

**CHARACTERIZATION OF NEUTROSOPHIC NOWHERE DENSE SETS**

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**ABSTRACT**

*In this paper, the concept of neutrosophic nowhere dense set is introduced and characterizations of neutrosophic nowhere dense sets are studied.*

*Keywords: Neutrosophic dense set; neutrosophic open hereditarily irresolvable; neutrosophic nowhere dense set.*

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**1. INTRODUCTION AND PRELIMINARIES**

The fuzzy concept has imposed a great influence in almost all branches of mathematics since the introduction of fuzzy sets by L. A. Zadeh [14]. The theory of fuzzy topological space was introduced and developed by C. L. Chang [6] and since then various notions in classical topology have been extended into the context of fuzzy topological space. The idea of "intuitionistic fuzzy set" was first published by Atanassov [1] and some research in this respect have been done by him and his colleagues [2, 3, 4]. Later, this concept was generalized to "intuitionistic L - fuzzy sets" by Atanassov and Stoeva [5]. The concept of nowhere dense set in intuitionistic fuzzy topological space introduced by S. S. Thakur and R. Dhavaseelan in [13]. F. Smarandache introduced the important and useful concepts of neutrosophy and neutrosophic set [[11], [12]]. The concepts of neutrosophic crisp set and neutrosophic crisp topological space were introduced by A. A. Salama and S. A. Alblowi [10]. The Basic definitions and Proposition related to neutrosophic topological spaces was introduced and discussed by Dhavaseelan et al. [8].

In this paper, we introduce the concept of neutrosophic nowhere dense set and study its fundamental properties. Her we mention some well-known notions which will be used in what follows.

**Definition 1.1:** Let T,I,F be real standard or non standard subsets of  $]0^-,1^+[$ , with  $sup_T = t_{sup}, inf_T = t_{inf}$   
 $sup_I = i_{sup}, inf_I = i_{inf}$   $sup_F = f_{sup}, inf_F = f_{inf}$   $n - sup = t_{sup} + i_{sup} + f_{sup}$   
 $n - inf = t_{inf} + i_{inf} + f_{inf}$ . T,I,F are neutrosophic components.

**Definition 1.2:** Let X be a nonempty fixed set. A neutrosophic set (briefly NS) A is an object having the form  $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$  where  $\mu_A(x)$ ,  $\sigma_A(x)$  and  $\gamma_A(x)$  which represents the degree of membership function (namely  $\mu_A(x)$ ), the degree of indeterminacy (namely  $\sigma_A(x)$ ) and the degree of nonmembership (namely  $\gamma_A(x)$ ) respectively of each element  $x \in X$  to the set A.

**Remark 1.1:**

- A neutrosophic set  $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X\}$  can be identified to an ordered triple  $\langle \mu_A, \sigma_A, \gamma_A \rangle$  in  $]0^-, 1^+[$  on X.
- For the sake of simplicity, we shall use the symbol  $A = \langle \mu_A, \sigma_A, \gamma_A \rangle$  for the neutrosophic set  $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X\}$ .

**Definition 1.3:** Let X be a nonempty set and the neutrosophic sets A and B in the form

$A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X\}$ ,  $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle : x \in X\}$ . Then

- $A \subseteq B$  iff  $\mu_A(x) \leq \mu_B(x)$ ,  $\sigma_A(x) \leq \sigma_B(x)$  and  $\gamma_A(x) \geq \gamma_B(x)$  for all  $x \in X$ ;
- $A = B$  iff  $A \subseteq B$  and  $B \subseteq A$ ;
- $\bar{A} = \{\langle x, \gamma_A(x), \sigma_A(x), \mu_A(x) \rangle : x \in X\}$ ; [Complement of A]
- $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle : x \in X\}$ ;
- $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x) \rangle : x \in X\}$ ;
- $[\ ]A = \{\langle x, \mu_A(x), \sigma_A(x), 1 - \mu_A(x) \rangle : x \in X\}$ ;
- $\langle \rangle A = \{\langle x, 1 - \gamma_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X\}$ .

**Definition 1.4:** Let  $\{A_i : i \in J\}$  be an arbitrary family of neutrosophic sets in X. Then

- $\bigcap A_i = \{\langle x, \wedge \mu_{A_i}(x), \wedge \sigma_{A_i}(x), \vee \gamma_{A_i}(x) \rangle : x \in X\}$ ;
- $\bigcup A_i = \{\langle x, \vee \mu_{A_i}(x), \vee \sigma_{A_i}(x), \wedge \gamma_{A_i}(x) \rangle : x \in X\}$ .

Since our main purpose is to construct the tools for developing neutrosophic topological spaces, we must introduce the neutrosophic sets  $0_N$  and  $1_N$  in X as follows:

**Definition 1.5:**  $0_N = \{\langle x, 0, 0, 1 \rangle : x \in X\}$  and  $1_N = \{\langle x, 1, 1, 0 \rangle : x \in X\}$ .

**Definition 1.6:** [8] A neutrosophic topology (briefly NT) on a nonempty set X is a family T of neutrosophic sets in X satisfying the following axioms:

- $0_N, 1_N \in T$ ,
- $G_1 \cap G_2 \in T$  for any  $G_1, G_2 \in T$ ,
- $\cup G_i \in T$  for arbitrary family  $\{G_i | i \in \Lambda\} \subseteq T$ .

In this case the ordered pair  $(X, T)$  or simply X is called a neutrosophic topological space (briefly NTS(X)) and each neutrosophic set in T is called a neutrosophic open set (briefly NOS). The complement  $\bar{A}$  of a NOS A in X is called a neutrosophic closed set (briefly NCS) in X.

**Definition 1.7:** [8] Let A be a neutrosophic set in a NTS(X). Then

$$Nint(A) = \bigcup \{G \mid G \text{ is an NOS in X and } G \subseteq A\}$$

is called the neutrosophic interior of A ;

$$Ncl(A) = \bigcap \{G \mid G \text{ is an NCS in X and } G \supseteq A\}$$

is called the neutrosophic closure of A .

**Definition 1.8:** Let X be a nonempty set. If  $r, t, s$  be real standard or non standard subsets of  $]0^-, 1^+[$  then the neutrosophic set  $x_{r,t,s}$  is called a neutrosophic point (briefly NP) in X given by

$$x_{r,t,s}(x_p) = \begin{cases} (r, t, s), & \text{if } x = x_p \\ (0, 0, 1), & \text{if } x \neq x_p \end{cases}$$

for  $x_p \in X$  is called the support of  $x_{r,t,s}$ , where  $r$  denotes the degree of membership value,  $t$  denotes the degree of indeterminacy and  $s$  is the degree of non-membership value of  $x_{r,t,s}$ .

## 2. NEUTROSOPHIC NOWHERE DENSE SETS

**Definition 2.1:** A neutrosophic set  $A$  in  $NTS (X, T)$  is called neutrosophic dense if there exists no neutrosophic closed set  $B$  in  $(X, T)$  such that  $A \subset B \subset 1_N$

**Definition 2.2:** A neutrosophic set  $A$  in  $NTS (X, T)$  is called neutrosophic nowhere dense set if there exists no  $NOS, U$  in  $(X, T)$  such that  $U \subset Ncl(A)$ . That is  $NintNcl(A) = 0_N$ .

**Example 2.1:** Let  $X = \{a, b, c\}$ . Define the neutrosophic sets  $A, B$  and  $C$  as follows:

$A = \langle x, (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.5}) \rangle$   $B = \langle x, (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}), (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}), (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3}) \rangle$ , and  
 $C = \langle x, (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.4}), (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.4}), (\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.4}) \rangle$ . Then the family  $T = \{0_N, 1_N, A\}$  is an  $NT$  on  $X$ . Thus,  $(X, T)$  is an Neutrosophic Topology.

Now  $NintNcl(\bar{A}) = 0_N, NintNcl(\bar{B}) = 0_N, NintNcl(\bar{C}) = 0_N, NintNcl(B) = 1_N \neq 0_N, NintNcl(\bar{C}) = 1_N \neq 0_N$ . and  $\bar{A}, \bar{B}$  and  $C$  are neutrosophic nowhere dense sets in  $(X, T)$ .  $B$  and  $\bar{C}$  are not neutrosophic nowhere dense set in  $(X, T)$ .

**Proposition 2.1:** Let  $A$  be a neutrosophic set. If  $A$  is a neutrosophic closed set in  $(X, T)$  with  $Nint(A) = 0_N$ , then  $A$  is a neutrosophic nowhere dense set in  $(X, T)$ .

**Proof:** Let  $A$  be a neutrosophic closed set in  $(X, T)$ . Then  $Ncl(A) = A$ . Now  $Nint(Ncl(A)) = Nint(A) = 0_N$  and hence  $A$  is a neutrosophic nowhere dense set  $A$  in  $(X, T)$ .

**Proposition 2.2:** Let  $A$  be a neutrosophic set. If  $A$  is a neutrosophic nowhere dense set in  $(X, T)$ , then  $Nint(A) = 0_N$ .

**Proof:** Let  $A$  be a neutrosophic nowhere dense set in  $(X, T)$ . Now  $A \subseteq Ncl(A)$  implies that  $Nint(A) \subseteq NintNcl(A) = 0_N$ . Hence we have  $Nint(A) = 0_N$ .

**The converse of Proposition 2.2, need not be true as shown in Example 2.2.**

**Example 2.2:** Let  $X = \{a, b, c\}$ . Define the neutrosophic sets

$A = \langle x, (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.5}) \rangle$   $B = \langle x, (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}), (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}) \rangle$ ,  
 $C = \langle x, (\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.4}) \rangle$ , and  $D = \langle x, (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}), (\frac{a}{0.7}, \frac{b}{0.5}, \frac{c}{0.5}) \rangle$ . Clearly  
 $T = \{0_N, 1_N, A, B, C\}$  is an  $NT$  on  $X$ . Thus  $(X, T)$  is an  $NT$ . Now  $Nint(D) = 0_N$ ,

where as  $NintNcl(D) = B \neq 0_N$ . Also  $Ncl(D) = \bar{C} \neq D$ . Hence  $D$  is not a neutrosophic nowhere dense set in  $(X, T)$ . Also  $D$  is not a neutrosophic closed set in  $(X, T)$ .

**Remark 2.1:** The complement of a neutrosophic nowhere dense set need not be a neutrosophic nowhere dense set. See Example 2.3.

**Example 2.3:** Let  $X = \{a, b, c\}$ . Define the neutrosophic sets

$A = \langle x, (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.5}) \rangle$  and  $B = \langle x, (\frac{a}{0.4}, \frac{b}{0.3}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.3}, \frac{c}{0.5}), (\frac{a}{0.6}, \frac{b}{0.7}, \frac{c}{0.5}) \rangle$ . Clearly  $T = \{0_N, 1_N, A\}$  is a neutrosophic topology on  $X$ . Thus  $(X, T)$  is an  $NT$ . Now  $B$  is a neutrosophic nowhere dense set in  $(X, T)$  where as  $\bar{B}$  is not a neutrosophic nowhere dense set, since  $NintNcl(\bar{B}) = Nint(1_N) = 1_N \neq 0_N$ .

**Proposition 2.3:** If  $A$  is a neutrosophic dense,  $NOS$  in  $(X, T)$ , such that  $B \subseteq \bar{A}$ , then  $B$  is a neutrosophic nowhere dense set in  $(X, T)$ .

**Proof:** Let  $A$  be a neutrosophic open set in  $(X, T)$  such that  $Ncl(A) = 1_N$ . Now  $B \subseteq \bar{A}$  implies that  $Ncl(B) \subseteq Ncl(\bar{A}) = \bar{A}$ . Then we have  $NintNcl(B) \subseteq Nint(\bar{A}) = \overline{Ncl(A)} = 0_N$  and hence  $NintNcl(B) = 0_N$ . Therefore  $B$  is a neutrosophic nowhere dense sets in  $(X, T)$ .

**Proposition 2.4:** If  $A$  is a neutrosophic closed set in  $(X, T)$ , then  $A$  is a neutrosophic nowhere dense set in  $(X, T)$  if and only if  $Nint(A) = 0_N$ .

**Proof:** Let  $A$  be a neutrosophic closed set in  $(X, T)$ , with  $Nint(A) = 0_N$ . Then by Proposition 2.1,  $A$  is a neutrosophic nowhere dense set in  $(X, T)$ . Conversely, let  $A$  be a neutrosophic nowhere dense set in  $(X, T)$ . Then  $NintNcl(A) = 0_N$  which implies that  $Nint(A) = 0_N$ . Since  $A$  is a neutrosophic closed,  $Ncl(A) = A$ . The proofs of following propositions are obvious.

**Definition 2.3:** Let  $A$  be a neutrosophic set. The  $NT$ ,  $(X, T)$  is called neutrosophic open hereditarily irresolvable if  $NintNcl(A) \neq 0_N$ , then  $Nint(A) \neq 0_N$  for any non-zero neutrosophic set in  $(X, T)$

**Proposition 2.5:** If  $(X, T)$  is a neutrosophic open hereditarily irresolvable space, any non zero neutrosophic set  $A$  with  $Nint(A) = 0_N$  is a neutrosophic nowhere dense set in  $(X, T)$ .

**Proof:** Let  $A$  be a non zero neutrosophic set in a neutrosophic open hereditarily irresolvable space  $(X, T)$  with  $Nint(A) = 0_N$ . Suppose that  $NintNcl(A) \neq 0_N$ . Since  $(X, T)$  is neutrosophic open hereditarily irresolvable space,  $Nint(A) \neq 0_N$ , which is contradiction to  $Nint(A) = 0_N$ . Hence we must have  $NintNcl(A) = 0_N$  and therefore  $A$  is a neutrosophic nowhere dense set in  $(X, T)$ .

**Proposition 2.6:** If  $A$  is a neutrosophic nowhere dense set in  $(X, T)$ , then  $\bar{A}$  is a neutrosophic dense set in  $(X, T)$ .

**Proof:** Let  $A$  be a neutrosophic nowhere dense set in  $(X, T)$ . Then by Proposition 2.2, we have,  $Nint(A) = 0_N$ .

Now  $Ncl(\bar{A}) = \overline{Nint(A)} = 1_N$ . Therefore  $\bar{A}$  is a neutrosophic dense set in  $(X, T)$ .

**Proposition 2.7:** If  $A$  is an intuitionsitic fuzzy dense and neutrosophic open set in  $(X, T)$ , then  $\bar{A}$  is a neutrosophic nowhere dense set in  $(X, T)$ .

**Proof:** Let  $A$  be a neutrosophic open set in  $(X, T)$  such that  $Ncl(A) = 1_N$ .

Now  $NintNcl(\bar{A}) = \overline{NclNint(A)} = \overline{Ncl(A)} = 0_N$ . Hence  $\bar{A}$  is a neutrosophic nowhere dense set in  $(X, T)$ .

**Proposition 2.8:** If  $A$  is a neutrosophic nowhere dense set in  $(X, T)$ , then  $Ncl(A)$  is a neutrosophic nowhere dense set in  $(X, T)$ .

**Proof:** Let  $Ncl(A) = B$ . Now  $NintNcl(B) = NintNcl(Ncl(A)) = NintNcl(A) = 0_N$ . Hence  $B = Ncl(A)$  is a neutrosophic nowhere dense set in  $(X, T)$ .

**Proposition 2.9:** If  $A$  is a neutrosophic nowhere dense set in  $(X, T)$ , then  $\overline{Ncl(A)}$  is a neutrosophic dense set in  $(X, T)$ .

**Proof:** By Proposition 2.8, we have  $Ncl(A)$  is a neutrosophic nowhere dense set in  $(X, T)$ . By Proposition 2.6, we have  $\overline{Ncl(A)}$  is a neutrosophic dense set in  $(X, T)$ .

**Proposition 2.10:** Let  $A$  be a neutrosophic dense set in a neutrosophic topological space  $(X, T)$ . If  $B$  is a neutrosophic set in  $(X, T)$ , then  $B$  is a neutrosophic nowhere dense set in  $(X, T)$  if and only if  $A \cap B$  is a neutrosophic nowhere dense set in  $(X, T)$ .

**Proof:** Let  $B$  be a neutrosophic nowhere dense set in  $(X, T)$ .

Now

$$NintNcl(A \cap B) = Nint(Ncl(A)) \cap Nint(Ncl(B)) = Nint(1_N) \cap Nint(Ncl(B)) = NintNcl(B) = 0_N.$$

Therefore  $A \cap B$  is a neutrosophic nowhere dense set in  $(X, T)$ .

Conversely, let  $A \cap B$  be a neutrosophic dense set in  $(X, T)$ . Then  $NintNcl(A \cap B) = 0_N$  implies that  $Nint(Ncl(A)) \cap Nint(Ncl(B)) = 0_N$ . Hence  $Nint(1_N) \cap Nint(Ncl(B)) = 0_N$  and therefore  $Nint(Ncl(B)) = 0_N$  which means that  $B$  is a neutrosophic nowhere dense set in  $(X, T)$ .

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