

GENERALIZATION
OF SOFT $\tau_1\tau_2\tau_3$ - α -NORMALITIES IN SOFT TRITOPOLOGICAL SPACES

DR. G. VASUKI¹, DR. B. AMUDHAMBIGAI² AND K. RATHIKA³

¹Associate Professor, Department of Mathematics,
Sri Sarada College for Women (Autonomous), Salem-16, India.

²Assistant Professor, Department of Mathematics,
Sri Sarada College for Women (Autonomous), Salem-16, India.

³Research Scholar, Department of Mathematics,
Sri Sarada College for Women (Autonomous), Salem-16, India.

E-mail: vasukigj@gmail.com¹, rbamudha@yahoo.co.in² and ksrathima@gmail.com³.

ABSTRACT

In this paper, the concepts of soft $\tau_1\tau_2\tau_3$ - α -open sets and soft $\tau_1\tau_2\tau_3$ - α -continuity in soft tritopological spaces are introduced. Also various generalization of soft $\tau_1\tau_2\tau_3$ - α -normal spaces and their properties are investigated in soft tritopological spaces.

Keywords: soft $\tau_1\tau_2\tau_3$ - α -open sets, soft $\tau_1\tau_2\tau_3$ - α -continuous functions, soft almost $\tau_1\tau_2\tau_3$ - α -normal spaces, soft almost $\tau_1\tau_2\tau_3$ - α - β -normal spaces, soft $\tau_1\tau_2\tau_3$ - α - k -normal spaces and soft $\tau_1\tau_2\tau_3$ - α - α -normal spaces.

1. INTRODUCTION

The concept of α -open set was introduced by O. Njastad [10] in 1995. The study of tritopological space was first initiated by Martin M.Kovar [6]. S.Palanimmal [11] study of tritopological spaces. N.F.Hameed and Moh.Yahya Abid [5] gives the definition of 123 open set in tritopological space. The concept of soft topological space is introduced in [13]. Normality plays a prominent role in general topology and several generalized notions of normality such as almost normal [14], k -normal [12, 15], almost β -normal [4]. In [1] A.V.Arhangels'skii and Ludwig introduced the concept of α -normal and β -normal spaces and Eva. Murtinova in [9] provided an example of $\alpha\beta$ -normal Tychonoff space which is not normal. In this paper, we introduce soft $\tau_1\tau_2\tau_3$ - α -open sets and soft $\tau_1\tau_2\tau_3$ - α -continuity in soft tritopological spaces are introduced. Also various generalization of soft $\tau_1\tau_2\tau_3$ - α -normal spaces and their properties are investigated in soft tritopological spaces.

2 PRELIMINARIES

In this section, the basic concepts about soft tri topological spaces are studied.

Definition 2.1: [8] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denotes the power set of X and A be a nonempty subset of E . A pair (F, A) is called a soft set over X , where F is a mapping given by $F: A \rightarrow P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X . For $e \in A$, $F(e)$ may be considered as the set of e -approximate elements of the soft set (F, A) . Clearly, a soft set is not a set.

Definition 2.2: [7] The complement of a soft set (F, A) over X is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $F^c: A \rightarrow P(X)$ is a mapping given by $F^c(\alpha) = X - F(\alpha)$ for all $\alpha \in A$.

Definition 2.3: [3] Let (F, E) be a soft set over X . The soft set (F, E) is called a soft point, denoted by (x_e, E) , if for the element $e \in E$, $F(e) = x$ and $F(e') = \emptyset_E$ for all $e' \in E - \{e\}$ (briefly, denoted by x_e).

Definition 2.4: [13] Let τ be the collection of soft sets over X , then τ is said to be a soft topology on X , if

- (1) \emptyset, X belong to τ .
- (2) The union of any number of soft sets in τ belongs to τ .
- (3) The intersection of any two soft sets in τ belongs to τ .

Then triple (X, τ, E) is called a soft topological space over X .

Definition 2.5: [2] Let (X, τ_1, E) , (X, τ_2, E) and (X, τ_3, E) be the three soft topological spaces on X . Then $(X, \tau_1, \tau_2, \tau_3, E)$ is called a soft tritopological space. The three soft topological spaces (X, τ_1, E) , (X, τ_2, E) and (X, τ_3, E) are independently satisfy the axioms of a soft topological space.

The members of τ_1 are called soft open sets and the complement of τ_1 open sets are called soft closed sets. And the members of τ_2 are called soft open sets and the complement of τ_2 open sets are called soft closed sets. Similarly, the members of τ_3 are called soft open sets and the complement of τ_3 open sets are called soft closed sets.

Definition 2.6: [2] Let $(X, \tau_1, \tau_2, \tau_3, E)$ be a soft tritopological space and (F, E) is soft set in X , then (F, E) is called a soft $\tau_1\tau_2\tau_3$ -open set if $(F, E) = (A, E) \cup (B, E) \cup (C, E)$, where $(A, E) \in \tau_1$, $(B, E) \in \tau_2$ and $(C, E) \in \tau_3$. The complement of a soft $\tau_1\tau_2\tau_3$ -open set is called a soft $\tau_1\tau_2\tau_3$ -closed.

Definition 2.7: [2] Let $(X, \tau_1, \tau_2, \tau_3, E)$ be a soft tritopological space and (F, E) is a soft set in X , then the soft $\tau_1\tau_2\tau_3$ -interior of (F, E) , denoted by $s.\tau_1\tau_2\tau_3$ -int (F, E) is defined by $s.\tau_1\tau_2\tau_3$ -int $(F, E) = \cup\{(B, E) : (B, E) \subseteq (F, E) \text{ and } (B, E) \text{ is soft } \tau_1\tau_2\tau_3\text{-open}\}$.

Definition 2.8: [2] Let $(X, \tau_1, \tau_2, \tau_3, E)$ be a soft tritopological space and (F, E) is a soft set in X , then the soft $\tau_1\tau_2\tau_3$ -closure of (F, E) , denoted by $s.\tau_1\tau_2\tau_3$ -cl (F, E) is defined by $s.\tau_1\tau_2\tau_3$ -cl $(F, E) = \cup\{(C, E) : (C, E) \supseteq (F, E) \text{ and } (C, E) \text{ is soft } \tau_1\tau_2\tau_3\text{-closed}\}$.

Definition 2.9: [2] Let $(X, \tau_1, \tau_2, \tau_3, E)$ be a soft tritopological space and (F, E) is a soft set in X , then (F, E) is called soft $\tau_1\tau_2\tau_3$ - α -open set (or soft tri- α -open) if $(F, E) \subseteq s.\tau_1\tau_2\tau_3$ -int $(s.\tau_1\tau_2\tau_3$ -cl $(s.\tau_1\tau_2\tau_3$ -int $(F, E)))$. The complement of a soft $\tau_1\tau_2\tau_3$ - α -open set is called a soft $\tau_1\tau_2\tau_3$ - α -closed set.

3. Soft $\tau_1\tau_2\tau_3$ - α -normalities

In this section, we introduce the concepts of soft $\tau_1\tau_2\tau_3$ - α -continuous functions, soft $\tau_1\tau_2\tau_3$ - α -regular open sets, soft $\tau_1\tau_2\tau_3$ - α - k -normal spaces, soft almost $\tau_1\tau_2\tau_3$ - α -normal spaces, soft $\tau_1\tau_2\tau_3$ - α -normal spaces and soft almost $\tau_1\tau_2\tau_3$ - α - β -normal spaces in soft tritopological spaces are introduced and study of some their properties. Also the characterization of soft almost $\tau_1\tau_2\tau_3$ - α - β -normal space established.

Definition 3.1: Let $(X, \tau_1, \tau_2, \tau_3, E)$ be a soft tritopological space. For any soft set (A, E) over X , the **soft $\tau_1\tau_2\tau_3$ - α -interior of (A, E)** (briefly, $s.\tau_1\tau_2\tau_3$ - α -int) is defined as follows $s.\tau_1\tau_2\tau_3$ - α -int $(A, E) = \cup\{(F, E) : (F, E) \subseteq (A, E) \text{ and } (F, E) \text{ is a soft } \tau_1\tau_2\tau_3\text{-}\alpha\text{-open set over } X\}$.

Definition 3.2: Let $(X, \tau_1, \tau_2, \tau_3, E)$ be a soft tritopological space. For any soft set (A, E) over X , the **soft $\tau_1\tau_2\tau_3$ - α -closure of (A, E)** (briefly, $s.\tau_1\tau_2\tau_3$ - α -cl) is defined as follows $s.\tau_1\tau_2\tau_3$ - α -cl $(A, E) = \cap\{(F, E) : (F, E) \supseteq (A, E) \text{ and } (F, E) \text{ is a soft } \tau_1\tau_2\tau_3\text{-}\alpha\text{-closed set over } X\}$.

Definition 3.3: Let $(X, \tau_1, \tau_2, \tau_3, E_1)$ and $(Y, \sigma_1, \sigma_2, \sigma_3, E_2)$ be two soft tritopological spaces. Any function $f : (X, \tau_1, \tau_2, \tau_3, E_1) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3, E_2)$ is called **soft $\tau_1\tau_2\tau_3$ - α -continuous** if $f^{-1}(U, E_2)$ is soft $\tau_1\tau_2\tau_3$ -open set in $(X, \tau_1, \tau_2, \tau_3, E_1)$ for each soft $\tau_1\tau_2\tau_3$ -open set (U, E_2) in $(Y, \sigma_1, \sigma_2, \sigma_3, E_2)$.

Definition 3.4: Let $(X, \tau_1, \tau_2, \tau_3, E_1)$ and $(Y, \sigma_1, \sigma_2, \sigma_3, E_2)$ be two soft tritopological space. Any function $f : (X, \tau_1, \tau_2, \tau_3, E_1) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3, E_2)$ is called **soft $\tau_1\tau_2\tau_3$ - α -continuous** if $f^{-1}(U, E_2)$ is soft $\tau_1\tau_2\tau_3$ - α -open set in $(X, \tau_1, \tau_2, \tau_3, E_1)$ for each soft $\tau_1\tau_2\tau_3$ -open set (U, E_2) in $(Y, \sigma_1, \sigma_2, \sigma_3, E_2)$.

Definition 3.5: Let $(X, \tau_1, \tau_2, \tau_3, E_1)$ and $(Y, \sigma_1, \sigma_2, \sigma_3, E_2)$ be two soft tritopological spaces. Any function $f : (X, \tau_1, \tau_2, \tau_3, E_1) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3, E_2)$ is called **soft $\tau_1\tau_2\tau_3$ - α -open (soft $\tau_1\tau_2\tau_3$ - α -closed)** if $f(U, E_1)$ is soft $\sigma_1\sigma_2\sigma_3$ - α -open set (soft $\tau_1\tau_2\tau_3$ - α -closed set) in $(Y, \sigma_1, \sigma_2, \sigma_3, E_2)$ for each soft $\tau_1\tau_2\tau_3$ - α -open set (soft $\tau_1\tau_2\tau_3$ - α -closed set) (U, E_1) in $(X, \tau_1, \tau_2, \tau_3, E_1)$.

Definition 3.6: A soft tritopological space $(X, \tau_1, \tau_2, \tau_3, E)$ is said to be a **soft $\tau_1\tau_2\tau_3$ - $\alpha_{1/2}$ space** if every soft $\tau_1\tau_2\tau_3$ - α -open set in $(X, \tau_1, \tau_2, \tau_3, E)$ is $\tau_1\tau_2\tau_3$ -open set.

Definition 3.7: Let $(X, \tau_1, \tau_2, \tau_3, E)$ be a soft tritopological space. Any soft set (A, E) over X is said to be **soft $\tau_1\tau_2\tau_3-\alpha$ -regular open** if $(A, E) = s\tau_1\tau_2\tau_3-\alpha\text{-int}(s\tau_1\tau_2\tau_3-\alpha\text{-cl}(A, E))$. The complement of a soft $\tau_1\tau_2\tau_3-\alpha$ -regular open set is called a soft $\tau_1\tau_2\tau_3-\alpha$ -regular closed.

Definition 3.8: A soft tritopological space $(X, \tau_1, \tau_2, \tau_3, E)$ is said to be **soft $\tau_1\tau_2\tau_3-\alpha$ -k-normal** if for every pair of disjoint soft regular $\tau_1\tau_2\tau_3-\alpha$ -closed sets (A, E) and (B, E) over X , there exist disjoint soft $\tau_1\tau_2\tau_3-\alpha$ -open sets (U, E) and (V, E) over X such that $(A, E) \subseteq (U, E)$ and $(B, E) \subseteq (V, E)$.

Definition 3.9: A soft tritopological space $(X, \tau_1, \tau_2, \tau_3, E)$ is said to be **soft almost $\tau_1\tau_2\tau_3-\alpha$ -normal** if for every pair of disjoint soft $\tau_1\tau_2\tau_3-\alpha$ -closed sets (A, E) and (B, E) over X , one of which is soft $\tau_1\tau_2\tau_3-\alpha$ -regular closed, there exist disjoint soft $\tau_1\tau_2\tau_3-\alpha$ -open sets (U, E) and (V, E) over X such that $(A, E) \subseteq (U, E)$ and $(B, E) \subseteq (V, E)$.

Definition 3.10: A soft tritopological space $(X, \tau_1, \tau_2, \tau_3, E)$ is said to be **soft $\tau_1\tau_2\tau_3-\alpha-\alpha$ -normal** if for any two disjoint soft $\tau_1\tau_2\tau_3-\alpha$ -closed sets (A, E) and (B, E) over X , there exist disjoint soft $\tau_1\tau_2\tau_3-\alpha$ -open sets (U, E) and (V, E) over X such that $s\tau_1\tau_2\tau_3-\alpha\text{-cl}((A, E) \cap (U, E)) = (A, E)$ and $s\tau_1\tau_2\tau_3-\alpha\text{-cl}((B, E) \cap (U, E)) = (B, E)$.

Definition 3.11: A soft tritopological space $(X, \tau_1, \tau_2, \tau_3, E)$ is said to be **soft almost $\tau_1\tau_2\tau_3-\alpha-\beta$ -normal** if for every pair of disjoint soft $\tau_1\tau_2\tau_3-\alpha$ -closed sets (A, E) and (B, E) over X , one of which is soft $\tau_1\tau_2\tau_3-\alpha$ -regular closed, there exist disjoint soft $\tau_1\tau_2\tau_3-\alpha$ -open sets (U, E) and (V, E) over X such that $s\tau_1\tau_2\tau_3-\alpha\text{-cl}((A, E) \cap (U, E)) = (A, E)$, $s\tau_1\tau_2\tau_3-\alpha\text{-cl}((B, E) \cap (U, E)) = (B, E)$ and $s\tau_1\tau_2\tau_3-\alpha\text{-cl}(U, E) \cap s\tau_1\tau_2\tau_3-\alpha\text{-cl}(V, E) = \emptyset_E$.

Definition 3.12: A soft tritopological space $(X, \tau_1, \tau_2, \tau_3, E)$ is said to be **soft semi- $\tau_1\tau_2\tau_3-\alpha$ -normal** if for every soft $\tau_1\tau_2\tau_3-\alpha$ -closed (A, E) over X contained in soft $\tau_1\tau_2\tau_3-\alpha$ -open set (U, E) over X , there exists a soft $\tau_1\tau_2\tau_3-\alpha$ -regular open set (V, E) over X such that $(A, E) \subset (V, E) \subset (U, E)$.

Definition 3.13: Any soft tritopological space $(X, \tau_1, \tau_2, \tau_3, E)$ is called a **soft $\tau_1\tau_2\tau_3$ -Hausdorff space** if for any two distinct soft points $x_e, y_e \in (X, E)$ for all $e \in E$, there exist soft $\tau_1\tau_2\tau_3$ -open sets (U, E) and (V, E) over X such that $x_e \in (U, E)$, $y_e \in (V, E)$ and $(U, E) \cap (V, E) = \emptyset_E$.

Definition 3.14: A soft $\tau_1\tau_2\tau_3$ -Hausdorff space $(X, \tau_1, \tau_2, \tau_3, E)$ is said to be **soft extremally $\tau_1\tau_2\tau_3-\alpha$ -disconnected** if the soft $\tau_1\tau_2\tau_3-\alpha$ -closure of every $\tau_1\tau_2\tau_3-\alpha$ -open set in $(X, \tau_1, \tau_2, \tau_3, E)$ is soft $\tau_1\tau_2\tau_3-\alpha$ -open.

Proposition 3.1: Every soft almost $\tau_1\tau_2\tau_3-\alpha$ -normal space is soft almost $\tau_1\tau_2\tau_3-\alpha-\beta$ -normal.

Proof: Let $(X, \tau_1, \tau_2, \tau_3, E)$ be a soft almost $\tau_1\tau_2\tau_3-\alpha$ -normal space. Let (A, E) and (B, E) be two disjoint soft $\tau_1\tau_2\tau_3-\alpha$ -closed sets in $(X, \tau_1, \tau_2, \tau_3, E)$ one of which (say (A, E)) is soft $\tau_1\tau_2\tau_3-\alpha$ -regular closed. Since $(X, \tau_1, \tau_2, \tau_3, E)$ is soft almost $\tau_1\tau_2\tau_3-\alpha$ -normal, there exist disjoint soft $\tau_1\tau_2\tau_3-\alpha$ -open sets (W, E) and (V, E) over X containing (A, E) and (B, E) respectively. Since $(W, E) \cap (V, E) = \emptyset_E$, $(W, E) \cap (s\tau_1\tau_2\tau_3-\alpha\text{-cl}(V, E)) = \emptyset_E$. Let $(U, E) = s\tau_1\tau_2\tau_3-\alpha\text{-int}(A, E)$. Then $s\tau_1\tau_2\tau_3-\alpha\text{-cl}((A, E) \cap (U, E)) = (A, E)$, $s\tau_1\tau_2\tau_3-\alpha\text{-cl}((B, E) \cap (V, E)) = (B, E)$ and $s\tau_1\tau_2\tau_3-\alpha\text{-cl}(U, E) \cap s\tau_1\tau_2\tau_3-\alpha\text{-cl}(V, E) = \emptyset_E$. So, $(X, \tau_1, \tau_2, \tau_3, E)$ is soft almost $\tau_1\tau_2\tau_3-\alpha-\beta$ -normal.

Remark: The converse of Proposition 3.1 need not be true as shown in Example 3.1.s

Example 3.1: Let $X = \{a, b, c, d\}$ and $E = \{e\}$. Let $\tau_1 = \{\emptyset_E, X, (F_1, E), (F_2, E), (F_3, E)\}$, $\tau_2 = \{\emptyset_E, X, (F_4, E), (F_5, E)\}$ and $\tau_3 = \{\emptyset_E, X, (F_6, E), (F_7, E)\}$, where $(F_1, E) = \{(e, \{b\})\}$, $(F_2, E) = \{(e, \{a, d\})\}$, $(F_3, E) = \{(e, \{a, b, d\})\}$, $(F_4, E) = \{(e, \{b, d\})\}$, $(F_5, E) = \{(e, \{b, c, d\})\}$, $(F_6, E) = \{(e, \{b, c\})\}$ and $(F_7, E) = \{(e, \{a, b, c\})\}$ are soft open sets over X . Clearly τ_1, τ_2 and τ_3 are soft topological spaces and so $(X, \tau_1, \tau_2, \tau_3, E)$ is soft tritopological space. The collection of soft $\tau_1\tau_2\tau_3-\alpha$ -open sets over X is $\{\emptyset_E, X, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E)\}$. Let $(A, E) = \{(e, \{b, c\})\}$, then (A, E) is soft $\tau_1\tau_2\tau_3-\alpha$ -regularly closed set. For the soft $\tau_1\tau_2\tau_3-\alpha$ -closed set $(B, E) = \{(e, \{d\})\}$, there exist disjoint soft $\tau_1\tau_2\tau_3-\alpha$ -open sets $(U, E) = \{(e, \{b\})\}$ and $(V, E) = \{(e, \{a, d\})\}$. Hence, $(X, \tau_1, \tau_2, \tau_3, E)$ is soft almost $\tau_1\tau_2\tau_3-\alpha-\beta$ -normal space but soft almost $\tau_1\tau_2\tau_3-\alpha$ -normal space.

Proposition 3.2: Every soft tritopological space $(X, \tau_1, \tau_2, \tau_3, E)$ which is both soft extremally $\tau_1\tau_2\tau_3-\alpha$ -disconnected and soft almost $\tau_1\tau_2\tau_3-\alpha-\beta$ -normal is soft almost $\tau_1\tau_2\tau_3-\alpha$ -normal.

Proof: Let $(X, \tau_1, \tau_2, \tau_3, E)$ be a soft extremally $\tau_1\tau_2\tau_3-\alpha$ -disconnected and soft almost $\tau_1\tau_2\tau_3-\alpha-\beta$ -normal space. Let (A, E) be a soft $\tau_1\tau_2\tau_3-\alpha$ -regular closed set over X which is disjoint from the soft $\tau_1\tau_2\tau_3-\alpha$ -closed set (B, E) over X . By soft almost $\tau_1\tau_2\tau_3-\alpha-\beta$ -normality of $(X, \tau_1, \tau_2, \tau_3, E)$, there exist disjoint soft $\tau_1\tau_2\tau_3-\alpha$ -open sets (U, E) and (V, E) over X such that $s\tau_1\tau_2\tau_3-\alpha\text{-cl}((A, E) \cap (U, E)) = (A, E)$, $s\tau_1\tau_2\tau_3-\alpha\text{-cl}((B, E) \cap (V, E)) = (B, E)$ and $s\tau_1\tau_2\tau_3-\alpha\text{-cl}(U, E) \cap s\tau_1\tau_2\tau_3-\alpha\text{-cl}(V, E) = \emptyset_E$. Thus $(A, E) \subset s\tau_1\tau_2\tau_3-\alpha\text{-cl}(U, E)$ and $(B, E) \subset s\tau_1\tau_2\tau_3-\alpha\text{-cl}(V, E)$. By the soft extremally $\tau_1\tau_2\tau_3-\alpha$ -disconnectedness of $(X, \tau_1, \tau_2, \tau_3, E)$, $s\tau_1\tau_2\tau_3-\alpha\text{-cl}(U, E)$ and $s\tau_1\tau_2\tau_3-\alpha\text{-cl}(V, E)$ are

disjoint soft $\tau_1\tau_2\tau_3$ - α -open sets containing (A, E) and (B,E) respectively. Hence, $(X, \tau_1, \tau_2, \tau_3, E)$ is soft almost $\tau_1\tau_2\tau_3$ - α -normal.

Proposition 3.3: For any soft tritopological space $(X, \tau_1, \tau_2, \tau_3, E)$, the following statements are equivalent:

1. $(X, \tau_1, \tau_2, \tau_3, E)$ is soft almost $\tau_1\tau_2\tau_3$ - α - β -normal.
2. Whenever $(F, E), (G, E)$ are disjoint soft $\tau_1\tau_2\tau_3$ - α -closed sets over X and (F, E) is soft $\tau_1\tau_2\tau_3$ - α -regular closed, there is a soft $\tau_1\tau_2\tau_3$ - α -open set (V, E) over X such that $(G, E) = s\tau_1\tau_2\tau_3$ - α - $cl((V, E) \cap (G, E))$ and $(F, E) \cap s\tau_1\tau_2\tau_3$ - α - $cl(V, E) = \emptyset_E$.
3. Whenever (F, E) is soft $\tau_1\tau_2\tau_3$ - α -closed over X, (U, E) is a soft $\tau_1\tau_2\tau_3$ - α -regular open set over X and $(F, E) \subseteq (U, E)$, there is a soft $\tau_1\tau_2\tau_3$ - α -open set (V, E) over X such that $(F, E) = s\tau_1\tau_2\tau_3$ - α - $cl((V, E) \cap (F, E)) \subseteq s\tau_1\tau_2\tau_3$ - α - $cl(V, E) \subseteq (U, E)$.

Proof:

(1) \Rightarrow (2): Suppose that $(F, E), (G, E)$ are disjoint soft $\tau_1\tau_2\tau_3$ - α -closed set over X and (F, E) is soft $\tau_1\tau_2\tau_3$ - α -regular closed. Since $(X, \tau_1, \tau_2, \tau_3, E)$ is soft almost $\tau_1\tau_2\tau_3$ - α - β -normal, there exist soft $\tau_1\tau_2\tau_3$ - α -open sets (U, E) and (V, E) over X such that $(F, E) = s\tau_1\tau_2\tau_3$ - α - $cl((U, E) \cap (F, E)) \subseteq s\tau_1\tau_2\tau_3$ - α - $cl(U, E)$,

$(G, E) = s\tau_1\tau_2\tau_3$ - α - $cl((G, E) \cap (V, E)) \subseteq s\tau_1\tau_2\tau_3$ - α - $cl(V, E)$ and $s\tau_1\tau_2\tau_3$ - α - $cl((U, E) \cap s\tau_1\tau_2\tau_3$ - α - $cl(V, E)) = \emptyset_E$. Then $(F, E) \cap s\tau_1\tau_2\tau_3$ - α - $cl(V, E) = \emptyset_E$.

(2) \Rightarrow (1): Suppose that $(F, E), (G, E)$ are disjoint soft $\tau_1\tau_2\tau_3$ - α -closed sets over X and (F, E) is soft $\tau_1\tau_2\tau_3$ - α -regular closed. By the assumption, there exists a soft $\tau_1\tau_2\tau_3$ - α -open set (V, E) over X such that $(G, E) = s\tau_1\tau_2\tau_3$ - α - $cl((V, E) \cap (G, E))$ and $(F, E) \cap s\tau_1\tau_2\tau_3$ - α - $cl(V, E) = \emptyset_E$. Let $(U, E) = s\tau_1\tau_2\tau_3$ - α - $int(F, E)$. Then $(F, E) = s\tau_1\tau_2\tau_3$ - α - $cl((U, E) \cap (F, E))$ and $s\tau_1\tau_2\tau_3$ - α - $cl(U, E) \cap s\tau_1\tau_2\tau_3$ - α - $cl(V, E) = (F, E) \cap s\tau_1\tau_2\tau_3$ - α - $cl(V, E) = \emptyset_E$.

(1) \Rightarrow (3): Suppose that (F, E) is soft $\tau_1\tau_2\tau_3$ - α -closed and (U, E) is soft $\tau_1\tau_2\tau_3$ - α -regular open over X and $(F, E) \subseteq (U, E)$. Since (U, E) is soft $\tau_1\tau_2\tau_3$ - α -regular open, $X_E - (U, E)$ is soft $\tau_1\tau_2\tau_3$ - α -regular closed set over X.

Since $(X, \tau_1, \tau_2, \tau_3, E)$ is soft almost β - $\tau_1\tau_2\tau_3$ - α -normal, there are soft $\tau_1\tau_2\tau_3$ - α -open sets (O, E) and (V, E) over X such that $X_E - (U, E) = s\tau_1\tau_2\tau_3$ - α - $cl((O, E) \cap (X_E - (U, E))) \subseteq s\tau_1\tau_2\tau_3$ - α - $cl(O, E)$, $(F, E) = s\tau_1\tau_2\tau_3$ - α - $cl((V, E) \cap (F, E)) \subseteq s\tau_1\tau_2\tau_3$ - α - $cl(V, E)$ and $s\tau_1\tau_2\tau_3$ - α - $cl(O, E) \cap s\tau_1\tau_2\tau_3$ - α - $cl(V, E) = \emptyset_E$. Then $(X_E - (U, E)) \cap s\tau_1\tau_2\tau_3$ - α - $cl(V, E) = \emptyset_E$ which means that $s\tau_1\tau_2\tau_3$ - α - $cl(V, E) \subseteq (U, E)$.

(3) \Rightarrow (2): Suppose that $(F, E), (G, E)$ are disjoint soft $\tau_1\tau_2\tau_3$ - α -closed sets over X and (F, E) is soft $\tau_1\tau_2\tau_3$ - α -regular closed. Then $(G, E) \subseteq X_E - (F, E)$ and $(X_E - (F, E))$ is soft $\tau_1\tau_2\tau_3$ - α -regular open set. By the hypothesis, there is a soft $\tau_1\tau_2\tau_3$ - α -open set (V, E) over X such that $(G, E) = s\tau_1\tau_2\tau_3$ - α - $cl((V, E) \cap (G, E)) \subseteq s\tau_1\tau_2\tau_3$ - α - $cl(V, E) \subseteq X_E - (F, E)$. Then $(F, E) \cap s\tau_1\tau_2\tau_3$ - α - $cl(V, E) = \emptyset_E$.

Proposition 3.4: A soft tritopological space $(X, \tau_1, \tau_2, \tau_3, E)$ is soft almost $\tau_1\tau_2\tau_3$ - α -normal if and only if it is both soft almost $\tau_1\tau_2\tau_3$ - α - β -normal and soft $\tau_1\tau_2\tau_3$ - α -k-normal.

Proof: Let $(X, \tau_1, \tau_2, \tau_3, E)$ be soft almost $\tau_1\tau_2\tau_3$ - α - β -normal and soft $\tau_1\tau_2\tau_3$ - α -k-normal. Let (A, E) and (B, E) be two disjoint soft $\tau_1\tau_2\tau_3$ - α -closed sets over X in which (A, E) is soft $\tau_1\tau_2\tau_3$ - α -regular closed. By soft almost $\tau_1\tau_2\tau_3$ - α - β -normality of $(X, \tau_1, \tau_2, \tau_3, E)$, there exist disjoint soft $\tau_1\tau_2\tau_3$ - α -open sets (U, E) and (V, E) over X such that $s\tau_1\tau_2\tau_3$ - α - $cl((A, E) \cap (U, E)) = (A, E)$, $s\tau_1\tau_2\tau_3$ - α - $cl((B, E) \cap (V, E)) = (B, E)$ and $s\tau_1\tau_2\tau_3$ - α - $cl(U, E) \cap s\tau_1\tau_2\tau_3$ - α - $cl(V, E) = \emptyset_E$. Thus $(A, E) \subseteq s\tau_1\tau_2\tau_3$ - α - $cl(U, E)$ and $(B, E) \subseteq s\tau_1\tau_2\tau_3$ - α - $cl(V, E)$. Here $s\tau_1\tau_2\tau_3$ - α - $cl(U, E)$ and $s\tau_1\tau_2\tau_3$ - α - $cl(V, E)$ are disjoint soft $\tau_1\tau_2\tau_3$ - α -regular closed sets over X. So by soft $\tau_1\tau_2\tau_3$ - α -k-normality of $(X, \tau_1, \tau_2, \tau_3, E)$, there exist disjoint soft $\tau_1\tau_2\tau_3$ - α -open sets (W_1, E) and (W_2, E) over X such that $s\tau_1\tau_2\tau_3$ - α - $cl(U, E) \subseteq (W_1, E)$ and $s\tau_1\tau_2\tau_3$ - α - $cl(V, E) \subseteq (W_2, E)$. Hence, $(X, \tau_1, \tau_2, \tau_3, E)$ is soft almost $\tau_1\tau_2\tau_3$ - α -normal. Necessity of the statement can be proved by using Proposition 3.1 and by using similar proof for soft $\tau_1\tau_2\tau_3$ - α -k-normal space.

Proposition 3.5: Every soft tritopological space $(X, \tau_1, \tau_2, \tau_3, E)$ which is both soft $\tau_1\tau_2\tau_3$ - α -seminormal and soft almost $\tau_1\tau_2\tau_3$ - α - β -normal is soft $\tau_1\tau_2\tau_3$ - α - α -normal.

Proof: Let $(X, \tau_1, \tau_2, \tau_3, E)$ be a soft semi $\tau_1\tau_2\tau_3$ - α -normal and soft β - $\tau_1\tau_2\tau_3$ - α -normal. Let (A, E) and (B, E) be two disjoint soft $\tau_1\tau_2\tau_3$ - α -closed sets over X. Thus $(A, E) \subseteq (X_E - (B, E))$. By soft $\tau_1\tau_2\tau_3$ - α -seminormality of $(X, \tau_1, \tau_2, \tau_3, E)$, there exists a soft $\tau_1\tau_2\tau_3$ - α -regular open set (F, E) over X such that $(A, E) \subseteq (F, E) \subseteq (X_E - (B, E))$. Now (A, E) and $X_E - (F, E)$ are disjoint soft $\tau_1\tau_2\tau_3$ - α -closed sets over X in which $X_E - (F, E)$ is a soft $\tau_1\tau_2\tau_3$ - α -regular closed set containing (B, E) . Since $(X, \tau_1, \tau_2, \tau_3, E)$ is soft almost $\tau_1\tau_2\tau_3$ - α - β -normal, there exist disjoint soft almost $\tau_1\tau_2\tau_3$ - α -open sets (U, E) and (V, E) over X such that

$s\tau_1\tau_2\tau_3$ - α - $cl((A,E) \cap (U,E)) = (A,E)$, $s\tau_1\tau_2\tau_3$ - α - $cl((V,E) \cap (X_E - (F,E))) = (X_E - (F,E))$ and $s\tau_1\tau_2\tau_3$ - α - $cl(U,E) \cap s\tau_1\tau_2\tau_3$ - α - $cl(V,E) = \emptyset_E$. So $(A,E) = s\tau_1\tau_2\tau_3$ - α - $cl((A,E) \cap (U,E)) \subset s\tau_1\tau_2\tau_3$ - α - $cl(U,E)$ and $(X_E - (F,E)) = s\tau_1\tau_2\tau_3$ - α - $cl((X_E - (F,E)) \cap (V,E)) \subset s\tau_1\tau_2\tau_3$ - α - $cl(V,E)$. Thus (U,E) and $(W,E) = X_E - (s\tau_1\tau_2\tau_3$ - α - $cl(U,E))$ are two disjoint soft $\tau_1\tau_2\tau_3$ - α -open sets over X such that $s\tau_1\tau_2\tau_3$ - α - $cl((A,E) \cap (U,E)) = (A,E)$ and $(B,E) \subset (W,E)$. Therefore, $s\tau_1\tau_2\tau_3$ - α - $cl((W,E) \cap (B,E)) = (B,E)$ and $(X, \tau_1, \tau_2, \tau_3, E)$ is soft $\tau_1\tau_2\tau_3$ - α -normal space.

Proposition 3.6: Suppose that $(X, \tau_1, \tau_2, \tau_3, E_1)$ and $(Y, \sigma_1, \sigma_2, \sigma_3, E_2)$ are any two soft tritopological spaces, $(X, \tau_1, \tau_2, \tau_3, E_1)$ is soft almost $\tau_1\tau_2\tau_3$ - α - β -normal and $f: (X, \tau_1, \tau_2, \tau_3, E_1) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3, E_2)$ is onto, soft $\tau_1\tau_2\tau_3$ - α -continuous, soft $\tau_1\tau_2\tau_3$ - α -open, soft $\tau_1\tau_2\tau_3$ - α -closed and $(Y, \sigma_1, \sigma_2, \sigma_3, E_2)$ is soft $\sigma_1\sigma_2\sigma_3$ - $\alpha_{1/2}$ space. Then $(Y, \sigma_1, \sigma_2, \sigma_3, E_2)$ is soft almost $\sigma_1\sigma_2\sigma_3$ - α - β -normal.

Proof: Suppose that $(F, E_2), (G, E_2)$ are disjoint soft $\sigma_1\sigma_2\sigma_3$ - α -closed sets over Y and (F, E_2) is soft $\tau_1\tau_2\tau_3$ - α -regular closed. Since $(Y, \sigma_1, \sigma_2, \sigma_3, E_2)$ is soft $\sigma_1\sigma_2\sigma_3$ - $\alpha_{1/2}$ space, $(F, E_2), (G, E_2)$ are soft $\sigma_1\sigma_2\sigma_3$ -closed sets over Y . Since f is soft $\tau_1\tau_2\tau_3$ - α -continuous, $f^{-1}(F, E_2)$ and $f^{-1}(G, E_2)$ are disjoint soft $\tau_1\tau_2\tau_3$ - α -closed sets over X . Clearly $f^{-1}(F, E_2) = s\sigma_1\sigma_2\sigma_3$ - α - $cl(f^{-1}(s\sigma_1\sigma_2\sigma_3$ - α - $int(F, E_2)))$. Suppose that (W, E_1) is soft $\tau_1\tau_2\tau_3$ - α -open over X such that $(W, E_1) \cap f^{-1}(F, E_2) \neq \emptyset_E$. Then $f(W, E_1)$ is soft $\sigma_1\sigma_2\sigma_3$ - α -open over Y and $f(W, E_1) \cap (F, E_2) = f(W, E_1) \cap s\sigma_1\sigma_2\sigma_3$ - α - $cl(s\sigma_1\sigma_2\sigma_3$ - α - $int(F, E_2)) \neq \emptyset_E$ which implies that $f(W, E_1) \cap (s\sigma_1\sigma_2\sigma_3$ - α - $int(F, E_2)) \neq \emptyset_E$. Hence $(W, E_1) \cap f^{-1}(s\sigma_1\sigma_2\sigma_3$ - α - $int(F, E_2)) \neq \emptyset_E$ and so $f^{-1}(F, E_2) = s\sigma_1\sigma_2\sigma_3$ - α - $cl(f^{-1}(s\sigma_1\sigma_2\sigma_3$ - α - $int(F, E_2)))$. Therefore, $f^{-1}(F, E_2)$ is soft $\tau_1\tau_2\tau_3$ - α -regular closed set. So there exist a soft $\tau_1\tau_2\tau_3$ - α -open set (U, E_1) over X such that $f^{-1}(G, E_2) = s\tau_1\tau_2\tau_3$ - α - $cl(f^{-1}(G, E_2) \cap (U, E_1))$ and $s\tau_1\tau_2\tau_3$ - α - $cl(U, E_1) \cap f^{-1}(F, E_2) = \emptyset_E$, $f(s\tau_1\tau_2\tau_3$ - α - $cl(U, E_1) \cap (F, E_2)) = \emptyset_E$. Also, note that $f(U, E_1)$ is soft $\sigma_1\sigma_2\sigma_3$ - α -open and $f(s\tau_1\tau_2\tau_3$ - α - $cl(U, E_1))$ is soft $\sigma_1\sigma_2\sigma_3$ - α -closed. Since $f(s\tau_1\tau_2\tau_3$ - α - $cl(U, E_1))$ is soft $\sigma_1\sigma_2\sigma_3$ - α -closed set containing $f(U, E_1)$, $s\tau_1\tau_2\tau_3$ - α - $cl(f(U, E_1)) \subseteq f(s\tau_1\tau_2\tau_3$ - α - $cl(U, E_1))$. So $s\sigma_1\sigma_2\sigma_3$ - α - $cl(f(U, E_1) \cap (F, E_2)) = \emptyset_E$. It remains to show that $(G, E_2) = s\sigma_1\sigma_2\sigma_3$ - α - $cl((G, E_2) \cap f(G, E_2))$. Let $y_e \in (G, E_2)$ for $e \in E$ and (O, E_2) be soft $\tau_1\tau_2\tau_3$ - α -open set over Y containing y_e . Then $f^{-1}(y) \subseteq [f^{-1}(G, E_2) \cap f^{-1}(O, E_2)]$.

Since $f^{-1}(G, E_2) = s\tau_1\tau_2\tau_3$ - α - $cl(f^{-1}(G, E_2) \cap (U, E_1))$, $f^{-1}(G, E_2) \cap (U, E_1) \cap f^{-1}(O, E_2) \neq \emptyset_E$. Hence, $(G, E_2) \cap f(U, E_1) \cap (O, E_2) = f(f^{-1}(G, E_2)) \cap f(U, E_1) \cap f(f^{-1}(O, E_2)) \supseteq f[f^{-1}(G, E_2) \cap (U, E_1) \cap f^{-1}(O, E_2)] \neq \emptyset_E$, as desired.

REFERENCES

1. A.V. Arhangel'skii, L. Ludwig, On a α -normal and β -normal spaces, comment Math.Univ. Carolin.42:3 (2001), 507-519.
2. Asmhan Flieh Hassan, Soft tritopological space, Internat.J.Comput.Appl. (0975-8887)
3. S. Bayramov, C. Gunduz, Soft locally compact spaces paracompsct spaces. Ann. Fuzzy Math.Inf.2013,3, 171-185.
4. A.K. Das, Pritiba Bhat, J.K. Tarlir, on simultaneous generalization of β -normality and almost normality, 31:2(2017), 425-430
5. N.F. Hameed and Mohammad Yahya Abid, Certain types of separation axioms in tritopological spaces, Iraqi journal of science, 2011, volume 52(2), 212-217.
6. M. Kovar, On 3-Topological version of Thet- Reularity, Internat.J. Matj, Sci., 2000, 23(6), 393- 398.
7. P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003), 555-562.
8. D.Molodtsov, Soft set theory first results, Comput. Math. Appl. 37 (1999), 19-31.
9. E. Murtinova, A β -normal Tycono space which is not normal, Comment. Math. Univ. Carolin. 43:1 (2002) 159-164.
10. O. Njastad, On some classes of nearly open sets, Pacific J. Math., 1965, 15, 961-970.
11. S.Palaniammal, Study of Tri topological spaces, Ph.D Thesis, 2011.
12. E.V. Schepin, Real-valued functions, and spaces close to normal, Sib. Matem. Journ. 13:5 (1972) 1182-1196.
13. M. Shabir, M. Naz, On soft topological spaces, Comput. Math. Appl. 61(2011) 1786 -1799.
14. M.K. Singal, S.P. Arya, Almost normal and almost completely regular spaces, Glasnik Mat. 5(25) (1970), 141-152.
15. M.K. Singal, A.R. Singal, Mildly normal spaces, Kyungpook Math J. 13 (1973) 27-31.

Source of support: National Conference on "New Trends in Mathematical Modelling" (NTMM - 2018), Organized by Sri Sarada College for Women, Salem, Tamil Nadu, India.