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β-OPEN SET IN FUZZY ROUGH TOPOLOGICAL SPACE

S. PADMAPRIYA
Assistant Professor of Mathematics,
Padmavani Arts and Science College for Women, Salem, Tamil Nadu, India,

R. PACHIYAMMAL
Mathematics, Padmavani Arts and Science College for women, Salem, India.

E-mail: priyasathi17@gmail.com

ABSTRACT

The purpose of this paper is to introduce the concepts of an fuzzy rough set, fuzzy roughβ-open set, fuzzy roughβ-closed set, fuzzy rough β-interior, fuzzy rough β-closure are introduced and studied. Some interesting properties are also discussed.

Keywords: *An fuzzy rough set, fuzzy rough β-open set, fuzzy rough β-closed set, fuzzy rough β-interior, fuzzy rough β-closure.*

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1. INTRODUCTION

Z.Pawlak[8] introduced the definition of fuzzy rough set. S.Nanda and S.Majumdar [7] studied the concepts of fuzzy rough sets. The theory of fuzzy topological spaces was introduced and developed by C.L.Chang[2].The concept of fuzzy sets was introduced by Zadeh [7]. In this paper the concepts of fuzzy rough topological spaces ,fuzzy rough β-open sets, fuzzy rough β-closed sets, fuzzy rough β-interiors, fuzzy rough β-closures sets are introduced and studied. Some interesting properties are also discussed.

And later Atanassov [1] generalized the idea to intuitionistic fuzzy sets. On the otherhand, Coker [2] introducedthe notions of an intuitionistic fuzzy topological spaces, intuitionistic fuzzy continuity, intuitionistic fuzzy compactness and some other related concepts. Roy, A.R and P.K.Maji [5] was studied the de_nition of fuzzy soft sets. Maji P.K., R. Biswas and A.R.Roy [3] was introduced the de_nition of an intuitionistic fuzzy soft sets. Necla Turanli and A. HaydarEs [4] was introduced and studied the concept of an intuitionistic fuzzysoft topological spaces. In this paper, the concepts of an intuitionistic fuzzy soft βopen set, intuitionistic fuzzy soft β-closed set, intuitionistic fuzzy soft β-interior, intuitionistic fuzzy soft β-closure, intuitionistic fuzzy soft β-continuous function, intuitionistic fuzzy soft β-compact, intuitionistic fuzzy soft almost β-compact and intuitionistic fuzzy soft nearly β-compact are introduced and studied. Some interesting properties are also discussed.

2. PRELIMINARIES

Let U be any non empty set and let $\mathcal{B}$ be a complete subalgebra of the Boolean algebra $\mathcal{P}(U)$ of subsets of U. The pair $(U,\mathcal{B})$ is called rough universe. Consider a rough set $X = (X_L, X_U) \in \mathcal{B}^2$with $X_L \subset X_U$.

Definition 2.1: A fuzzy rough set $A = (A_L, A_U)$ in X is characterized by a pair of maps $A_L: X_L \longrightarrow I$ and $A_U: X_U \longrightarrow I$ with $A_L(x) \leq A_U(x)$ for every $x \in X_U$. The collection of all fuzzy rough sets in X is denoted by $\mathbf{FRS}(\boldsymbol{X})$.

Definition 2.2: For any two fuzzy rough sets $A = (A_L, A_U)$ and $B = (B_L, B_U)$in X.

(i) $A = B$iff
$A_L(x) = \mu_{B_L}(x)$for every $x \in X_L$ and
$A_U(x) = \mu_{B_U}(x)$for every $x \in X_U$.

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(ii)$A \subseteq B$ iff
$A_L(x) \leq \mu_{B_L}(x)$ for every $x \in X_L$ and
$A_U(x) \leq \mu_{B_U}(x)$ for every $x \in X_U$.

If $\{A_i : i \in J\}$ be any family of fuzzy rough sets in X, where $A = (A_L, A_U)$ then $E = \cup_i A_i$ iff
$E_L(x) = sup_{i \in J} A_{L_i}(x)$ for every $x \in X_L$ and
$E_U(x) = sup_{i \in J} A_{U_i}(x)$ for every $x \in X_U$.

Similarly, $F = \cap_i A_i$ iff
$F_L(x) = inf_{i \in J} A_{L_i}(x)$ for every $x \in X_L$ and
$F_U(x) = inf_{i \in J} A_{U_i}(x)$ for every $x \in X_U$.

Definition 2.3: Let $A = (A_L, A_U)$ be a fuzzy rough sets in X. Then the complement A' of A is defined by ordered pairs (A'_L, A'_U) of membership functions where
$A'_L(x) = 1 - A_L(x)$ for every $x \in X_L$ and
$A'_U(x) = 1 - A_U(x)$ for every $x \in X_U$.

Proposition 2.1: If A, B, C, D and B_i $i \in J$ are FRS in X, then
(i)$A \subset B$ and $C \subset D$ implies $A \cup C \subset B \cup D$,
(ii)$A \subset B$ and $B \subset C$ implies $A \subset C$,
(iii)$A \cap B \subset A$,$A \cup B \supset B$,
(iv)$A \cup (\cap_i B_i) = \cap_i (A \cup B_i)$ and $A \cap (\cup_i B_i) = \cup_i (A \cap B_i)$,
(v)$A \subset B \Longrightarrow A' \subset B'$,
(vi)$(\cup_i B_i)' = \cap_i B_i'$ and $(\cap_i B_i)' = \cup_i B_i'$.

Definition 2.4: The null fuzzy rough set and whole fuzzy rough set in X are defined by $\tilde{0} = (0_L, 0_U)$ and $\tilde{1} = (1_L, 1_U)$.

Proposition 2.2: If A be any fuzzy rough set in X then
(i)$\tilde{0} \subset A \subset \tilde{1}$ and (ii)$(\tilde{0})' = \tilde{1}, (\tilde{1})' = \tilde{0}$.

Definition 2.5: Let $(V, \mathcal{B})$ and $(V_1, \mathcal{B}_1)$ be two rough universes and $f: (V, \mathcal{B}) \longrightarrow (V_1, \mathcal{B}_1)$. Let $A = (A_L, A_U)$ be a fuzzy rough set in X. Then $Y = f(X) \in \mathcal{B}_1{}^2$ and $Y_L = f(X_L)$, $Y_U = f(X_U)$. The image of f, denoted by
$f(A) = \big(f(A_L), f(A_U)\big)$ is defined by
$f(A_L)(y) = \vee \{A_L(x): x \in X_L \cap f^{-1}(y)\}$ for every $y \in Y_L$, and
$f(A_U)(y) = \vee \{A_U(x): x \in X_U \cap f^{-1}(y)\}$ for every $y \in Y_U$.

Definition 2.6: Let $B = (B_L, B_U)$ be a fuzzy rough set in Y where $Y = (Y_L, Y_U) \in \mathcal{B}_1{}'$ is a rough set. $f^{-1}(Y) \in \mathcal{B}_1{}^2$, where $X_L = f^{-1}(Y_L)$, $X_U = f^{-1}(Y_U)$. Then the inverse image of B under f, denoted by $f^{-1}(B) = \big(f^{-1}(B_L), f^{-1}(B_U)\big)$ is defined by
$f^{-1}(B_L)(x) = B_L\big(f(x)\big)$ for every $x \in X_L$ and
$f^{-1}(B_U)(x) = B_U\big(f(x)\big)$ for every $x \in X_U$.

Proposition 2.3: If $f: (V, \mathcal{B}) \longrightarrow (V_1, \mathcal{B}_1)$ be a mapping then for all FRS $A, A_1, A_2 \in X$, we have
(i)$f(B') \supset \big(f(B)\big)'$,
(ii)$A_1 \subset A_2 \Longrightarrow f(A_1) \subset f(A_2)$.

Proposition 2.4: If $f: (V, \mathcal{B}) \longrightarrow (V_1, \mathcal{B}_1)$ be a mapping such that $f^{-1}: (V_1, \mathcal{B}_1) \longrightarrow (V, \mathcal{B})$.Then for all FRS, $B_i \in Y, i \in J$, we have
(i) $f^{-1}(B') = \big(f^{-1}(B)\big)'$,
(ii) $B_1 \subset B_2 \Longrightarrow f^{-1}(B_1) \subset f^{-1}(B_2)$,
(iii) $f^{-1}(\cup_i B_i) = \cup_i f^{-1}(B_i)$,
(iv)$f^{-1}(\cap_i B_i) = \cap_i f^{-1}(B_i)$,
(v) $f(\cup_i B_i) = \cup_i f(B_i)$,
(vi) $f(\cap_i B_i) \subset \cap_i f(B_i)$.

Proposition 2.5: If $f: (V, \mathcal{B}) \longrightarrow (V_1, \mathcal{B}_1)$ be a mapping such that $f^{-1}: (V_1, \mathcal{B}_1) \longrightarrow (V, \mathcal{B})$.Then for all FRS A in X and B in Y, we have
(i)$B = f\big(f^{-1}(B)\big)$, (ii) $A \subset f^{-1}\big(f(A)\big)$.

3. Fuzzy rough β-open set

In this section the concepts of Fuzzy rough β-open set,Fuzzy rough β −closed set, Fuzzy rough β −interior,Fuzzy roughβ −closure are introduced and some of the proposition are discussed.

Definition 3.1: Let (X,τ) be an fuzzy rough topological space. Let $A=(A_L,A_U)$ be an fuzzy rough set in fuzzy rough topological space (X,τ). Then A said to be **Fuzzy rough β-open set.**

$A \subseteq cl(int(cl(A)))$

The complement of an fuzzy rough β-open set is said to be **Fuzzy rough β −closed set.**

Ex:

$T=\{\tilde{0},\tilde{1},((.2,.3),(.4,.3)),((.3,.5),(.4,.5))\}$

$A=((.3,.5),(.5,.5))$

$cl(A)=((.7,.5),(.6,.5))$

$int(cl(A))=((.3,.5),(.4,.5))$

$cl\left(int(cl(A))\right)=((.7,.5),(.6,.5))$

$A \subseteq cl\left(int(cl(A))\right)$

$((.3,.5),(.5,.5)) \subseteq ((.7,.5),(.6,.5))$

Definition 3.2: Let (X,τ) be an fuzzy rough topological space. Let $A=(A_L,A_U)$ be an fuzzy rough set in Fuzzy rough topological space (X,τ).The **fuzzy rough β −interior of$A$** is denoted and defined by

$FR\beta-int(A)=\cup\{B; B=(B_L,B_U)$is a fuzzy rough β −open set in X and $B\subseteq A\}$.

Definition 3.3: Let (X,τ) be a fuzzy rough topological space. Let $A=(A_L,A_U)$ be an fuzzy rough set in Fuzzy rough topological space (X,τ).The **fuzzy rough β −closureof A** is denoted and defined by

$FR\beta-cl(A)=\cap\{B; B=(B_L,B_U)$is a fuzzy rough β −closed set in X and $A\subseteq B\}$.

Proposition 3.1: Let (X,τ) be a fuzzy rough topological space. For any two fuzzy rough sets $A=(A_L,A_U)$ and $B=(B_L,B_U)$ of an fuzzy rough topological space (X,τ) then the following statements are true.

(i)FR$\beta-cl(0_\sim)=0_\sim$

(ii)$A\subseteq B \Longrightarrow FR\beta-cl(A)\subseteq FR\beta-cl(B)$

(iii)FR$\beta-cl[FR\beta-cl(A)]=FR\beta-cl(A)$

(iv) $FR\beta-cl(A\cup B)\supseteq[FR\beta-cl(A)]\cup[FR\beta-cl(B)]$

(v) $FR\beta-cl(A\cap B)\subseteq[FR\beta-cl(A)]\cap[FR\beta-cl(B)]$

Proof:

(i) Since $0_\sim$ itself is a fuzzy rough β closed set.

FR$\beta-cl(0_\sim)=0_\sim$

(ii)$A\subseteq B$

$FR\beta-cl(A)=\cap\{K; K=(K_L,K_U)$is a fuzzy rough β closed set in X and $A\subseteq K\}$

$\subseteq\cap\{K; K=(K_L,K_U)$is a fuzzy rough β closed set in X and $B\subseteq K\}\subseteq FR\beta-cl(B)$

(iii) Since $FR\beta-cl(A)$is a fuzzy rough β closed set in X

FR$\beta-cl[FR\beta-cl(A)]=FR\beta-cl(A)$

(iv) $FR\beta-cl(A\cup B)=\cap\{K; K=(K_L,K_U)$is a fuzzy rough β closed set in X and $(A\cup B)\subseteq K\}$ $\supseteq[\cap\{K; K=(K_L,K_U)$is a fuzzy rough β closed set in X and $A\subseteq K\}]\cup[\cap\{K; K=(K_L,K_U)$is a fuzzy rough β closed set in X and $B\subseteq K\}]\supseteq[FR\beta-cl(A)]\cup[FR\beta-cl(B)]$

(v) $FR\beta-cl(A\cap B)=\cap\{C; C=(C_L,C_U)$is a fuzzy rough β closed set in X and $(A\cap B)\subseteq C\}\subseteq[\cap\{C; C=(C_L,C_U)$is a fuzzy rough β closed set in X and $A\subseteq C\}]\cup[\cap\{C; C=(C_L,C_U)$is a fuzzy rough β closed set in X and $B\subseteq C\}]\subseteq[FR\beta-cl(A)]\cap[FR\beta-cl(B)]$

Proposition 3.2: Let (X,τ) be a fuzzy rough topological space. Let $A=(A_L,A_U)$ and $B=(B_L,B_U)$ are fuzzy rough sets in fuzzy rough topological space (X,τ). Then the following statements are true.

(i) $FR\beta$-$int(A)$ is the largest fuzzy rough β open set contained in A.

(ii) If A is a fuzzy rough β open set then $A=FR\beta$-$int(A)$.

(iii) If A is a fuzzy rough β open set then $FR\beta$-$int[FR\beta$-$int(A)]=FR\beta$-$int(A)$.

(iv) $1_\sim$-$FR\beta$-$int(A)=FR\beta$-$cl(1_\sim$-A).

(v) $1_\sim$-$FR\beta$-$cl(A)=FR\beta$-$int(1_\sim$-A).

(vi) If $A \subseteq B \Longrightarrow FR\beta - int(A) \subseteq FR\beta - int(B)$.
(vii) $[FR\beta - int(A)] \cup [FR\beta - int(B)] \subseteq FR\beta - int(A \cup B)$
(viii) $[FR\beta - int(A)] \cap [FR\beta - int(B)] \supseteq FR\beta - int(A \cap B)$.

Proof: The proof of the (i) and (ii) are trivial.
The proof of the (iii) follows from (i) and (ii).

(iv) $1_{\sim}$-$FR\beta$-$int(A)$
$=1_{\sim} - \cup \{B; B = (B_L, B_U)$ is a fuzzy rough βopen set in $X$and $B \subseteq A\}$
$= \cap \{B; B = (B_L, B_U)$ is a fuzzy rough β closed set in X and $B \supseteq 1_{\sim}$-A$\}$
$= FR\beta$-$cl(1_{\sim}$-A).

(v) $1_{\sim}$-$FR\beta$-$cl(A)$
$=1_{\sim} - \cap \{B; B = (B_L, B_U)$ is a fuzzy rough β open set in $X$and $B \supseteq A\}$
$= \cup \{B; B = (B_L, B_U)$ is a fuzzy rough β open set in X and $B \subseteq 1_{\sim}$-A$\}$
$= FR\beta$-$int(1_{\sim}$-A).

(vi) $A \subseteq B$
$FR\beta - int(A) = \cup \{C: C = (C_L, C_U)$is a fuzzy rough β open set in X and $C \subseteq A\}$
$\subseteq \cup \{C: C = (C_L, C_U)$is a fuzzy rough β open set in X and $C \subseteq B\}$
$\subseteq FR\beta - int(B)$.

(vii) $FR\beta - int(A \cup B)$
$= \cup \{C: C = (C_L, C_U)$ is a fuzzy rough β open set in $X$and $C \subseteq (A \cup B)\}$
$\supseteq [\cup \{C: C = (C_L, C_U)$ is a fuzzy rough β open set in $X$and $C \subseteq A\}] \cup [\cup \{C: C = (C_L, C_U)$ is a fuzzy rough β open set in X and $C \subseteq B\}]$
$\supseteq [FR\beta - int(A)] \cup [FR\beta - int(B)]$.

(viii)$FR\beta - int(A \cap B)$
$= \cup \{C: C = (C_L, C_U)$ is a fuzzy rough β open set in $X$and $C \subseteq (A \cap B)\}$
$\subseteq [\cup \{C: C = (C_L, C_U)$ is a fuzzy rough β open set in $X$and $C \subseteq A\}] \cap [\cup \{C: C = (C_L, C_U)$ is a fuzzy rough β open set in X and $C \subseteq B\}]$
$\subseteq [FR\beta - int(A)] \cap [FR\beta - int(B)]$.

CONCLUSION

It is well known that various types of functions play a significant role in the theory of classical point set topology and engineering, economics etc. A great number of paper dealing with such functions have appeared and a good many of them have been extended to the fuzzy topological spaces, rough topological space and fuzzy rough topological space by workers. The purpose of the present paper is to define fuzzy rough β-opes sets and obtain several basic properties.

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