Volume 9, No. 3, March - 2018 (Special Issue)
International Journal of Mathematical Archive-9(3), 2018, 86-90
IJMA Available online through www.ijma.info ISSN 2229 – 5046

ON $S\alpha\omega$-CLOSED SETS AND $S\alpha\omega$-$T_{\frac{i\omega}{8}}$ $(i = 1, 3, 5)$ SPACES IN SOFT TOPOLOGICAL SPACES

G. VASUKI[1], B. AMUDHAMBIGAI[2] AND R. SUBHA SHREE[3]

[1]Associate Professor, Department of Mathematics,
Sri Sarada College for Women (Autonomous) , Salem-16, India.

[2]Assistant Professor, Department of Mathematics,
Sri Sarada College for Women (Autonomous) , Salem-16, India.

[3]Research Scholar, Department of Mathematics,
Sri Sarada College for Women (Autonomous), Salem-16, India.

E-mail: vasukigj@yahoo.com[1], rbamudha@yahoo.co.in[2] and subhashreeraghu15@g.mail.com[3].

ABSTRACT

In this paper the concepts of $s\alpha\omega$-closed sets and $s\alpha\omega$- $kernel$ sets are introduced. Then a characterization of $s\alpha\omega$-kernal set is established. Also the concepts of $s\alpha\omega$ -$T_{\frac{\omega}{4}}$ spaces, $s\alpha\omega$-$T_{\frac{3\omega}{8}}$ spaces and $s\alpha\omega$-$T_{\frac{5\omega}{8}}$ spaces are established and their properties are investigated.

***Key Words**: $s\alpha\omega$-closed sets, $s\alpha\omega$-kernal sets, $s\alpha\omega$ -$T_{\frac{\omega}{4}}$ spaces, $s\alpha\omega$ - $T_{\frac{3\omega}{8}}$ sspaces and $s\alpha\omega$-$T_{\frac{5\omega}{8}}$ spaces.*

***2010 Mathematics Subject Classification:** 06D72, 54A40.*

1. INTRODUCTION

Any examination work should bring about expansion to the current learning of a specific idea. Such an exertion extends the extent of the idea as well as urges others to investigate more current thoughts. Here the researchers have prevailed in their insight building exertion by presenting a new class of soft sets called Soft $\alpha\omega$-Closed sets and soft $S\alpha\omega$-$T_{\frac{i\omega}{8}}$ $(i = 1,3,5)$ spaces in soft topological spaces.

Nandhini and kalaiselvi [3] proposed the idea of α-closed sets in topological spaces. The idea of ω-closed sets were introduced by Nimala Rebecca paul [4] and as of late M. Parimala and Biju [5] considered αω-closed sets in topological spaces. Sabir Hussain and Bashir Ahmad [6] presented the thought of soft topological spaces which are characterized over an initial universe with a settled arrangement of parameter.

In this paper the concepts of $s\alpha\omega$-closed sets and $s\alpha\omega$- kernel sets are introduced. Then a characterization of is established. Also the concepts of $s\alpha\omega$ -$T_{\frac{\omega}{4}}$ spaces, $s\alpha\omega$ - $T_{\frac{3\omega}{8}}$ spaces and $s\alpha\omega$-$T_{\frac{5\omega}{8}}$ spaces are established and their properties are investigated.

2. PRELIMINARIES

In this section, some basic concepts of soft sets have been studied. Also, throughout this paper, X is a non-empty set and E is the set of parameters. Then (G, E) denoted soft sets over X.

CONFERENCE PAPER
National Conference dated 26th Feb. 2018, on "New Trends in Mathematical Modelling" (NTMM - 2018), Organized by Sri Sarada College for Women, Salem, Tamil Nadu, India.

Definition 2.1 [8]: Let τ be the collection of soft sets over X, then τ is said to be a **soft topology** on X if

i) Φ, X belongs to τ;
ii) the union of any number of soft sets in τ belongs to τ;
iii) the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a **soft topological space** over X.

Definition 2.2 [9]: The soft set $(F, A) \tilde{\in} SS(U)_A$ is called a **soft point** in U_A, denoted by e_F, if for the element $e \in A$, $F(e) \neq \phi$ and $F(\grave{e}) = \phi$, for all $\grave{e} \in A - \{e\}$, where $SS(X)_A$ denotes the family of soft sets over X with the set of parameters A.

Definition 2.3 [9]: A soft set (G, A) in a soft topological space (U, τ, A) is called a **soft neighborhood** (briefly: nbd) of the soft point $e_F \tilde{\in} U_A$, if there exists a soft open set (H, A) such that $e_F \tilde{\in} (H, A) \tilde{\subseteq} (G, A)$.

Definition 2.4 [4]: A soft set (A, E) is called a **soft ω-closed set** in a soft topological space $(X, \bar{\tau}, E)$, if $\overline{(A, E)} \tilde{\subseteq} (G, E)$ whenever $(A, E) \tilde{\subseteq} (G, E)$ and (G, E) is soft semi-open in X.

Definition 2.5 [8]: Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. Then the **soft closure** of (F, E) denoted by $\overline{(F, E)}$ is the intersection of all soft closed super sets of (F, E)

Definition 2.6 [7]: If (S, E) is a soft subset of a soft topological space $(X, \tilde{\tau}, E)$. Then the **soft interior** of (S, E) is the union of all soft open sets in $\tilde{X}$ which are contained in (S, E) and is denoted by int(S, E)

Definition 2.7 [3]: In a soft topological spaces(X, τ, E), a soft set (F, E) over X is called a **soft α-open** if $(F, E) \tilde{\subseteq} int(cl(int(F, E)))$.

Definition 2.8 [7]: If $(X, \tilde{\tau}, E)$ is a soft topological space and $(S, E) \tilde{\subseteq} \tilde{X}$ Then the soft omega closure (briefly **soft ω-closure)** of (S, E) is the intersection of all soft ω-closed sets in $\tilde{X}$ which are contains (S, E) and is denoted by $cl_\omega(S, E)$.

3. Soft $\alpha\omega$-closed set, soft $\alpha\omega$-kernal set, soft $\alpha\omega$- $T_{\frac{\omega}{4}}$, soft $\alpha\omega$- $T_{\frac{3\omega}{8}}$, soft $\alpha\omega$- $T_{\frac{5\omega}{8}}$

In this section, the concepts of $s\alpha\omega$-closed sets and $s\alpha\omega$- kernel sets are introduced. Then a characterization of $s\alpha\omega$-kernal set is established. Also the concepts of $s\alpha\omega$ -$T_{\frac{\omega}{4}}$ spaces, $s\alpha\omega$ - $T_{\frac{3\omega}{8}}$ spaces and $s\alpha\omega$-$T_{\frac{5\omega}{8}}$ spaces are established and their properties are investigated.

Definition 3.1: Let (X, τ, E) be a soft topological space. A soft set (A, E) over X is called a **soft $\alpha\omega$-closed set** (briefly, $s\alpha\omega$-closed set) if $s\omega cl(A, E) \tilde{\subseteq} (G, E)$ whenever $(A, E) \tilde{\subseteq} (G, E)$ and (G, E) is soft α-open in (X, τ, E).The complement of a $s\alpha\omega$-closed set is $s\alpha\omega$-open set.

Definition 3.2: Let (X, E, τ) be a soft topological space. For any soft set (A, E), the **soft $\alpha\omega$-closure** of (A, E) is denoted and defined as $s\omega cl(A, E)$ $=\tilde{\cap}$ $\{(A, E): (A, E) \subseteq (G, E), (G, E)$ is soft $\alpha\omega$-closed$\}$.

Definition 3.3: Let (X, E, τ) be a soft topological space over X. Let (A, E) be a soft set over X. Then the **soft $\alpha\omega$-interior** of (A, E) is denoted and defined as $s\omega cl(A, E)$ $=\tilde{\cup}$ $\{(G, E): (A, E) \subseteq (G, E), (G, E)$ is soft $\alpha\omega$-open $\}$.

Definition 3.4: A soft topological space (X, τ, E) is called a **soft $\alpha\omega$-T_0 space** (briefly, $s\alpha\omega$-T_0 space) if for every pair of two distinct soft points e_F, e_G of (X, τ, E), there exists a soft $\alpha\omega$-open set (A, E) over X which contains one of the soft points and not the other.

Definition 3.5: Let (X, τ, E) be a soft topological space. For any soft set (A, E) over X, then **soft $\alpha\omega$-kernal** of (A, E) denoted by $s\alpha\omega$-$ker(A, E)$ is the set $\tilde{\cap}$ $\{(G, E): (G, E)$ is $s\alpha\omega$-open and $(A, E) \tilde{\subseteq} (G, E)\}$.

Definition 3.6: Let (X, τ, E) be a soft topological space. Any soft set (A, E) over X is called a **soft $\alpha\omega$-kernal** set if $(A, E) = s\alpha\omega$-$ker(A, E)$.

Definition 3.7: Let (X, τ, E) be a soft topological space. Any soft set (A, E) over X is said to be **soft $\alpha\omega\lambda^*$-closed** (briefly, **$s\alpha\omega\lambda^*$-closed**) if $(A, E) = (G, E) \tilde{\cap} s\alpha\omega cl(H, E)$,
where $(G, E)$is a soft $\alpha\omega$-kernal set and (H, E) is a soft set over X.

Definition 3.8: Any soft topological space (X, τ, E) is called a **soft $\alpha\omega$-$T_{\frac{\omega}{4}}$ space** (briefly, **$s\alpha\omega$- $T_{\frac{\omega}{4}}$ space)** if for every finite soft set (A, E) over X and for every soft point e_B in $X_E - (A, E)$, there exists a soft set (G, E) over X with $(A, E) \tilde{\subseteq} (G, E)$ and $(G, E) \tilde{\cap} (B, E) = \Phi_E$, where (B, E) over X is a soft neighborhood of e_B such that (G, E) is either $s\alpha\omega$-open or $s\alpha\omega$-closed.

Definition 3.9: Any soft topological space (X, τ, E) is called a **soft $\alpha\omega$-$T_{\frac{3\omega}{8}}$ space** (briefly, **$s\alpha\omega$- $T_{\frac{3\omega}{8}}$ space)** if for every countable soft set (A, E) over X and for every soft point e_B in $X_E - (A, E)$, there exists a soft set (G, E) over X with $(A, E) \tilde{\subseteq} (G, E)$ and $(G, E) \tilde{\cap} (B, E) = \Phi_E$, where (B, E) over X is a soft neighborhood of e_B such that (G, E) is either $s\alpha\omega$-open or $s\alpha\omega$-closed.

Definition 3.10: Any soft topological space (X, τ, E) is called a **soft $\alpha\omega$-$T_{\frac{5\omega}{8}}$ space** (briefly, **$s\alpha\omega$- $T_{\frac{5\omega}{8}}$ space)** if for any soft set (A, E) over X and for every soft point e_B in $X_E - (A, E)$, there exists a soft set (G, E) over X with $(A, E) \tilde{\subseteq} (G, E)$ and $(G, E) \tilde{\cap} (B, E) = \Phi_E$, where (B, E) over X is a soft neighborhood of e_B such that (G, E) is either $s\alpha\omega$-open or $s\alpha\omega$-closed.

Remark: Let (X, τ, E) be a soft topological space. For any two soft sets (A, E) and (G, E)over X, the following conditions hold:

i) $s\alpha\omega int\,(G, E) \tilde{\leq} (G, E) \tilde{\leq} s\alpha\omega cl\,(G, E)$
ii) $s\alpha\omega cl\big(s\alpha\omega cl\,(G, E)\big) = s\alpha\omega cl(G, E)$.
iii) If $(G, E) \tilde{\subseteq} (A, E)$, then $s\alpha\omega cl(G, E) \tilde{\subseteq} s\alpha\omega cl(A, E)$.

Proposition 3.1: Let (X, τ, E) be a soft topological space over X. Any soft set (F, E) over X is $s\alpha\omega\lambda^*$- closed if and only if $(F, E) = (K, E) \tilde{\cap}\ s\alpha\omega cl(F, E)$, where (K, E) is a soft $\alpha\omega$-kernal set over X.

Proof: Let (F, E) be $s\alpha\omega\lambda^*$- closed, then $(F, E) = (K, E) \tilde{\cap}\ s\alpha\omega cl(G, E)$, where (K, E) is a soft $\alpha\omega$-kernal set and (G, E) is a soft set over X. Since $(F, E) \tilde{\subset} (K, E)$ and $(F, E) \tilde{\subset} s\alpha\omega cl(G, E)$, it follows that $s\alpha\omega cl(F, E) \tilde{\subset} s\alpha\omega cl\big(s\alpha\omega cl(G, E)\big) = s\alpha\omega cl(G, E)$.
Therefore, $(F, E) \tilde{\subset} (K, E) \tilde{\cap}\ s\alpha\omega cl(F, E) \tilde{\subset} (K, E) \tilde{\cap}\, s\alpha\omega cl(G, E) = (F, E)$
and hence, $(F, E) = (K, E) \tilde{\cap}\ s\alpha\omega cl(G, E)$.

Conversely, let $(F, E) = (K, E) \tilde{\cap}\ s\alpha\omega cl(F, E)$, where (F, E) is a soft set over X and (K, E) is a $s\alpha\omega$ kernal(K, E). Hence , (F, E) is $s\alpha\omega\lambda^*$- closed.

Proposition 3.2: A soft topological space (X, τ, E) is $s\alpha\omega$- $T_{\frac{\omega}{4}}$ if and only if every finite soft set over X is $s\alpha\omega\lambda^*$-closed.

Proof: Let (X, τ, E) be a $s\alpha\omega$- $T_{\frac{\omega}{4}}$ space and (P, E) be a finite soft set over X. So for every soft point e_B in $X_E - (P, E)$, there exists a soft set (G, E) over X with $(P, E) \tilde{\subseteq} (G, E)$ and $(G, E) \tilde{\cap} (B, E) = \Phi_E$, where (B, E) over X is a soft neighborhood of e_B such that (G, E) is either $s\alpha\omega$-open or $s\alpha\omega$-closed. Let (H, E) be the intersection of all such $s\alpha\omega$-open sets (G, E), that is $(H, E) = \tilde{\cap}\, \{(U, E); (U, E)$ is $s\alpha\omega$-open and $(H, E) \tilde{\subseteq} (U, E)\}$= $s\alpha\omega$ ker(H, E).

Let (L, E) be the intersection of all such $s\alpha\omega$-closed sets (G, E), that is, $(L, E) = s\alpha\omega cl(G, E)$. Therefore, $(H, E) \tilde{\cap} (L, E) = (H, E) \tilde{\cap}\, s\alpha\omega cl(G, E) = (P, E)$. So (P, E) is $s\alpha\omega\lambda^*$- closed.

Conversely, let (P, E) be a finite soft set over X and by assumption it is $s\alpha\omega\lambda^*$- closed. Let e_B be a soft point in $X_E - (P, E)$. Then by Proposition 3.1, $(P, E) = (K, E) \tilde{\cap}\, s\alpha\omega cl(P, E)$ where (K, E) is a $s\alpha\omega$-ker(K, E). If e_B is not a soft point in $s\alpha\omega cl(P, E)$, then there exists a $s\alpha\omega$-closed set (G, E) containing (P, E) such that $(B, E) \tilde{\not\subset} (G, E)$. Again if e_B is a soft point in $s\alpha\omega cl(P, E)$, then e_B is not a soft point in (K, E). Then there exists some $s\alpha\omega$-open set (G, E) containing (K, E) such that e_B is not a soft point in (G, E) and also (G, E) contains (P, E). Hence, (X, τ, E) is a $s\alpha\omega$- $T_{\frac{\omega}{4}}$ space.

Proposition 3.3: Let (X, τ, E) be a soft topological space. Then every soft $\alpha\omega$- $T_{\frac{\omega}{4}}$ space is a soft $\alpha\omega$- T_0 space.

Proof: Let (X, τ, E) be a soft $\alpha\omega$- $T_{\frac{\omega}{4}}$ space and e_A, e_B be any two distinct soft points over X. Since (X, τ, E) is soft $\alpha\omega$- $T_{\frac{\omega}{4}}$, for every soft point e_A in $X_E - e_B$ there exists a soft set (G, E) containing e_B for each e_B such that (G, E) is either $s\alpha\omega$-open or $s\alpha\omega$-closed and e_A is not a soft point in(G, E). This implies that (X, τ, E) is a soft $\alpha\omega$- T_0 space.

Proposition 3.4: Let (X, τ, E) be a soft topological space. Then (X, τ, E) is soft $\alpha\omega$- $T_{\frac{3\omega}{8}}$ if and only if every countable soft set (F, E) is soft $\alpha\omega\lambda^*$-closed.

Proof: Let (X, τ, E) be a $s\alpha\omega$- $T_{\frac{3\omega}{8}}$ space and (P, E) be a countable soft set over X. So for every soft point e_B in $X_E - (P, E)$, there exists a soft set (G, E) over X with $(P, E) \tilde{\subseteq} (G, E)$ and $(G, E) \tilde{\cap} (B, E) = \Phi_E$, where (B, E) over X is a soft neighborhood of e_B such that (G, E) is either $s\alpha\omega$-open or $s\alpha\omega$-closed. Let (H, E) be the intersection of all such $s\alpha\omega$-open sets (G, E), that is $(H, E) = \tilde{\cap}\{(U, E); (U, E)$ is $s\alpha\omega$-open and $(H, E) \tilde{\subseteq} (U, E)\}$= $s\alpha\omega$-ker(H, E).

Let (L, E) be the intersection of all such $s\alpha\omega$-closed sets (G,E), that is, $(L, E) = s\alpha\omega cl(G, E)$. Therefore $(H, E) \tilde{\cap} (L, E) = (H, E) \tilde{\cap} s\alpha\omega cl(G, E)$ = (P, E). So (P, E) is $s\alpha\omega\lambda^*$- closed.

Conversely, let (P, E) be a countable soft set over X and by assumption it is $s\alpha\omega\lambda^*$- closed. Let e_B be a soft point in $X_E - (P, E)$. Then by Proposition 3.1, $(P, E) = (K, E) \tilde{\cap} s\alpha\omega cl(P, E)$, where (K, E) is a $s\alpha\omega$-ker(K, E). If e_B is not a soft point in $s\alpha\omega cl(P, E)$. Then there exists a $s\alpha\omega$-closed set (G, E) containing (P, E) such that $(B, E) \tilde{\not\subset} (G, E)$. Again if e_B is a soft point in $s\alpha\omega cl(P, E)$, then e_B is not a soft point in (K, E). Then there exists some $s\alpha\omega$-open set (G, E) containing (K, E) such that e_B is not a soft point in (G, E) and also (G, E) contains (P, E). Hence, (X, τ, E) is a $s\alpha\omega$- $T_{\frac{3\omega}{8}}$ space.

Proposition 3.5: Let (X, τ, E) be a soft topological space. Then (X, E, τ) is soft $\alpha\omega$- $T_{\frac{5\omega}{8}}$ if and only if every soft set (F, E) is soft $\alpha\omega\lambda^*$-closed.

Proof: Let (X, τ, E) be a $s\alpha\omega$- $T_{\frac{5\omega}{8}}$ space and (P, E) be any soft set over X. So for every soft point e_B in $X_E - (P, E)$, there exists a soft set (G, E) over X with $(P, E) \tilde{\subseteq} (G, E)$ and $(G, E) \tilde{\cap} (B, E) = \Phi_E$, where (B, E) over X is a soft neighborhood of e_B such that (G, E) is either $s\alpha\omega$-open or $s\alpha\omega$-closed. Let (H, E) be the intersection of all such $s\alpha\omega$-open sets (G, E), that is, $(H, E) = \tilde{\cap}\{(U, E); (U, E)$ is $s\alpha\omega$-open and $(H, E) \tilde{\subseteq} (U, E)\}$= $s\alpha\omega$-ker(H, E).

Let (L, E) be the intersection of all such $s\alpha\omega$-closed sets (G, E), that is, $(L, E) = s\alpha\omega cl(G, E)$. Therefore, $(H, E) \tilde{\cap} (L, E) = (H, E) \tilde{\cap} s\alpha\omega cl(G, E)$ = (P, E). So (P, E) is $s\alpha\omega\lambda^*$- closed.

Conversely, let (P, E) be any soft set over X and by assumption it is $s\alpha\omega\lambda^*$- closed. Let e_B be a soft point in $X_E - (P, E)$. Then by Proposition 3.1, $(P, E) = (K, E) \tilde{\cap} s\alpha\omega cl(P, E)$, where (K, E) is a $s\alpha\omega$-ker(K, E). If e_B is not a soft point in $s\alpha\omega cl(P, E)$. Then there exists a $s\alpha\omega$-closed set (G, E) containing (P, E) such that $(B, E) \tilde{\not\subset} (G, E)$. Again if e_B be a soft point in $s\alpha\omega cl(P, E)$, then e_B is not a soft point in (K, E). Then there exists some $s\alpha\omega$-open set (G, E) containing (K, E) such that e_B is not a soft point in (G, E) and also (G, E) contains (P, E). Hence, (X, τ, E) is a $s\alpha\omega$- $T_{\frac{5\omega}{8}}$ space.

CONCLUSION

In the present study, we have introduced the concepts of $s\alpha\omega$-closed sets and $s\alpha\omega$-kernal set. Then the concepts of $s\alpha\omega$ -$T_{\frac{\omega}{4}}$ spaces, $s\alpha\omega$ - $T_{\frac{3\omega}{8}}$ spaces and $s\alpha\omega$-$T_{\frac{5\omega}{8}}$ spaces are established and their properties are investigated. In future these findings may be extended to a new dimension on soft $\alpha\omega$-closed and soft $\alpha\omega$-open sets in soft topological spaces.

ACKNOWLEDGEMENT

The authors would like to express their sincere gratitude to the referees for the various suggestions towards the betterment of this paper.

REFERENCES

1. Amar Kumar Banerjee and Jagannath Pal, Semi λ^*-Closed Sets and New Separation Axioms in Alexandroff Spaces, Math.Gn (2017).
2. Georgiou D. N , Megaritis A. C. and Petropoulos V. I., On Soft Topological Spaces, Applied Mathematics and Information Sciences. 7, No. 5, 1889-1901 (2013).
3. Nandhini T, Kalaiselvi A, Soft $\hat{g}$-Closed Sets in Soft Topological Spaces, International Journal of Innovation Research in Science, Engineering and Technology, Vol. 3, Issue 7, July 2014.

4. Nirmala Rebecca paul, Remarks on soft ω-closed sets in soft topological spaces, Bol. Soc. Paran. Mat, Vol: 33 1 2015
5. Parimala M, Udhayakumar R, Jeevitha R, Biju V, ON αω -Closed Sets in Topological Spaces, International Journal of Pure and Applied Mathematics, Vol. 115, 2017.
6. Sabir Hussain and Bashir Ahmad, Soft separation axioms in soft topological spaces, Hacettepe Journal of Mathematics and Statistics, Volume 44 (3) (2015).
7. Sabiha I. Mahmood, On Weakly Soft Omega Open Functions and Weakly Soft Omega Closed Functions in Soft Topological Spaces, Iraqi Journal of Science, 2017, Vol. 58.
8. Shabir M, Naz M, On Soft Topological spaces, Computers And Mathematics with Application 61 (2011).
9. Zorlutana I, Akdag N, Min W.K, Atmaca S, Remarks on Soft Topological Spaces, Annals of Fuzzy Mathematics and Informatics 3(2) (2012).

Source of support: National Conference on "New Trends in Mathematical Modelling" (NTMM - 2018), Organized by Sri Sarada College for Women, Salem, Tamil Nadu, India.

