

$(1, \alpha)$ – DERIVATIONS IN Γ – NEAR RING

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ABSTRACT

In this paper, we introduce the notion of $(1, \alpha)$ - derivation of Γ – near ring and give some generalizations of [1]. The purpose of this paper is to prove the following two assertions: (i) Let M be a semiprime Γ – near ring, U be a subset of M such that $0 \in U, U\Gamma M \subseteq U$ and d be a $(1, \alpha)$ -derivation of M . If d acts as homomorphism on U or as antihomomorphism on U under certain conditions on α , then $d(U) = \{0\}$. (ii) Let M be a prime Γ – near ring, U be a nonzero semigroup ideal of M , and d be a $(1, \alpha)$ derivation on M . If $d(x + y - x - y) = 0$ for all $x, y \in U$, then $(M, +)$ is abelian.

Key words: prime Γ – near ring, semiprime Γ – near ring, semigroup ideal, $(1, \alpha)$ – derivation.

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1. INTRODUCTION

The notion of a Γ – ring a concept more general than a ring was defined by Nobusawa [6]. As a generalization of near rings, Γ – near ring introduced by Satyanarayana [7]. The derivations in Γ – near rings has been introduced by Bell and Mason [3]. They obtained some basic properties of derivations in, Γ – near ring. In [2] Argac defined a two sided α – derivation of a Γ – near ring. In a similar way, we introduce the notion of a $(1, \alpha)$ – derivation of Γ – near ring and give some Generalizations of [1, 2].

2. PRELIMINARIES

Throughout this paper, M stands for a zero symmetric right Γ – near ring. In this section, we collect all basic concepts and results in Γ – near rings mostly from Mustafa Kazaz and Akin Alkan [5] which are required for our study.

Definition 2.1[5]: A Γ – near ring M is a triple $(M, +, \Gamma)$, where

- (i) $(M, +)$ is a group (not necessarily abelian),
- (ii) Γ is a non empty set of binary operations on M such that $(M, +, \gamma)$ is a near ring for each $\gamma \in \Gamma$.
- (iii) $(x\beta y)\gamma z = x\beta(\gamma yz)$ for all $x, y, z \in M$ and $\beta, \gamma \in \Gamma$

Definition 2.2[5]: A Γ – near ring M is said to be **zero-symmetric** Γ – near ring if $0\gamma x = 0$ for all $x \in M$ and $\gamma \in \Gamma$.

Definition 2.3[5]: A Γ – near ring M is said to be **prime Γ – near ring** if $x\Gamma M\Gamma y = \{0\}$ for $x, y \in M$ implies $x = 0$ or $y = 0$ and **semiprime Γ – near ring** if $x\Gamma M\Gamma x = \{0\}$ for $x \in M$ implies $x = 0$.

Definition 2.4[4]: A **derivation** on M is defined to be an additive endomorphism d of M satisfying the product rule
 $d(x\gamma y) = d(x)\gamma y + x\gamma d(y)$ for all $x, y \in M$ and $\gamma \in \Gamma$, or equivalently
 $d(x\gamma y) = x\gamma d(y) + d(x)\gamma y$ for all $x, y \in M$ and $\gamma \in \Gamma$

Definition 2.5[5]: If M and M' are two Γ – near rings, then a mapping $f: M \rightarrow M'$ such that $f(x + y) = f(x) + f(y)$ and $f(x\gamma y) = f(x)\gamma f(y)$ (resp. $f(x\gamma y) = f(y)\gamma f(x)$) for all $x, y \in M$ and $\gamma \in \Gamma$, is called a Γ – near rings **homomorphism** (resp. an **anti-homomorphism**) on M .

Definition 2.6[5]: Let S be a nonempty subset of M and let d be a derivation on M . If $d(x\gamma y) = d(x)\gamma d(y)$ (resp. $d(x\gamma y) = d(y)\gamma d(x)$) for all $x, y \in S$ and $\gamma \in \Gamma$, then d is said to act as a **homomorphism** (resp. an **anti-homomorphism**) on S .

Definition 2.7[5]: An additive endomorphism $d: M \rightarrow M$ of a Γ – near ring M is called a (α, β) derivation on M if there exist two functions $\alpha, \beta: M \rightarrow M$ such that the following product rule holds: $d(x\gamma y) = d(x)\gamma\alpha(y) + \beta(x)\gamma d(y)$ for all $x, y \in M$ and $\gamma \in \Gamma$. One can easily show that if d is (α, β) derivation on M Such that $\alpha(x + y) = \alpha(x) + \alpha(y)$ and $\beta(x + y) = \beta(x) + \beta(y)$, then $d(x\gamma y) = \beta(x)\gamma d(y) + d(x)\gamma\alpha(y)$.

Definition 2.8[5]: An additive mapping $d: M \rightarrow M$ is called a two-sided α – derivation if d is $(\alpha, 1)$ derivation as well as $(1, \alpha)$ -derivation. We should note that if $\alpha = 1$, then a two sided α – derivation is just a derivation.

Definition 2.9 [4]: A nonempty subset U of M is called a **semigroup right ideal** (resp. **semigroup left ideal**) if $U\Gamma M \subset M(\Gamma U)$ is called **semigroup ideal** if it is both right and left semigroup ideal.

3. (1, α)- derivation in prime and semiprime Γ – near rings

We need the following Lemmas to prove the main theorem of this section.

Lemma 3.1: Let M be a prime Γ – near ring and let U be a nonzero semigroup ideal of M . If $a + b = b + a$ for all $a, b \in U$, then $(M, +)$ is abelian.

Proof: By the hypothesis, we have $x\gamma a + y\gamma a - x\gamma a - y\gamma a = 0$ for all $a \in U$, $x, y \in M$ and $\gamma \in \Gamma$. Then we get $(x + y - x - y)\gamma a = 0$ for all $a \in U$, $x, y \in M$ and $\gamma \in \Gamma$. It means that $(x + y - x - y)\Gamma M \Gamma U \subseteq (x + y - x - y)\Gamma U = 0$.

Since U is a non zero semigroup ideal then $x + y - x - y = 0$ for all $x, y \in M$ by the primeness of M . Thus $(M, +)$ is Abelian.

Lemma 3.2: Let M be right Γ – near ring, d a $(1, \alpha)$ - derivation of M and U a multiplicative semigroup of M which contains 0. If d acts as an anti-homomorphism on U and $\alpha(x) = x$, then $x\gamma 0 = 0$ for all $x \in U$.

Proof: Since $0\gamma x = 0$ for all $x \in U$ and $\gamma \in \Gamma$, d acts as an anti-homomorphism on U , it is clear that $d(0\gamma x) = d(x)\gamma d(0) = d(x)\gamma 0$ for all $x \in U$. Taking $x\gamma 0$ instead of x one, can obtain $d(x)\gamma 0 + \alpha(x)\gamma d(0) = \alpha(x)\gamma d(0) = 0$ for all $x \in U$. Thus we have $x\gamma 0 = 0$ for all $x \in U$.

Lemma 3.3: Let M be a Γ – near ring and U be a multiplicative subsemigroup of M . If d is a $(1, \alpha)$ – derivation of M such that $\alpha(x\gamma y) = \alpha(x)\gamma\alpha(y)$ for all $x, y \in U$ and $\alpha(U) = U$, then $n\mu(d(x)\gamma y + \alpha(x)\gamma d(y)) = n\mu d(x)\gamma y + n\mu\alpha(x)\gamma d(y)$ for all $n, x, y \in U, \gamma, \mu \in \Gamma$.

Proof: Since $d(x\gamma y) = d(x)\gamma y + \alpha(x)\gamma d(y)$. From the associative law $d(n\mu(x\gamma y)) = d(n)\mu(x\gamma y) + \alpha(n)\mu d(x\gamma y) = d(n)\mu(x\gamma y) + \alpha(n)\mu(d(x)\gamma y + \alpha(x)\gamma d(y))$ since $(U) = U$, we have

$$d(n\mu(x\gamma y)) = d(n)\mu(x\gamma y) + n\mu(d(x)\gamma y + \alpha(x)\gamma d(y)) \tag{1}$$

On other hand for all $n, x, y \in U$ and $\gamma, \mu \in \Gamma$.

$$\begin{aligned} d((n\mu x)\gamma y) &= d(n\mu x)\gamma y + \alpha(n\mu x)\gamma d(y) \\ &= d(n)\mu(x\gamma y) + \alpha(n)\mu(d(x)\gamma y + \alpha(x)\gamma d(y)) \\ &= d(n)\mu(x\gamma y) + n\mu(d(x)\gamma y + \alpha(x)\gamma d(y)) \text{ since } \alpha(U) = U \end{aligned} \tag{2}$$

Since U is a semigroup ideal, comparing (1) and (2)

$$n\mu(d(x)\gamma y + \alpha(x)\gamma d(y)) = n\mu d(x)\gamma y + n\mu\alpha(x)\gamma d(y)$$

Lemma 3.4: Let M be a prime Γ – near ring and U be a non zero semigroup ideal of M . Let d be a non-zero $(1, \alpha)$ derivation on M such that $\alpha(x\gamma y) = \alpha(x)\gamma\alpha(y)$ for all $x, y \in U, \gamma \in \Gamma$. If $x \in M$ and $d(U)\gamma x = \{0\}$ then $x = 0$.

Proof: Assume that $d(U)\gamma x = \{0\}$. Since U is a non-zero semigroup ideal of M , $d(u\mu y)\gamma x = 0$ for all $x, y \in M, u \in U$ and $\gamma, \mu \in \Gamma$. Hence $0 = [d(u)\mu y + \alpha(u)\mu d(y)]\gamma x = \alpha(u)\mu d(y)\gamma x$ for all $u \in U, y \in M$. $\Rightarrow \alpha(u)\Gamma d(y)\Gamma x = 0$ for all $\gamma, \mu \in \Gamma$. Since M is prime, $\alpha(u) = 0$ or $x = 0$. If $\alpha(u) = 0$ for all $u \in U$, then $\alpha = 0$. This is not possible. There for $x = 0$.

Lemma 3.5: Let M be a prime Γ – near ring and U be a non zero semigroup ideal of M and a non-zero $(1, \alpha)$ – derivation on M . If $d(x + y - x - y) = 0$ for all $x, y \in U$ then $\alpha(x + y - x - y)\gamma d(z) = 0$ for all $x, y \in U$ and $z \in M$.

Proof: Assume that $d(x + y - x - y) = 0$ for all $x, y \in U$. Let us take $y\gamma z$ and $x\gamma z$ instead of y and x respectively (where $Z \in M$ and $\gamma \in \Gamma$) We obtain

$$\begin{aligned} d(x\gamma z + y\gamma z - x\gamma z - y\gamma z) &= d((x + y - x - y)\gamma z) = 0 \\ \Rightarrow d(x + y - x - y)\gamma z + \alpha(x + y - x - y)\gamma d(z) &= \alpha(x + y - x - y)\gamma d(z) = 0 \text{ for all } x, y \in U, z \in M, \gamma \in \Gamma. \end{aligned}$$

Therefore $\alpha(x + y - x - y)\gamma d(z) = 0$ for all $\gamma \in \Gamma$.

Lemma 3.6: Let M be a Γ – near ring U be a multiplicative subsemigroup of M . Let d be $(1, \alpha)$ - derivation of M such That $\alpha(x\gamma y) = \alpha(x)\gamma\alpha(y)$ for all $x, y \in U$ and $\alpha(U) = U$.

- (i) If d acts as a homomorphism on U then,
 $d(y)\mu x\gamma d(y) = \alpha(y)\mu x\gamma d(y) = d(y)\mu x\gamma y$ for all $x, y \in U$ and $\mu, \gamma \in \Gamma$.
- (ii) If d acts as a anti- homomorphism on U then,
 $d(y)\gamma x\gamma d(y) = x\gamma\alpha(y)\gamma d(y) = d(y)\gamma y\gamma x$ for all $x, y \in U$ and $\mu, \gamma \in \Gamma$.

Proof:

(i) Let d acts as a homomorphism on U . Then

$$d(x\gamma y) = d(x)\gamma y + \alpha(x)\gamma d(y) = d(y)\mu x\gamma y \text{ for all } x, y \in U, \gamma \in \Gamma \tag{3}$$

Substituting $y\mu x$ for x in equation (3)

$$d(y)\mu x\gamma y = d(y)\mu x\gamma y + \alpha(y\mu x)\gamma d(y) = d(y)\mu d(x\gamma y) \text{ for all } x, y \in U, \mu, \gamma \in \Gamma \tag{4}$$

By the Lemma 3.3

$$d(y)\mu d(x\gamma y) = d(y)\mu d(x)\gamma y + d(y)\mu \alpha(x)\gamma d(y) \text{ for all } x, y \in U, \mu, \gamma \in \Gamma.$$

Using this relation in equation (4), we get

$$\alpha(y)\mu \alpha(x)\gamma d(y) = d(y)\mu \alpha(x)\gamma d(y)$$

Since $\alpha(U) = U$, we have

$$\alpha(y)\mu x\gamma d(y) = d(y)\mu x\gamma d(y) \text{ for all } x, y \in U, \mu, \gamma \in \Gamma.$$

Similarly taking $y\mu x$ instead of y in equation (3) we obtain

$$d(x)\gamma d(y\mu x) = d(x)\gamma y\mu x + \alpha(x)\gamma d(y\mu x) = d(x\gamma y)\mu d(x) \text{ for all } x, y \in U, \mu, \gamma \in \Gamma. \tag{5}$$

On the other hand $d(x\gamma y)\mu d(x) = (d(x)\gamma y + \alpha(x)\gamma d(y))\mu d(x) = d(x)\gamma y\mu x + \alpha(x)\gamma d(y)\mu d(x)$. using this relation in (5) we get $d(y)\mu x\gamma d(y) = \alpha(y)\mu x\gamma d(y) = d(y)\mu x\gamma y$ for all $x, y \in U, \mu, \gamma \in \Gamma$

(ii) since d acts as a anti- homomorphism on U , we have

$$\begin{aligned} d(x\gamma y) &= d(x)\gamma y + \alpha(x)\gamma d(y) \\ \Rightarrow d(y)\gamma d(x) &= d(x)\gamma y + \alpha(x)\gamma d(y) \text{ for all } x, y \in U, \gamma \in \Gamma \end{aligned} \tag{6}$$

Taking $x\gamma y$ for y in equation (6) and by the hypothesis we get

$$\begin{aligned} d(x\gamma y)\gamma d(x) &= d(x)\gamma x\gamma y + \alpha(x)\gamma d(x\gamma y) \\ \Rightarrow d(x)\gamma y\gamma d(x) &= d(x)\gamma x\gamma y \end{aligned} \tag{7}$$

Similarly taking $x\gamma y$ instead of x in equation (6) and by hypothesis

$$\begin{aligned} d(y)\gamma d(x\gamma y) &= d(x\gamma y)\gamma y + \alpha(x\gamma y)\gamma d(y) \\ \Rightarrow d(y)\gamma \alpha(x)\gamma d(y) &= \alpha(x\gamma y)\gamma d(y) \end{aligned}$$

Since $\alpha(U) = U$, we have

$$d(y)\gamma x\gamma d(y) = x\gamma\alpha(y)\gamma d(y)$$

Theorem 3.7: Let M be a semiprime Γ – near ring and U be a subset of M such that $0 \in U$ and $U\Gamma M \subseteq U$. Let d be a $(1, \alpha)$ – derivation on M such that $\alpha(U) = U$ and $\alpha(x\gamma y) = \alpha(x)\gamma\alpha(y) \forall x, y \in U$

- (i) If d acts as a homomorphism on U then $d(U) = \{0\}$
- (ii) If d acts as a anti- homomorphism on U and $\alpha(0) = 0$ then $d(U) = \{0\}$

Proof: Suppose d acts as a homomorphism on U . By the Lemma 3.6, we have

$$d(y)\mu x\gamma d(y) = d(y)\mu x\gamma y \text{ for all } x, y \in U \text{ and } \mu, \gamma \in \Gamma \tag{8}$$

Multiply (8) by $d(z)$ where $z \in U$ and using the hypothesis that d act as an homomorphism on U together with Lemma 3.6 we get

$$\begin{aligned} d(y)\mu x \gamma (d(y)\mu z + \alpha(y)\mu d(z)) &= d(y)\mu x \gamma \gamma \mu d(z) \\ \Rightarrow d(y)\mu x \gamma d(y)\mu z + d(y)\mu x \gamma \alpha(y)\mu d(z) &= d(y)\mu x \gamma \gamma \mu d(z) \text{ by the Lemma 3.3} \end{aligned}$$

Since $\alpha(U) = U$, we get

$$\begin{aligned} d(y)\mu x \gamma d(y)\mu z + d(y)\mu x \gamma \gamma \mu d(z) &= d(y)\mu x \gamma \gamma \mu d(z) \\ \Rightarrow d(y)\mu x \gamma d(y)\mu z &= 0 \text{ for all } x, y \in U \text{ and } \mu, \gamma \in \Gamma \end{aligned} \quad (9)$$

Taking $z\eta m$ instead of x where $m \in M$

We get $d(y)\mu z \eta m \gamma d(y)\mu z = 0$ for all $x, y, z \in U, m \in M$ and $\mu, \eta, \gamma \in \Gamma$.

In particular

$$\Rightarrow d(y)\mu z \Gamma M \Gamma d(y)\mu z = 0$$

By the semiprimeness of M we conclude that

$$d(y)\mu z = \{0\} \quad (10)$$

Substitute $y\eta n$ for y in equation (10)

$$d(y\eta n)\mu z = 0 \quad (11)$$

Left multiply (11) by $d(z)$ where $z \in U$, we get

$$\begin{aligned} d(z)\beta d(y\eta n)\mu z &= 0 \\ \Rightarrow d(z)\beta d(y)\eta n \mu z + d(z)\beta \alpha(y)\eta d(n)\mu z &= 0 \text{ by the Lemma 3.3} \end{aligned}$$

Since the second summand is zero by (11) we get

$$d(z)\beta d(y)\eta n \mu z = 0$$

By the hypothesis and by using (10)

$$\alpha(z)\beta d(y)\eta n \mu z = 0$$

Since $\alpha(U) = U$, we have

$$z\beta d(y)\eta n \mu z = 0 \text{ for all } x, y \in U, n \in M$$

Substitute $z = z\beta d(y)$ in above equation and since M is semiprime

$$z\beta d(y) = 0 \text{ for all } y, z \in U$$

Since

$$\begin{aligned} \alpha(U) &= U, \\ \alpha(z)\beta d(y) &= 0 \end{aligned} \quad (12)$$

Combining (10) and (12) we get

$$d(y\beta z) = 0 \text{ for all } y, z \in U$$

Replace y by $z\gamma m$

$$d(z\gamma m\beta z) = 0 \text{ for all } m \in M \text{ and } \gamma, \beta \in \Gamma, z \in U$$

By the hypothesis and by (12)

$$\begin{aligned} d(z)\gamma m\beta d(z) &= 0 \\ \Rightarrow d(z)\Gamma M \Gamma d(z) &= 0 \end{aligned}$$

Hence

$$d(z) = \{0\} \text{ for all } z \in U \quad (13)(ii)$$

Now assume that d acts as an anti- homomorphism on U . By the Lemma 3.6, we have

$$x \gamma \alpha(y) \gamma d(y) = d(y) \gamma x \gamma d(y) \quad (14)$$

$$d(y) \gamma \gamma x = d(y) \gamma x \gamma d(y) \quad (15)$$

Replace x by $x\gamma d(y)$ in (14), we get

$$x\gamma d(y) \gamma \alpha(y) \gamma d(y) = d(y) \gamma x \gamma d(y) \gamma y + d(y) \gamma x \gamma \alpha(y) \gamma d(y) \text{ by the Lemma 3.3}$$

Since $\alpha(U) = U$, we have

$$x\gamma d(y) \gamma \gamma d(y) = d(y) \gamma x \gamma d(y) \gamma \alpha(y) + d(y) \gamma x \gamma \gamma d(y) \quad (16)$$

Substitute $x\gamma y$ for x equation (14)

$$x\gamma y\gamma\alpha(y)\gamma d(y) = d(y)\gamma x\gamma y\gamma d(y) \tag{17}$$

multiply equation (14) by $\alpha(y)$

$$x\gamma\alpha(y)\gamma d(y)\gamma\alpha(y) = d(y)\gamma x\gamma d(y)\gamma\alpha(y) \tag{18}$$

Replace x by y in equation (14)

$$y\gamma\alpha(y)\gamma d(y) = d(y)\gamma y\gamma d(y)$$

Multiply by x above relation by

$$x\gamma y\gamma\alpha(y)\gamma d(y) = x\gamma d(y)\gamma y\gamma d(y) \tag{19}$$

Using (17) (18) and (19) in (16)

$$\begin{aligned} x\gamma y\gamma\alpha(y)\gamma d(y) &= x\gamma\alpha(y)\gamma d(y)\gamma\alpha(y) + x\gamma y\gamma\alpha(y)\gamma d(y) \\ x\gamma\alpha(y)\gamma d(y)\gamma\alpha(y) &= 0 \end{aligned}$$

In particular,

$$x\gamma\alpha(y)\gamma d(y)\gamma\alpha(y) = 0 \text{ where } n \in M$$

Hence $\alpha(y)\gamma d(y)\gamma\alpha(y)\gamma M\gamma\alpha(y)\gamma d(y)\gamma\alpha(y) = \{0\}$

By the semiprimeness of

$$\alpha(y)\gamma d(y)\gamma\alpha(y) = 0 \text{ for all } x, y \in U \tag{20}$$

According to (18) we get

$$d(y)\gamma x\gamma d(y)\gamma\alpha(y) = 0 \text{ for all } x, y \in U \tag{21}$$

Replacing x by $\alpha(y)\gamma x\gamma n$

$$\begin{aligned} d(y)\gamma\alpha(y)\gamma x\gamma n\gamma d(y)\gamma\alpha(y)\gamma x &= 0 \\ \Rightarrow d(y)\gamma\alpha(y)\gamma x &= 0 \text{ for all } x, y \in U, n \in M \end{aligned}$$

Since $\alpha(U) = U$, we get

$$d(y)\gamma y\gamma x = 0 \text{ for all } x, y \in U \tag{22}$$

Using (22) in (15) we obtain

$$d(y)\gamma x\gamma d(y) = 0 \text{ for all } x, y \in U$$

And so we have

$$d(y)\gamma x\gamma n\gamma d(y)\gamma x = 0 \text{ for all } x, y \in U, n \in M$$

Hence $d(y)\gamma x = 0$ for all $x, y \in U$ (23)

Therefore

$$x\gamma d(z)\gamma d(y\gamma n)\gamma x = 0 \text{ for all } x, y, z \in U, n \in M$$

By the hypothesis and using Lemma 3.3

$$x\gamma d(z)\gamma d(y)\gamma n\gamma x + 0 = 0$$

Since $\alpha(U) = U$ the second summand is zero by (13)

Hence $x\gamma d(z)\gamma d(y)\gamma M\gamma x = 0$

By the semiprimeness of M we get

$$\begin{aligned} 0 &= x\gamma d(z)\gamma d(y) \\ &= x\gamma d(z)\gamma y + x\gamma\alpha(z)\gamma d(y) \\ &= x\gamma\alpha(z)\gamma d(y) = 0 \end{aligned}$$

By the semiprimeness of M ,

$$\alpha(z)\gamma d(y) = 0 \text{ for all } z, y \in U \tag{24}$$

Combining (23) and (24) we have

$$0 = d(x\gamma y) \text{ for all } x, y \in U$$

Replace y by $x\gamma n$. By hypothesis and Lemma 3.3

$$0 = d(x)\gamma n\gamma d(x) + \alpha(x)\gamma d(n)\gamma d(x)$$

Since the second summand is zero we get

$$d(x)\gamma n\gamma d(x) = 0$$

Therefore $d(x) = 0$ for all $x \in U$.
 i.e., $d(U) = 0$

Corollary 3.8: Let M be a semi prime Γ – near ring and d a $(1, \alpha)$ – derivation of M Such that α is onto and $\alpha(x\gamma y) = \alpha(x)\gamma\alpha(y)$ for all $x, y \in M$

- (i) If d acts as a homomorphism on M , then $d = 0$
- (ii) If d acts as a anti- homomorphism on M such that $\alpha(0) = 0$ then $d = 0$

Proof: Take $U = M$ in above theorem we get

- (i) $d(M) = 0 \Rightarrow d = 0$
- (ii) $\alpha(0) = 0$ then $d(M) = 0 \Rightarrow d = 0$

Corollary 3.9: Let M be a prime Γ – near ring and U a non zero subset of M such that $0 \in U$ and $U\Gamma M \subseteq U$. Let d be a $(1, \alpha)$ – derivation of M Such that $\alpha(U) = U$ and $\alpha(x\gamma y) = \alpha(x)\gamma\alpha(y)$ for all $x, y \in U$

- (i) If d acts as a homomorphism on U , then $d = 0$
- (ii) If d acts as an anti- homomorphism on U and $\alpha(0) = 0$ then $d = 0$

Proof: By the theorem 3.7, we have $d(x) = 0$ for all $x \in U$. Then $d(x\gamma n) = d(x)\gamma n + \alpha(x)\gamma d(n) = \alpha(x)\gamma d(n) = 0$ for all $x \in U, n \in M$. Replace x by $x\gamma m$, $\alpha(x\gamma m)\gamma d(n) = \alpha(x)\gamma\alpha(m)\gamma d(n) = 0$ for all $x \in U, n, m \in M$ and $\gamma \in \Gamma$. Hence $x\Gamma M \Gamma d(n) = 0$. By the primeness of M we have $x = 0$ or $d(n) = 0$ for all $x \in U$. Since U is non zero we have $d(n) = 0$ for all $n \in M$.

Theorem 3.10: Let M be a prime Γ – near ring U a non zero semigroup ideal of M and d a non zero $(1, \alpha)$ - derivation of M such that $\alpha(x\gamma y) = \alpha(x)\gamma\alpha(y)$ for all $x, y \in U$ and $\alpha(U) = U$. If $d(x + y - x - y) = 0$ for all $x, y \in U$ then $(M, +)$ is abelian.

Proof: Suppose that $d(x + y - x - y) = 0$ for all $x, y \in U$. By the Lemma 3.5, we have $\alpha(x + y - x - y)\gamma d(z) = 0$ for all $x, y \in U, z \in M$ and $\gamma \in \Gamma$. Since $d \neq 0$ it follows that $\alpha(x + y - x - y) = x + y - x - y = 0$ for all $x, y \in U$. Hence $(M, +)$ is abelian by Lemma 3.1.

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