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FUZZY BI-MATRIX GAME THEORY APPROACH ON ROAD ACCIDENTS

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ABSTRACT

T*he paper is focusing on the most important domains of road accidents in India. A bi-matrix game model was used to compare the main alternatives, injuries and fatal or death. Aiming to individualized this game is a simultaneous game for two players in which each player has a finite number of possible actions the game can be described by two matrices and describing the payoffs for both players. There were used the concepts of content analysis in road accidents. We collected and processed a broad range of experimental data in an instructional case study for road accidents. The quantitative models allowed computing balanced strategies and deriving metrics and calculations useful for future assisted tutoring platforms.*

Keywords*: Bi-matrix game, Nash equilibrium points, Fuzzy max-min solution.*

2010 AMS Classification: *03E72.*

INTRODUCTION

Game theory is a mathematical approach that evaluates decision interplay among marketers, primarily based on evaluation of various occasions [5]. One of the desires of game theory is the computing of ultimate factors for unique content material, in which marketers act together. One of these is Nash point proposed by American mathematician John Nash [6]. When we apply the game theory to model some realistic problems which we come across in real situations, we ought to realize the values of payoffs precisely. In keeping with the Ministry of road transport and highways, 1,50,785 humans were killed and the other, 4,94,624 were injured in 4,80,652 road crashes in India in 2016. This interprets into 1317 crashes and 413 deaths each day or 55 crashes and 17 deaths each hour. "This reflects an alarming fashion that whilst the variety of accidents has gone long past down, their severity has elevated resulting in lots of more deaths. One of the major reasons for the excessive number of fatalities is speeding by the way of drivers," said a senior road ministry official. Road crash fatalities increased by means of 3% in the year 2016 (from 1, 46, 133 in 2015 to 1, 50, 785 in 2016) and accident severity 2 expanded from 29.1 in 2015 to 31.4 in 2016. The range of fatal accidents has multiplied continually since 2005 and noticed a pointed rise from 1, 31, 726 in 2015 to 1, 36, 071 in 2016. There has been a decline inside the number of road crash instances and injured individuals with the aid of 4.1% and 1.1% respectively. The intention of this paper is to provide a model of the road accidents in India to study the drunk and drive, over speed accidents on the vehicles, and to evaluate the optimal Nash point. The proposed method of this study cannot be received with the analytical techniques, but can be decided on with a kind of approach and an answer that is nearer to the computed Nash point.In this paper, we take into account the fuzzy bi-matrix games, namely, the games in which the number of participants are two and fuzzy payoffs.

PRELIMINARIES

Definition: [2] Game theory is a tool used to analyze strategic behavior by taking into account how participants expect others to behave. Game theory is used to find the optimal outcome from a set of choices by analyzing the costs and benefits to each independent party as they compete with each other.

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Definition: [3] In game theory, a **Bi-matrix game** is a simultaneous game for two players in which each player has a finite number of possible actions.

Definition: [4] In game theory, the **Nash equilibrium**, is a solution concept of a non-cooperative game involving two or more players in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only their own strategy. If each player has chosen a strategy and no player can benefit by changing strategies while the other players keep their unchanged, then the current set of strategy choices and the corresponding payoffs constitutes a Nash equilibrium.

GAME THEORY AND ROAD ACCIDENTS

Game theory deals with the situations in which payoff depends not only on our own choices but also on the choices of others. In this optimization problem, our payoff depends only on our own choices. Though the total number of road accidents has been lower in 2016 over the previous eight years, the number of persons killed has seen sharp increase in 2016 over 2015. Road accidents being the result of inter-play of multiple factors, multi-prong measures are needed to reduce the number of accidents and fatalities. The strategy is under implementation and substantial progress has been made towards putting in place necessary resources, programs and legislation for improving road safety scenario in the country. Over the years, 2005 to 2016 only marginal changes have taken place in terms of percentage share in number of road accidents, number of persons killed and injured within the various categories of roads.

The aim of the study is to known different types of road accidents: the two major cautions are deaths and injuries using game theory and its statistics. In our research we have the two different types of factors such as drunk and drive and over speed road accidents in India.

Road accidents are a negative externality associated with expansion in road network, motorization and urbanization in the country. The share of two wheeler user killed in accidents 34.8 per cent in 2016. During 2016, other road users killed in road accidents are cars, taxies, vans and other light and medium motor vehicles (17.9 per cent), trucks (11.2 per cent), pedestrians (10.5 per cent), buses (6.6 per cent), auto rickshaws (4.7 per cent) and others motor vehicles (10.6 per cent). Road users of non motorized vehicles like animal drawn vehicles, cycle rickshaws, handcarts, and other persons killed in road accidents accounted for 2.0% of total persons killed during 2016. Based on the extant data reporting system wherein the factor responsible for accidents are reported on the basis of subjective judgment of the reporter, drivers' fault is single most important factors responsible for road accidents (84 per cent), killings (80.3 per cent) and injuries (83.9) on all roads in the country during 2016 [7].

THEORETICAL BACKGROUND

The total road accidents are caused due to many reasons such as over speed, drunk and drive, overtaking, using mobile phones etc., Thus for the total accidental units we may choose the two factors as over speed and drunk and drive. In the year 2016, the total accidental unit (AU) can be divided into four accidental sub units (ASU) such as injuries and deaths for the two factors over speed and drunk and drive. In game theory, the utility function for a company is the profit, which is intended to be maximized, or it could be the relation among the costs of any kind, that has to be minimized. The utility function has to be optimized. In this case the people's participative and non-participative role was to be estimated in the four accidental units. For every ASU it has the following concepts: content units as for the top five states, record units and frequency units of each state as in the year 2016. In this study in every accidental unit (AU) the accident causes injuries and death, then the role of the people are participative and non–participative. The accidental injuries are over speed and drunk and drive injuries (especially for every ASU). The same thing to be for death: both the over speed and drunk and drive deaths, especially for every ASU.

In drunk and drive injuries, the approach to be taken as for the top five states in India on the content units. The role of the people is more participative in this case. The record units have recorded the number of accidents due to the drunk and drive. The frequency is very important in this case, because the accidents are happened in 2016. The over speed injuries are to be the same of drunk and drive. In this case, the record units are very important for the amount of accidents causing injuries and deaths. In the drunk and drive deaths is the case where the total number people are died in the accidents and over speed deaths are to be noted by the same recorded units in 2016. The people's role in the drunk and drive injuries are to be participative and non-participative. Generally not all the people are participative in the over speed injuries on road accidents. In figure1: shows the stages of accidents and its cautions in India.

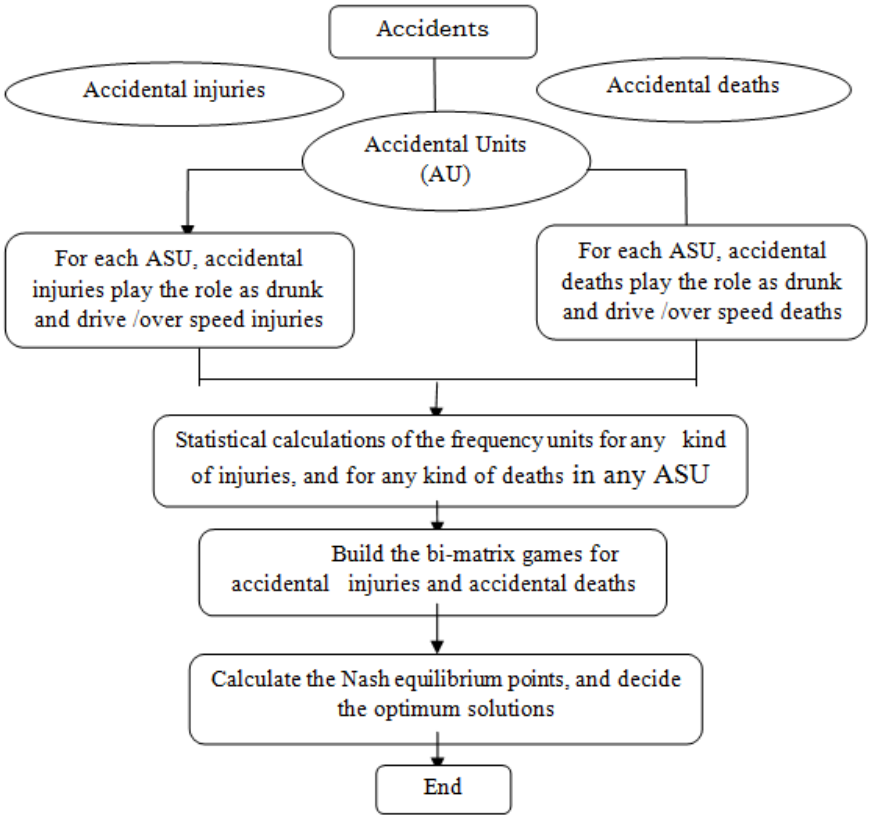

Figure-1: The stages of accidents and its cautions

Thus the necessary statistics are computed, which are followed by the Bi-matrix games and the results of the Nash points. Finally, the optimum results can be determined. The main aim is to build the Bi-matrix games: one for the injuries and the other for deaths. The accidental injuries may have two strategies: drunk and drive injuries and over speed injuries. (Figure: 2)

For the accidental injuries we define the following payoffs:

- A_{DI11} stands for the drunk and drive injuries with participative people
- A_{DI21} stands for the drunk and drive injuries with non-participative people
- A_{OI12} stands for the over speed injuries with participative people
- A_{OI22} stands for the over speed injuries with non–participative people.

The people can have the two strategies as participative and non-participative role. We define the payoffs:

- A_{AI11} stands for the average injuries on the participative role with the drunk and drive cases.
- A_{AI12} stands for the average injuries on the participative role with the over speed cases.
- A_{AI21} stands for the average injuries on the non-participative role with the drunk and drive cases
- A_{AI22} stands for the average injuries on the non-participative role with the over speed cases.

		Accidental injuries	
		Drunk and drive injuries	Over speed injuries
People	Participative	A_{AI11},A_{DI11}	A_{AI12},A_{OI12}
People	Non participative	A_{AI21}, A_{DI21}	A_{AI22}, A_{OI22}

Figure-2: Payoffs of the accidental injuries

For the construction of the bi-matrix game the accidents have the two strategies are drunk and drive and over speed deaths, (Figure: 3) having the payoffs:

- A_{DD11} stands for the drunk and drive deaths with participative people
- A_{DD21} stands for the drunk and drive deaths with non-participative people.
- A_{OD12} stands for the over speed deaths with participative people
- A_{OD22} stands for over speed deaths with non-participative people.

The people can choose the two different strategies: participative and non-participative. We define the payoffs:

- A_{AD11} stands for the average deaths on the participative role with drunk and drive cases
- A_{AD12} stands for the average deaths on the participative role with over speed cases.
- A_{AD21} stands for the average deaths on the non-participative role with drunk and drive cases
- A_{AD22} stands for the average deaths on the non- participative role with over speed cases.

		Accidental deaths	
		Drunk and drive death	Over speed death
People	Participative	A_{AD11},A_{DD11}	A_{AD12},A_{OD12}
	Non participative	A_{AD21}, A_{DD21}	A_{AD22}, A_{OD22}

Figure-3: Payoffs of the accidental deaths

For drunk and drive injuries the amount of A_{DI11} and A_{DI21} is

$$A_{DIm1} = \sum_{k=1}^{N} EV_k \tag{1}$$

where m = 1 or 2, N = number of states in the record units and EV_k is the specific frequency unit.

The difference between A_{DI11} and A_{DI21} is based on the participation of the people's.

For the over speed injuries the A_{OI12} and A_{OI22} can be calculated with the same equation (1). The difference between the payoff of the injuries, A_{DI11}, A_{DI21} and A_{OI12}, A_{OI22} is based on the sum of frequency units in drunk and drive injuries that is bigger than the over speed injuries.

For the people's payoffs we use the average amount of participation in all ASU. The amount of A_{AI11}, $A_{AI12,}$ A_{AI21} and A_{AI22} are different according to the people's participative and non participative role in the drunk and drive injuries and over speed injuries. For each case the utility of the people is calculated by:

$$A_{AIij} = \frac{\sum_{n=1} \frac{RR_n}{T_p}}{T_{ASU}} \tag{2}$$

where i, j =1 or 2, T_{ASU}= total number of ASU, T_p= total number of accidents,
RR_n = number of accidents in the states.

The death payoffs of the people are computed as:

$$A_{ADij} = \frac{\sum_{n=1} \frac{RR_n}{T_p}}{T_{ASU}} \tag{3}$$

where i, j =1 or 2 and it to be the same in the people's payoff of the injuries.

For the accidental payoffs in the accidental deaths we use the various degree of complexity in every ASUs. As for drunk and drive deaths in the accidental units shows the number of accidents. Here the people play a non-participative role. On the other hand, in an over speed deaths the people's participation is more active. The formula for computing the drunk and drive and over speed deaths are

$$A_{DDm1} = \frac{\sum_{s=1}^{T_k} H_s}{T_{ASU}} \tag{4}$$

where m = 1 or 2, T_{ASU}= total number of ASU, T_k = total number of states, H_s = number of accidents in each states. For over speed deaths A_{OD12}, A_{OD22} can also be calculated by using (4).

After a statistical determination of payoffs for the people and the accidents, we get the payoffs matrices for both of the games.

The injuries of drunk and drive and over speed accidents with the payoff matrix is

$$[A_{PI}] = \begin{bmatrix} A_{DI11} & A_{OI12} \\ A_{DI21} & A_{OI22} \end{bmatrix} \tag{5}$$

$$[A_{AI}] = \begin{bmatrix} A_{AI11} & A_{AI12} \\ A_{AI21} & A_{AI22} \end{bmatrix} \tag{6}$$

The deaths of drunk and drive and over speed accidents with the payoff matrix is

$$[A_{PD}] = \begin{bmatrix} A_{DD11} & A_{OD12} \\ A_{DD21} & A_{OD22} \end{bmatrix} \tag{7}$$

$$[A_{AD}] = \begin{bmatrix} A_{AD11} & A_{AD12} \\ A_{AD21} & A_{AD22} \end{bmatrix} \tag{8}$$

EXPERIMENTAL RESULTS

The experimental part of the study is based on the road accidents with the two factors such as injuries and deaths. The first type was meant to use two strategies for drunk and drive injuries and over speed injuries, and the second type was meant for drunk and drive and over speed deaths. The object of the study was injuries and deaths with the implementation of drunk and drive and over speed. The accidental unit (AU) can be divided into four accidental subunits (ASU) as injuries causing due to drink and drive and over speed, deaths causing due to drink and drive and over speed.

Most of the fatal accidents occur due to over speeding. A vehicle moving on high speed will have greater impact during the accident and hence may cause more injuries. During 2016, within the category of drivers' fault, accidents caused and persons killed due to 'Exceeding lawful speed', accounted for a high share of 66.5 per cent (2,68341 out of 4,03,598 accidents) and 61.0 per cent (73,896 out of 1,21, 126 deaths), respectively. However taking into account the total road accidents and total road accident killings, the share of over speeding comes to 55.9 per cent (2,68, 341 out of 4,80,652 accidents) and 49.0 per cent (73,896 out of 1,50,785 deaths) respectively.

Intake of alcohol and drugs by drivers reduces concentration and cause accidents and many times it proves fatal. Intake of alcohol/drugs by drivers resulted in 14,894 road accidents and 6,131 fatalities in 2016. Within the category of drivers' fault, intake of alcohol/drugs accounted for 3.7 per cent and 5.1 per cent respectively. However taking into account the total road accidents and total road accident killings, the share of intake of alcohol/drugs comes to 3.1 per cent (14,894 out of 4,80,652 accidents) and 4.1 per cent (6,131 out of 1,50, 785 deaths) respectively. The number of accidents in the record units is shown in Table 1[7]. The record units are taken by top five States in India are: Tamil Nadu, Madhya Pradesh, Karnataka, Maharashtra, and Kerala. The importance of these concepts for the accidental units is illustrated.

Total record units from the Table1 represent the payoffs of the accidents can be calculated with (1). The average of the accidents for each subunits ASU stands for payoffs of the drunk and drive and over speed participants of the people, based on (2) as shown in the below Table 2. The rates are given from 1 (minimum) to 10 (maximum).

Table-1: Frequency record units in each states based on record units appearances

Records units of the states	AU			
	Drunk and drive injuries		Over speed injuries	
	Participative	Non-participative	Participative	Non– participative
Tamil Nadu	169	508	41173	11613
Madhya Pradesh	1197	703	17574	12213
Karnataka	103	256	26399	12424
Maharashtra	122	183	11697	10374
Kerala	872	2033	1912	4064
Total record units	2463	3683	98755	38264

Table-2: The average of each condition at the end of each ASU

	AU			
	Participative	Non-participative	Participative	Non-participative
Average of each Subunits causing injuries	4.419	2.484	2.974	4.843

For the accidental deaths we used the same scenario with four ASU. The amount of payoffs for the accidental deaths is shown in Table 3 [7].

Table-3: Accidental rates over the deaths

	AU			
	Drunk and drive deaths		Over speed deaths	
	Participative	Non–participative	Participative	Non–participative
Tamil Nadu	21	49	8926	2518
Madhya Pradesh	307	170	3060	2128
Karnataka	56	68	5456	2572
Maharashtra	43	35	4127	3660
Kerala	894	1902	1750	3721
Total accidents	1324	2224	23328	14599

Table-4: Average of the accidental unit and the subunits

	AU			
	Drunk and drive deaths		Over speed deaths	
	Participative	Non-participative	Participative	Non-participative
Average of the accidental deaths	1.031	1.434	1.256	1.034
Average of each subunits causing deaths	3.310	5.560	5.832	3.649

The payoffs for the accidental units with injuries will be:

$$\begin{bmatrix} A_{DI11} & A_{OI12} \\ A_{DI21} & A_{OI22} \end{bmatrix} = \begin{bmatrix} 2.463 & 9.875 \\ 3.683 & 3.826 \end{bmatrix}$$

For average of the accidental injuries

$$\begin{bmatrix} A_{AI11} & A_{AI12} \\ A_{AI21} & A_{AI22} \end{bmatrix} = \begin{bmatrix} 4.419 & 2.974 \\ 2.484 & 4.843 \end{bmatrix}$$

The payoffs for the accidental units with deaths:

$$\begin{bmatrix} A_{DD11} & A_{OD12} \\ A_{DD21} & A_{OD22} \end{bmatrix} = \begin{bmatrix} 3.310 & 5.832 \\ 5.560 & 3.649 \end{bmatrix}$$

For average of the accidental deaths

$$\begin{bmatrix} A_{AD11} & A_{AD12} \\ A_{AD21} & A_{AD22} \end{bmatrix} = \begin{bmatrix} 1.031 & 1.256 \\ 1.434 & 1.034 \end{bmatrix}$$

FUZZY MAX-MIN SOLUTION [1]

Thus the matrix can be taken as a fuzzy matrix and we make use of the max-min solution methods in the fuzzy relation approach. The fuzzy matrix can be written as

$$\text{A} = \begin{bmatrix} (2.463,\ 3.073,\ 3.683) & (3.826,\ 6.851,\ 9.875) \\ (3.310,\ 4.435,\ 5.560) & (3.649,\ 4.741,\ 5.832) \end{bmatrix}$$

Here the *x*-axis $(x_1,\ x_2)$ is taken as injuries and deaths of the accidents and the *y*-axis $(y_1,\ y_2)$ is taken as drunk and drive and over speed accidents. In the fuzzy relation we can take $\alpha \in [0, 1]$, then the road environment α are climatic changes, road shapes etc,. The membership function is

$$\mu_A(x) = \begin{cases} 0 & x \le 3.073 \\ \dfrac{x - 3.073}{3.778} & 3.073 \le x \le 6.851 \\ 1 & x \ge 6.851 \end{cases}$$

The max-min solution as,

Min α subject to $\dfrac{3.683y_1 + 9.875y_2 - 3.073}{0.61y_1 + 3.024y_2 + 3.778} \le \alpha$

$$\frac{5.560y_1 + 5.832y_2 - 3.073}{1.125y_1 + 1.091y_2 + 3.778} \le \alpha$$

$y_1 + y_2 = 1$ and $y_1 \ge 0, y_2 \ge 0$.

We compute the max-min solution and obtained the solution by the fuzzy relation approach for different values of road environments α are

$\alpha = 0, \quad y_1 = 0.6829, \; y_2 = 0.3171$

$\alpha = 0.2, \quad y_1 = 0.7232, \; y_2 = 0.2768$

$\alpha = 0.4, \quad y_1 = 0.7578, \; y_2 = 0.2422$

$\alpha = 0.6, \quad y_1 = 0.7878, \; y_2 = 0.2122$

$\alpha = 0.8, \quad y_1 = 0.8140, \; y_2 = 0.1860$

$\alpha = 1, \quad y_1 = 0.8372, \; y_2 = 0.1628$

Here the above values are taken as the probabilities for drunk and drive and the over speed accidents making injuries and deaths in the accidental units (AU).

RESULTS

For drunk and drive we get the probability of 0.6829, it is approximately 70% of the accidental unit and 30% for over speed accidents making injuries and deaths for the first road environment $\alpha = 0$. In our experiment the AU has four ASU, so three of them should be for the drunk and drive and only one for over speed. The accident has four variants which are presented in Table 5 (DD represents drunk and drive and OS represents over speed).

Table-5: The order of drunk and drive alternating with over speed in the AU with four variants

Variants	ASU	ASU	ASU	ASU
1	DD	DD	DD	OS
2	DD	DD	OS	DD
3	DD	OS	DD	DD
4	OS	DD	DD	DD

In a similar manner, the probabilities of drunk and drive accidents and over speed accidents for different values of road environments α can be computed.

Thus from the above discussions the drunk and drive accidents may causes more injuries and deaths comparing to the over speed accidents.

CONCLUSION

This paper provided a framework for analyzing the road accidents in India, with the use of game theory. The level of information provided to the road accidents and it may affect the game structure. This paper tries to propose a methodology for injuries and deaths in the accidental units using the fuzzy bi- matrix game theory with two players. This research is one of the implementation of accidental platforms. For this purpose we used the data collections and processing. All this information was processed using statistical averages, and extracted from this payoffs for both accidents and people's role. In this case, the study of road accidents is based on drunk and drive and over speed accidents causing injuries and deaths with the people's participative and non- participative role. At last, the drunk and drive accidents may causes more injuries and deaths with the people's participative role and non-participative role. Then we may have the preventive measures to avoid the road accidents such as,

1. Education and awareness about road safety
2. Strict Enforcement of Law
3. Engineering: (a) Vehicle design (b) Road infrastructure

The control of all accidents is, in the first instance, the responsibility of the drivers and personnel of the affected means of transport. It is up to them to limit the resulting damage as much as possible.

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