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ON *gα-CLOSED SETS AND *gα-CONTINUOUS MAPS

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ABSTRACT

*The purpose of this paper is to introduce the concept of *gα-closed set. Besides studying some properties, the interrelations of the set introduced with other related sets are studied with necessary counter examples. Introducing *gα-continuous map, some interesting properties and characterizations are discussed.*

Keywords: *α semi-open set, $\hat{g}\alpha$-closed set, *gα-closed set, *gα-continuous map, *gα-closure, $T_{*g\alpha}$-space.*

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1. INTRODUCTION

α-open set was introduced and investigated by O. Njastad [12]. Cameron [4], Veerakumar. M. K. R. S. [17], Jafari. S, Noiri. T, Rajesh. N and Thivagar. M. L. [8] and Syed Ali Fathima and Mariasingam [14, 15, 16] introduced and investigated regular semi-open set, ĝ-closed set, *g-closed set, #rg-closed set, #rg-closure, #rg-continuous map and $T_{\#rg}$-space. We introduce a new class of set called*gα-closed set and study some of its properties. The concept of *gα-continuous map is introduced and some basic properties are characterized.

2. PRELIMINARIES

Throughout this paper X, Y and Z denote the topological spaces (X, T), (Y, S) and (Z, R) respectively on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a topological space X, the closure of A, interior of A, semi-closure of A, semipre-closure of A, the complement of A and #rg-closure of A are denoted by cl(A), int(A), scl(A), spcl(A), X\A and #rg-cl(A) respectively. We recall the following definitions and results.

Definition 2.1A subset A of a space X is called

(1) a semi-open set[10] if A ⊆clint (A) and a semi-closed set if intcl (A) ⊆A.
(2) a regular open set[13]if A = intcl (A) and a regular closed set if A = clint (A).
(3) aπ-open set[18]if A is a finite union of regular open sets.
(4) aα-open set [12] if A ⊆int(cl(int(A))) and a α-closed set if cl(int(cl(A))) ⊆ A.
(5) aregular semi-open set [4]if there is a regular open U such that U ⊆ A ⊆ cl(U).
(6) a semi-preopen set [1] if A ⊆ cl(int(cl(A))) and a semi-preclosed set if int(cl(int(A))) ⊆ A.

Definition 2.2:A subset A of a space X is called

(1) ageneralized closed set (briefly, g-closed) [9] if cl(A) ⊆ U whenever A ⊆ U and U is open in X.
(2) aweakly generalized closed set (briefly, wg-closed) [11] if cl(int(A)) ⊆ U whenever A ⊆U and U is open in X.
(3) aπ-generalized closed set (briefly, πg-closed) [7] if cl(A) ⊆ U whenever A ⊆ U and U is π-open in X.
(4) aĝ-closed set [17] if cl(A) ⊆ U whenever A ⊆ U and U is semi-open in X.
(5) a*g-closed set [8] if cl(A) ⊆ U whenever A ⊆ U and U is ĝ-open in X.
(6) a generalized semi-closed set (briefly, gs-closed) [2] if scl(A) ⊆ U whenever A ⊆ U and U is open in X.
(7) a generalized semi pre-closed set (briefly, gsp-closed) [6] if spcl(A) ⊆ U whenever A ⊆ U and U is open in X.

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Definition 2.3 [15]: For a subset A of a space X, #rg-cl(A) = ∩ {F: A ⊆ F, F is #rg closed in X} is called the #rg-closure of A.

Definition 2.4: A map f: (X, T) → (Y, S) is called
(1) continuous [3] if $f^{-1}(V)$ is closed set in X for every closed subset V of Y.
(2) π-continuous [7] if $f^{-1}(V)$ is π-closed set in X for every closed subset V of Y.
(3) πg-continuous [7] if $f^{-1}(V)$ is πg-closed set in X for every closed subset V of Y.
(4) wg-continuous [11] if $f^{-1}(V)$ is wg-closed set in X for every closed subset V of Y.
(5) gs-continuous [5] if $f^{-1}(V)$ is gs-closed set in X for every closed subset V of Y.
(6) gsp-continuous [6] if $f^{-1}(V)$ is gsp-closedset in X for every closed subset V of Y.
(7) #rg-continuous [16] if $f^{-1}(V)$ is #rg-closedset in X for every closed subset V of Y.

Definition 2.5: A space X is called
(1) $T_{1/2}$-space [9] if every g-closed set is closed.
(2) $T_{\#rg}$-space [14] if every #rg-closed set in it is closed.

Definition 2.6 [15]: Let (X, T) be a topological space and $T_{\#rg}$ = {V ⊆X : #rg-cl(X\V) = X\V}.

3 *gα-CLOSED SETS AND THEIR BASIC PROPERTIES

In this section we introduce and study *gα-closed sets.

Definition 3.1: A subset A of a space X is called a α semi-open set if there is a α-open U such that U ⊆ A ⊆cl(U).

Definition 3.2: A subset A of a space X is called a $\hat{g}\alpha$-closed set if cl(A)⊆U whenever A⊆U and U is α semi-open in X.

Definition 3.3: A subset A of a space X is called a *gα-closed set if cl(A) ⊆ U whenever A ⊆ U and U is$\hat{g}\alpha$-open in X.

The interrelations among the setintroduced and other related sets are exhibited below:

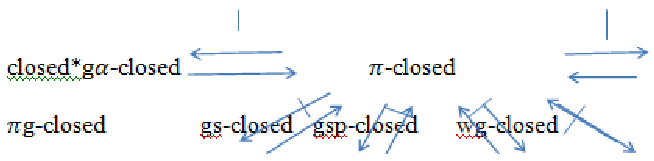

Diagram-1

The converse of the above interrelations need not be true as seen from the following examples.

Example 3.1: Let X = {a, b, c, d} and T = {ϕ, X, {a}, {b}, {a, b}, {a, b, c}}. Then
(i) The set A = {a, d} is*gα-closed but not closed in X.
(ii) The set A = {a, d} is *gα-closed but not π-closed in X.
(iii) The set A = {a, c} is πg-closed but not *gα-closed in X.
(iv) The set A = {c} is wg-closed but not *gα-closed in X.
(v) The set A = {a} is gs-closed but not *gα-closed in X.
(vi) The set A = {b} is gsp-closed but not*gα-closed in X.

Proposition 3.1: The union of two *gα-closed subsets of X is also a *gα-closedsubset of X.

Proof: Assume that A and B are *gα-closed sets in X. Let A∪ B ⊆ U and U be $\hat{g}\alpha$-open in X. Then A⊆ U and B ⊆ U and U is $\hat{g}\alpha$-open in X. Since A and B are *gα-closed sets in X, cl(A) ⊆ U and cl(B) ⊆ U. Hence, cl(A ∪ B) = cl(A) ∪ cl(B) ⊆ U. Therefore, A ∪ B is a *gα-closed set in X.

Proposition 3.2: The intersection of two *gα-closed subsets of X is also a *gα-closed subset of X.

Proposition 3.3: Let A be a *gα-closed set in X. Then cl(A)\A does not contain any non-empty $\hat{g}\alpha$-closed set in X.

Proof: Let U be a non-empty $\hat{g}\alpha$-closed subset of cl(A)\A. Now A⊆ X\U and X\U is $\hat{g}\alpha$-open in X. Since A is *gα-closed, cl(A) ⊆ X\U. Then U ⊆ X\cl(A). This is a contradiction since by assumption, U ⊆cl(A).

Proposition 3.4: Let A be a *gα-closed set in X. Then A is closed iffcl(A)\A is$\hat{g}\alpha$-closed.

Proposition 3.5: For every point x of a space X, X\{x} is *gα-closed (or) $\hat{g}\alpha$-open.

Proposition 3.6: Let A be a *gα-closed subset of (X, T) such that A ⊆ B ⊆cl(A). Then B is also a *gα-closed subset of (X, T).

Proof: Let B ⊆ U and U be $\hat{g}\alpha$-open in (X, T). Since A ⊆ B, A ⊆ U and U is a $\hat{g}\alpha$-open set in (X, T). Since A is *gα-closed, cl(A) ⊆ U. Then cl(B) ⊆ cl(cl(A)) = cl(A) ⊆ U. Therefore, cl(B) ⊆ U. Hence, B is *gα-closed.

The converse of the above Proposition need not be true as seen from the following Example.

Example 3.2: Let X = {a, b, c, d} and T = {ϕ, X, {a}, {b}, {a, b}, {a, b, c}}. Let A = {c, d}and B = {a, c, d}.Then A and B are *gα-closed sets in (X, T). But A = {c, d}⊆ B = {a, c, d} ⊈cl(A) = {c, d}.

Proposition 3.7: If a subset A of a topological space X is both $\hat{g}\alpha$-open and *gα-closed. Then A is a closed set.

Proposition 3.8: Let A be $\hat{g}\alpha$-open and *gα-closed in X. Suppose that F is closed in X. Then A ⋂ F is a *gα-closed set in X.

Proof: Let A be a $\hat{g}\alpha$-open and *gα-closed set in X and let F be a closed set in X. By Proposition 3.7, A is closed and so A⋂ F is closed. Since every closed set is *gα-closed, A ⋂ F is a *gα-closed set in X. Hence, A ⋂ F is a *gα-closed set in X.

Remark 3.1: If a subset A of a topological space X is
- (i) open and g-closed, then A is *gα-closed.
- (ii) π-open and πg-closed, then A is *gα-closed.
- (iii) semi-open and $\hat{g}$-closed, then A is *gα-closed.
- (iv) αsemi-open and $\hat{g}\alpha$-closed, then A is *gα-closed.
- (v) open and wg-closed, then A is *gα-closed.

Definition 3.4: A space X is called a $T_{*g\alpha}$-space if every *gα-closed set in it is closed.

Proposition 3.9: Every $T_{1/2}$-space is $T_{*g\alpha}$-space.

4*gα-CONTINUOUS MAPS

In this section we introduce and study *gα-continuous maps.

Definition 4.1: For a subset A of a space X, *gα-cl(A) = ⋂ {F : A ⊆ F, F is *gα-closed in X} is called the *gα-closure of A.

Definition 4.2: Let (X, T) be a topological space and $T_{*g\alpha}$ = {V ⊆X :*gα-cl(X\V) = X\V}.

Definition 4.3: A map f : (X, T)→ (Y, S) is called *gα-continuous if f^{-1}(V) is *gα-closedsetin X for every closed subset V of Y.

Remark 4.1: Let A and B be subsets of (X, T). Then
1) *gα-cl(ϕ) = ϕ and *gα-cl(X) = X.
2) If A ⊆ B, then *gα-cl (A) ⊆ *gα-cl (B).
3) A ⊆*gα-cl(A).
4) If A is *gα-closed, then *gα-cl(A) = A.

Proposition 4.1: Suppose $T_{*g\alpha}$is a topology. If A is *gα-closed in (X, T), then A is closed in (X, $T_{*g\alpha}$).

Proposition 4.2: A set A ⊆ X is *gα-open iff F ⊆int(A) whenever F ⊆ A and F is $\hat{g}\alpha$-closed.

Proof: Let A be a *gα-open set in X. Let F ⊆ A and F be $\hat{g}\alpha$-closed. Now X\A ⊆ X\F and X\F is $\hat{g}\alpha$-open. Since X\A is *gα-closed, cl(X\A) ⊆ X\F. Therefore, X\int(A) ⊆ X\F. Hence, F ⊆int(A).

Suppose F ⊆int(A) whenever F ⊆ A and F is $\hat{g}\alpha$-closed. Let X\F ⊆ U where U is $\hat{g}\alpha$-open. Then X\U ⊆ F ⊆ A, where X\U is $\hat{g}\alpha$-closed. By hypothesis, X\U ⊆int(A). This implies X\int(A) ⊆ U and so cl(X\A) ⊆ U, which implies X\A is *gα-closed. Therefore, A is *gα-open.

Proposition 4.3: Let X be a space in which every singleton set is $\hat{g}\alpha$-closed. Then f: (X, T)→ (Y, S) is $*g\alpha$-continuous iff $x \in int(f^{-1}(V))$ for every open subset V of Y which contains f(x).

Proof: Suppose f : (X, T)→ (Y, S) is $*g\alpha$-continuous. Fix $x \in X$ and an open set V in Y such that $f(x) \in V$. Then $f^{-1}(V)$ is $*g\alpha$-open. Since $x \in f^{-1}(V)$ and {x} is $\hat{g}\alpha$-closed, $x \in int(f^{-1}(V))$ by Proposition 4.2.

Suppose $x \in int(f^{-1}(V))$ for every open subset V of Y which contains f(x). Let V be an open set in Y. Suppose $F \subseteq f^{-1}(V)$ and F is $\hat{g}\alpha$-closed. Let $x \in F$. Then $f(x) \in V$ so that $x \in int(f^{-1}(V))$. This implies $F \subseteq int(f^{-1}(V))$. By Proposition 4.2, $f^{-1}(V)$ is $*g\alpha$-open. Hence, f is $*g\alpha$-continuous.

Proposition 4.4: Let f : (X, T) → (Y, S) be a map. Let (X, T) and (Y, S) be any two spaces such that $T_{*g\alpha}$ is a topology on X. Then the following statements are equivalent:

(i) For every subset A of X, $f(*g\alpha\text{-}cl(A)) \subseteq cl(f(A))$.

(ii) f : $(X, T_{*g\alpha})$ → (Y, S) is continuous.

Proof:

(i)⇒(ii): Suppose (i) holds. Let A be a closed set in Y. By (i), $f(*g\alpha\text{-}cl(f^{-1}(A))) \subseteq cl(f(f^{-1}(A))) \subseteq cl(A) = A$ and so $*g\alpha\text{-}cl(f^{-1}(A)) \subseteq f^{-1}(A)$. Also $f^{-1}(A) \subseteq *g\alpha\text{-}cl(f^{-1}(A))$. Hence, $*g\alpha\text{-}cl(f^{-1}(A)) = f^{-1}(A)$. This implies $(f^{-1}(A))^c \in T_{*g\alpha}$. Thus, $f^{-1}(A)$ is closed in $(X, T_{*g\alpha})$. Hence, f is continuous.

(ii)⇒(i): Suppose (ii) holds. Let A be a subset of X. Then cl(f(A)) is closed in Y. Since f: $(X, T_{*g\alpha})$ → (Y, S) is continuous, $f^{-1}(cl(f(A)))$ is closed in $(X, T_{*g\alpha})$. By Definition 4.2, $*g\alpha\text{-}cl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A)))$. Now $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(cl(f(A)))$, which implies $*g\alpha\text{-}cl(A) \subseteq *g\alpha\text{-}cl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A)))$. Therefore, $f(*g\alpha\text{-}cl(A)) \subseteq cl(f(A))$.

Proposition 4.5: The composition of two $*g\alpha$-continuous maps need not be a $*g\alpha$-continuous mapin general as seen from the following Example.

Example 4.1: Let X = Y = Z = {a, b, c, d}, T = {ϕ, X, {c}, {a, b}, {a, b, c}},S = {ϕ, Y, {a}, {b}, {a, b}} and R = {ϕ, Z, {a}, {b}, {a, b}, {a, b, c}}. Let f : (X, T) → (Y, S) be the identity map. Then clearly, f is $*g\alpha$-continuous. Let g : (Y, S) → (Z, R) be defined by g(a) = a, g(b) = b, g(c) = d and g(d) = c. Then clearly, g is $*g\alpha$-continuous. But their composition $g\circ f$: (X, T) → (Z, R) is not $*g\alpha$-continuous, since for the closed set {d} in (Z, R), $(g f)^{-1}(\{d\}) = f^{-1}(g^{-1}\{d\}) = f^{-1}\{c\} = \{c\}$, which is not $*g\alpha$-closed in (X, T).

Proposition 4.6: Let (X, T), (Y, S) and (Z, R) be topological spaces such that $S_{*g\alpha}$= S. Let f : (X, T) → (Y, S) and g : (Y, S) → (Z, R) be $*g\alpha$-continuous. Then their composition gf : (X, T) → (Z, R) is also a $*g\alpha$-continuous.

The interrelations among the map introduced and other related maps are exhibited below:

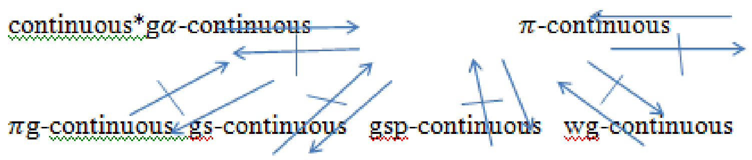

Diagram-2

Proposition 4.7: Let f : (X, T) → (Y, S) be a map. Then the following are equivalent:

(i) f is $*g\alpha$-continuous.

(ii) The inverse image of each open set in (Y, S) is $*g\alpha$-open in (X, T).

(iii) The inverse image of each closed set in (Y, S) is $*g\alpha$-closed in (X, T).

Proof:

(i)⇒(ii): Suppose (i) holds. Let V be open in Y. Then Y\V is closed in Y. Since f is $*g\alpha$-continuous, $f^{-1}(Y\setminus V)$ is $*g\alpha$-closed in X. But $f^{-1}(Y\setminus V) = X\setminus f^{-1}(V)$ which is $*g\alpha$-closed in X. Therefore, $f^{-1}(V)$ is $*g\alpha$-open in X. Hence, the inverse image of each open set in (Y, S) is $*g\alpha$-open in (X, T).

(ii)⇒(iii): Suppose (ii) holds. Let V be a closed set in Y. Then Y\V is open in Y. Since the inverse image of each open set in (Y, S) is $*g\alpha$-open in (X, T), $f^{-1}(Y\setminus V)$ is $*g\alpha$-open. But $f^{-1}(Y\setminus V) = X\setminus f^{-1}(V)$ which is $*g\alpha$-open. Therefore, $f^{-1}(V)$ is $*g\alpha$-closed in X. Hence, the inverse image of each closed set in (Y, S) is $*g\alpha$-closed in (X, T).

(iii)⇒(i): Suppose (iii) holds. Let V be a closed set in Y. Since, the inverse image of each closed set in (Y, S) is $*g\alpha$-closed in (X, T), $f^{-1}(V)$ is $*g\alpha$-closed in X. Hence, f is $*g\alpha$-continuous.

Proposition 4.8: If a mapf : (X, T) → (Y, S) is $*g\alpha$-continuous, then $f(*g\alpha\text{-}cl(A)) \subseteq cl(f(A))$ for every subset A of X.

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