 & -3 & -4 \\ -4 & -4 & -4 & -4 & -3 & -4 & -4 & -4 & -4 & -4 \end{bmatrix}$	$\begin{bmatrix} 26 \\ -38 \\ 29 \\ 32 \\ -35 \\ -39 \end{bmatrix}$

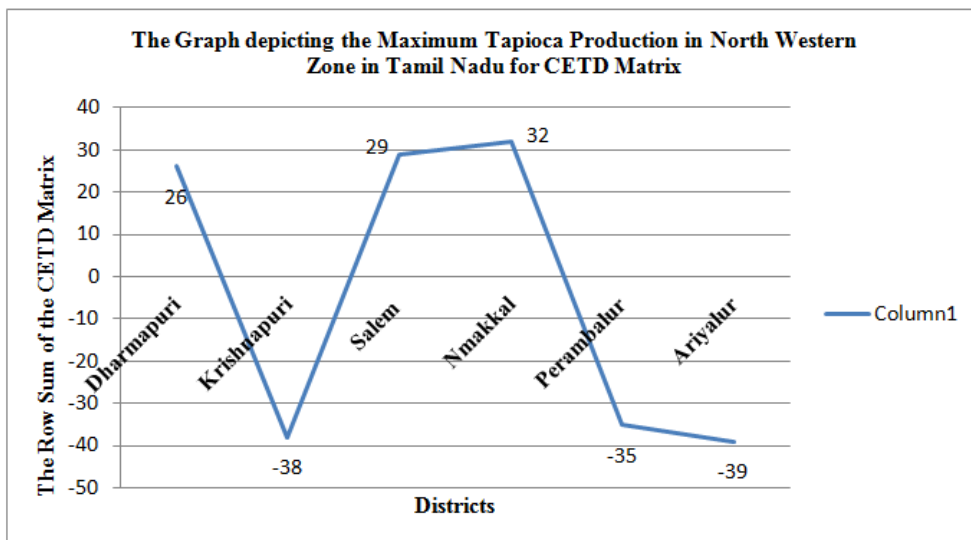
The graphical representations of the maximum tapioca production in North Western Zone in Tamil Nadu for different districts are given in the section 5.

5. GRAPHICAL REPRESENTATION OF THE MAXIMUM TAPIOCA PRODUCTION FOR DIFFERENT DISTRICTS

Graph-1



Graph-2



6. CONCLUSION AND SUGGESTIONS

Accordingly, through the analysis using the method of CETD matrix and its graph the researcher could observe that Namakkal district (P32) tops the list followed by Salem (P29) AND Dharmapuri (P26), in terms of production. This looks up will help the farmers to recognize how high production could be performed in the next coming season with the change in cropping pattern and diversification of crop towards tapioca production. Since tapioca productions has export intensity and fetch foreign exchange earnings to the Nation.

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**Source of support: National Conference on "New Trends in Mathematical Modelling" (NTMM - 2018),
Organized by Sri Sarada College for Women, Salem, Tamil Nadu, India.**