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ON I_g**-CLOSED SETS IN IDEAL TOPOLOGICAL SPACE

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ABSTRACT

*In this paper, we introduce and study the notion of I_g**-closed sets along with their properties Furthermore A_I^g-set is introduced and characterized.*

Keywords: *I_g**-closed set, τ_g^* - closed set, ⋆-g-closed set, A_I^g-set.*

INTRODUCTION

An ideal I on a topological space (X, τ) is a non-empty collection of subsets of X which satisfies the following properties : (1) A ∈ I and B ⊆ A implies B ∈ I, (2) A ∈ I and B ∈ I implies A ∪ B ∈ I. An ideal topological space is a topological space (X, τ) with an ideal I on X and is denoted by (X, τ, I). For a subset A ⊆ X, A*(I, τ) = {x ∈ X: A ∩ U ∉ I for every U ∈ τ(x)} is called the local function of A with respect to I and τ [7]. We simply write A* in case there is no chance for confusion. A Kuratowski closure operator cl*(.) for a topology τ*(I, τ) called the ⋆-topology, finer than τ is defined by cl*(A) = A ∪ A* [18]. A subset A of an ideal topological space (X, τ, I) is ⋆-closed [6], if A* ⊆ A and a subset A of an ideal topological space (X, τ, I) is said to be ⋆-perfect [5] if A* = A. A subset A of an ideal topological space (X, τ) is said to be g-closed [3, 8], if cl(A) ⊆ U whenever A ⊆ U and U is open. The complement of a g-closed set is g-open set [3, 8]. The collection of all g-open sets in a topological space (X, τ) is denoted by τ_g . For each subset A of X, A_g* (I, τ) = {x ∈ X: U_x ∩ A ∉ I. For every g-open set U_x containing x}, is called the g-local function of A [1] with respect to I and τ_g and is denoted by A_g*.

Also cl_g*(A) = A ∪ A_g* [1] is a Kurotowski closure operator for a topology, τ_g^* = {X–A: cl_g*(A) = A} [1] on X which is finer than τ_g and the g-interior of A denoted by Int_g(A) [1] is the union of all g-open sets contained in A. In this paper we introduced I_g**-closed sets and investigated some of their basic properties. Also we define A_I^g-set and study its properties.

PRELIMINARIES

Definition 0.1: A subset A of an ideal topological space (X, τ, I) is said to be
- (i) τ_g^* - closed [1] if A_g* ⊆ A .
- (ii) ⋆-g-closed [11] if cl(A) ⊆ U whenever A ⊆ U and U is ⋆-open in X.
- (iii) I_g-⋆-closed [9] if A* ⊆ U whenever A ⊆ U and U is ⋆-open in X.

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Definition: 0.2. A subset A of an ideal topological space (X, τ, I) is said to be

(i) I_g-closed [14] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(ii) rgI-closed [13] if $A_* \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

(iii) I_{rg}-closed [15] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

(iv) $I_{g\delta}$-closed [17] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is δ-open in X.

Definition 0.3: A subset A of a topological space (X, τ) is said to be

(i) rg-closed [16] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

(ii) gδ-closed [10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is δ-open in X.

Definition 0.4: A subset A of an ideal topological space (X, τ, I) is said to be

(i) I-locally ⋆-closed [12] if $A = U \cap V$ where U is open and V is ⋆-closed.

(ii) I-locally τ_g^*-closed [2] if $A = U \cap V$ where U is τ_g^*-open and V is τ_g^*-closed.

(iii) I-locally closed [4] if $A = U \cap V$ where U is open and V is ⋆-perfect.

Definition 0.5: [3] A space (X, τ, I) is called a T_1-space if every I_g-closed subset of X is ⋆-closed.

1. I_g^{**}- CLOSED SETS

In this section, a new class of generalized closed set called I_g^{**}- closed set is introduced and some properties of this notion have been studied in ideal topological space and several characterizations of this notion are derived.

Definition 1.1: A subset A of an ideal topological space (X, τ, I) is said to be an I_g^{**}- closed set if $A_g^* \subseteq U$ whenever $A \subseteq U$ and U is ⋆-open in X .

Example 1.2: Let X= {a, b, c, d}, τ = {φ, X, {a}, {a, c}} and I = {φ}. Then {a, b} is an I_g^{**}-closed set.

Example 1.3: Let X= {a, b, c}, τ = {φ, X, {a}, {a, b}} and I = {φ}. Then {b} is not an I_g^{**}-closed set.

Theorem 1.4: Every τ_g^*-closed set is an I_g^{**}-closed set.

Proof: Let A be a τ_g^*-closed set and U be a ⋆-open set containing A. Since A is τ_g^*-closed, $A_g^* \subseteq A$.
Hence $A_g^* \subseteq A \subseteq U$. Consequently A is an I_g^{**}-closed set.

Remark 1.5: The converse of the above theorem is not true as seen from the following example.

Example 1.6: Let X= {a, b, c, d}, τ = {φ, X, {b}, {a, b}, {b, c}, {a, b, c}, {a, b, d}} and I= {φ, {a}, {c}, {a, c}}. Then {b} is an I_g^{**}-closed set but not an τ_g^*-closed set

Theorem 1.7: Every ⋆-closed set is an I_g^{**}-closed set.

Proof: Let A be a ⋆-closed set and U be a ⋆-open set containing A. Since A is ⋆-closed set, $A^* \subseteq A$. By theorem 3.10 [1], $A_g^* \subseteq A^* \subseteq A \subseteq U$ and so $A_g^* \subseteq U$. Hence A is an I_g^{**}-closed set.

Remark 1.8: The converse of the above theorem is not true as seen from the following example..

Example 1.9: Let X= {a, b, c} ,τ = {φ, X, {a}, {a, b}} and I= {φ}. Then {a, c} is an I_g^{**}-closed set but not a ⋆-closed set.

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Theorem 1.10: Every ⋆-g-closed set is an I_g^{**}-closed set.

Proof: Let A be a ⋆-g-closed set such that A ⊆ U where U is ⋆-open. Since A is ⋆-g-closed, cl(A) ⊆ U. By theorem 3.10 [1], A_g^* ⊆ cl(A) ⊆ U and so A_g^* ⊆ U. Hence A is an I_g^{**}-closed set.

Remark 1.11: The converse of the above theorem is not true as seen from the following example.

Example 1.12: Let X= {a, b, c}, τ = {φ, X, {a}, {b, c}} and I= {φ, {a}, {b}, {a, b}}. Then {c} is an I_g^{**}-closed set but not a ⋆-g-closed set.

Theorem 1.13: Every I_g-⋆-closed set is a I_g**-closed set.

Proof: Let A be a I_g-⋆-closed set such that A ⊆ U where U is ⋆-open. Since A is

I_g-⋆-closed set, A* ⊆ U. By theorem 3.10 [1], A_g^* ⊆ A*. Hence A_g^* ⊆ U and hence A is an I_g^{**}-closed set.

Remark 1.14: The converse of the above theorem is not true as seen from the following example.

Example 1.15: Let X= {a, b, c, d}, τ = {φ, X, {a}, {b}, {a, b}} and I= {φ, {c}}. Then {a} is an I_g^{**}-closed set but not an I_g-⋆-closed set.

Remark 1.16: The following table shows the relations between I_g**-closed sets and other known existing closed sets in ideal topological space. where the symbol ”1”in a cell means that a set implies the other set and the symbol ”0” means that a set does not imply the other set.

sets	I_g^{**} - closed	τ_g^* - closed	⋆ - closed	⋆-g- closed	I_g-⋆-closed
I_g^{**} - closed	1	0	0	0	0
τ_g^* - closed	1	1	0	0	0
⋆ - closed	1	1	1	0	0
⋆-g- closed	1	1	0	1	1
I_g-⋆-closed	1	0	0	0	1

Remark 1.17: I_g-closed sets and I_g^{**}-closed sets are independent of each other as seen from the following example.

Example 1.18: Let X= {a, b, c, d}, τ = {φ, X, {a}, {b}, {a, b}} and I= {φ, {b}, {c}, {b, c}}. Then {a} is an I_g **-closed set but not an I_g-closed set.

Example 1.19: Let X= {a, b, c}, τ = {φ, X, {a, b}} and I= {φ, {a}, {b}, {a, b}}. Then {a, b} is an I_g -closed set but not an I_g**-closed set.

Remark: 1.20. I_{rg}-closed sets and I_g**-closed sets are independent of each other as seen from the following example.

Example 1.21: Let X= {a, b, c}, τ = {φ, X, {a}, {b}, {a, b}} and I= {φ}. Then {a, b} is an I_{rg} -closed set but not an I_g**-closed set.

Example 1.22: Let X= {a, b, c, d}, τ = {φ, X, {a}, {b}, {a, b}} and I= {φ, {b}, {c}, {b, c}}.Then {a} is an I_g **-closed set but not an I_{rg}-closed set.

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1.1 CHARACTERIZATIONS OF I_g **- CLOSED SETS

Theorem 1.23: Let (X, τ, I) be an ideal topological space. If A and B are I_g**-closed sets, then A ∪ B is an I_g**-closed set.

Proof: Let U be a ⋆-open such that A ∪ B ⊆ U .Then A ⊆ U and B ⊆ U. Since A and B are I_g**-closed sets, $A_g^* \subseteq U$ and $B_g^* \subseteq U$ and so $A_g^* \cup B_g^* \subseteq U$. By theorem 3.7 [1], $(A \cup B)_g^* = A_g^* \cup B_g^*$ which implies $(A \cup B)_g^* \subseteq U$. Consequently A ∪ B is an I_g**-closed set.

Theorem 1.24: Let (X, τ, I) be an ideal topological space and A ⊆ X. Then the following are equivalent.

(i) A is an I_g**-closed set

(ii) $cl_g^*(A) \subseteq U$ whenever A ⊆ U and U is ⋆-open in X.

(iii) For all $x \in cl_g^*(A)$, $cl(\{x\}) \cap A \neq \varnothing$.

Proof:

(i)⇒(ii): Let A be an I_g**-closed set and U be a ⋆-open set containing A. Then $A_g^* \subseteq U$ which implies $cl_g^*(A) = A \cup A_g^* \subseteq U$ therefore $cl_g^*(A) \subseteq U$.

(ii)⇒(iii): Suppose that $x \in cl_g^*(A)$. If $cl(\{x\}) \cap A = \varnothing$, then $A \subseteq (cl(\{x\}))^c$ where $(cl(\{x\}))^c$ is open. Since every open set is ⋆-open, $(cl(\{x\}))^c$ is ⋆-open .By (ii), $cl_g^*(A) \subseteq (cl(\{x\}))^c$ which is a contrary to $x \in cl_g^*(A)$.

(iii)⇒(i): Suppose A is not an I_g**-closed set. Then there exist a ⋆-open set U such that A ⊆ U and $A_g^* \not\subset U$. Then there exists $x \in A_g^*$ such that $x \notin U$ and hence {x} U = ∅. Since A ⊆ U, $cl(\{x\}) \cap A = \varnothing$ which is a contradiction to (iii). Hence A is an I_g**-closed set.

Theorem 1.25: If a subset A of an ideal topological space (X, τ, I) is an I_g**-closed set, then $cl_g^*(A) - A$ contains no non-empty ⋆-closed set.

Proof: Let A be an I_g**-closed set and U be a ⋆-closed subset of $cl_g^*(A) - A$. Then $U \subseteq cl_g^*(A) - A = A_g^* \cap A^c$ which implies $U \subseteq A_g^*$. Moreover $U \subseteq A^c$ implies A Uc where U^c is ⋆-open. Since A is an I_g**-closed set, $A_g^* \subseteq U^c$. Hence we have $U \subseteq (A_g^*)^c$ and so $U \subseteq A_g^* \cap (A_g^*)^c = \varnothing$. Consequently $cl_g^*(A) - A$ contains no non-empty ⋆-closed set.

Theorem 1.26: If a subset A of an ideal topological space (X, τ, I) is an I_g**-closed set, then $A_g^* - A$ contains no non-empty ⋆-closed set.

Proof: Since $cl_g^*(A) - A = A_g^* - A$, proof follows from theorem 1.25.

Theorem 1.27: Let (X, τ, I) be an ideal topological space and A ⊆ X. If $A \subseteq A_g^*$ and A is I_g**-closed, then A is

(i) rgI-closed.

(ii) I_g-closed.

(iii) rg-closed.

(iv) I_{rg}-closed.

(v) $I_{g\delta}$-closed.

(vi) gδ-closed.

Proof:

(i) Let A be an I_g**-closed set and U be a regular open set containing A. Then A ⊆ U where U is ⋆-open. Since A is I_g**-closed, $A_g^* \subseteq U$ and by theorem 3.10, $A_g^* = A^*$ and so A is rgI-closed.

Let A be a I_g**-closed set and A ⊆ U whenever U is an open set. Since every open set is ⋆-open, $A_g^* \subseteq U$ and by theorem 3.10 [1], $A_g^* = A^*$. Hence A is an I_g-closed set.

(ii) Let A be an I_g**-closed set and U be a regular open set containing A. Since regular open set is open and open set is ⋆-open, A ⊆ U where U is ⋆-open.

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Now A is an I_g**-closed and by theorem 3.10 [1], $A_g^* = cl(A) \subseteq U$. Hence A is a rg-closed set.

(iii) Let A be an I_g**-closed set. Suppose $A \subseteq U$ where U is regular open and consequently $A \subseteq U$ where U is $\star$-open. Since A is a I_g**-closed set $A_g^* \subseteq U$ By theorem 3.10 [1], $A_g^* = A^*$ which implies $A^* \subseteq U$. Hence A is a I_{rg}-closed set.

Let A be an I_g**-closed set and then $A \subseteq U$ where U is δ-open. Since every δ-open set is open and every open set is $*$-open, $A \subseteq U$ where U is $*$-open. Since A is an I_g**-closed set, $A_g^* \subseteq U$ and by theorem 3.10 [1], $A_g^* = A^*$ which implies $A^* \subseteq U$. Hence A is an $I_{g\delta}$-closed set.

(vi) Let A be an I_g**-closed set and U be a δ-open set. suchthat $A \subseteq U$ where U is $\star$-open. Now A is an I_g**-closed set implies $A_g^* \subseteq U$. By theorem 3.10 [1], $A_g^* = cl(A)$,which implies $cl(A) \subseteq U$. Hence A is a gδ-closed set.

Remark 1.28: The converses of the above theorem are not true as seen from the following example.

Example 1.29: Let X= {a, b, c}, τ = {φ, X, {a}, {b}, {a, b}} and I= {φ}. Then {a} is a rgI-closed set but not an I_g**-closed set.

Example 1.30: Let X= {a, b, c}, τ = {φ, X, {a, b}} and I={φ, {a}, {b}, {a, b}}. Then {a, b} is an I_g -closed set but not an I_g**-closed set.

Example 1.31: Let X= {a, b, c}, τ = {φ, X, {a}, {b}, {a, b}} and I= {φ}. Then {a, b} is a rg-closed set but not an I_g**-closed set.

Example 1.32: Let X= {a, b, c}, τ = {φ, X, {a}, {b}, {a, b}} and I= {φ}. Then {a, b} is an I_{rg}-closed but not an I_g**-closed set.

Example 1.33: Let X= {a, b, c}, τ = {φ, X, {c}, {a, c}} and I= {φ}. Then {a} is an $I_{g\delta}$ -closed set but not an I_g**-closed set.

Example 1.34: Let X= {a, b, c, d}, τ = {φ, X, {b}, {c, d}, {b, c, d}} and I= {φ, {a}, {b}, {a, b}}. Then {a, b} is a gδ-closed set but not an I_g**-closed set.

Theorem 1.35: If A is an I_g**-closed set of (X, τ, I) such that $A \subseteq B \subseteq A_g^*$, then B is also an I_g**-closed set.

Proof: Let $B \subseteq U$ where U is $\star$-open. Now $A \subseteq B \subseteq U$ implies $A \subseteq U$. Since A is an I_g**-closed set, $A_g^* \subseteq U$. Now $B \subseteq A_g^*$ and by theorem 3.7 [1], $B_g^* \subseteq (A_g^*)_g^* \subseteq A_g^* \subseteq U$ .Hence B is an I_g**-closed set.

Theorem 1.36: Let (X, τ, I) be an ideal topological space. For every $A \in I$ and $A \subseteq X$ where I = P(X), A is an I_g**-closed set.

Proof: Let $A \subseteq U$ where U is $\star$-open. By theorem 3.5 [1], $A_g^* = \varnothing \subseteq U$. Hence A is an I_g**-closed set.

Theorem 1.37: Let (X, τ, I) be an ideal topological space, then A_g^* is an I_g**-closed set for every subset A of X.

Proof: Let $A_g^* \subseteq U$ where U is a $\star$-open set in X. By theorem 3.7 [1], $(A_g^*)_g^* \subseteq A_g^* \subseteq U$. Hence A_g^* is an I_g**-closed set.

Theorem 1.38: Let (X, τ, I) be an ideal topological space. Then every ideal I is I_g**-closed set.

Proof: By theorem 3.12 [1], I is a τ_g^* -closed set. By theorem 1.4, I is an I_g**- closed set.

Theorem 1.39: Let (X, τ, I) be an ideal topological space and A be an I_g**-closed subset of X. Then the following are equivalent:

(a) A is τ_g^* -closed.

(b) $A_g^* - A$ is $\star$-closed.

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Proof:

(a) ⇒(b): Since A is a τ_g^*-closed set, $A_g^* \subseteq A$ which implies $A_g^* - A = \emptyset$. Therefore $A_g^* - A$ is a ⋆-closed set.

(b) ⇒(a): Suppose that $A_g^* - A$ is a ⋆-closed set. Since A is an I_g**-closed set.

By theorem 1.26, $A_g^* - A = \emptyset$ and so $A_g^* = A$ which implies $A_g^* \subseteq A$. Consequently A is a τ_g^*-closed set.

Theorem 1.40: Let (X, τ, I) be an ideal topological space and $A \subseteq X$. Then the following are equivalent:

(a) A is an I_g**-closed set.

(b) $cl_g^*(A) \cap F = \emptyset$ whenever $A \cap F = \emptyset$ and F is ⋆-closed.

Proof:

(a) ⇒(b): Suppose that $A \cap F = \emptyset$ and F is ⋆-closed. Then $A \subseteq X - F$ where $X - F$ is ⋆-open. Since A is an I_g**-closed set, $cl_g^*(A) \subseteq X - F$ which implies that $cl_g^*(A) \cap F = \emptyset$.

(b) ⇒ (a): Let U be a ⋆-open set containing A. Then $A \cap (X-U) = \emptyset$ where X–U is a ⋆-closed set. By (b), $cl_g^*(A) \cap (X-U) = \emptyset$ and so $cl_g^*(A) \subseteq U$ which implies A is an I_g**-closed set.

1.2 I_g** - OPEN SETS

In this section, we define I_g** -open sets and basic properties of this notion are derived.

Definition 1.41: A subset A of an ideal topological space (X, τ, I) is said to be I_g**-open if $X - A$ is an I_g**- closed set.

Theorem 1.42: Let (X, τ, I) be an ideal topological space then the following hold:

(i) Every τ_g^* -open set is I_g**-open but not conversely.

(ii) Every ⋆-open set is I_g**-open but not conversely.

(iii) Every ⋆-g-open set is I_g**-open but not conversely.

(iv) Every I_g-⋆-open set is I_g**-open but not conversely.

Theorem: 1.43: Intersection of two I_g**-open sets is I_g**-open.

Proof: Let A and B be two I_g**-open sets. Then A^c and B^c are I_g**-closed. By theorem1.23, $A^c \cup B^c$ is I_g**-closed and so $(A \cap B)^c$ is an I_g**-closed set. Hence $A \cap B$ is an I_g**-open set.

Theorem 1.44: Let (X, τ, I) be an ideal topological space and $A \subseteq X$, then A is I_g**-open iff $F \subseteq int_g^*(A)$ whenever $F \subseteq A$ and F is ⋆-closed.

Proof: Let A be an I_g**-open set and F is a ⋆-closed set such that $F \subseteq A$. Then $X-A \subseteq X - F$ where $X - F$ is ⋆-open. Since X–A is I_g**-closed and $cl_g^*(X - A) \subseteq X - F$
which implies $F \subseteq X - (cl_g^*(X - A))$. Hence $F \subseteq int_g^*(A)$.
Conversely, let U be a ⋆-open set such that $X-A \subseteq U$. Then $X-U \subseteq A$ where X–U is ⋆-closed and so $X-U \subseteq int_g^*(A)$ implies $cl_g^*(X-A) \subseteq U$ which implies X–A is I_g**-closed. Hence A is an I_g**-open set.

Theorem 1.45: Let (X, τ, I) be an ideal topological space. Then for each $x \in X$ either $\{x\}$ is ⋆-closed (or) I_g**-open.

Proof: Suppose $\{x\}$ is not a ⋆-closed set. Then $\{x\}^c$ is not a ⋆-open set and hence X is the only ⋆-open set containing $X - \{x\}$ and so $X - \{x\}$ is I_g**closed. Consequently $\{x\}$ is an I_g**-open set.

Theorem 1.46: In a T_1-space, every I_g-closed set is an I_g**-closed set.

Proof: Let (X, τ, I) be a T_1-space.Then every I_g-closed set in X is ⋆-closed. By theorem 1.7, the result follows.

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1.3 A_I^g-set

In this section, we introduce A_I^g-set and investigated some of its properties.

Definition 1.47: A subset A of an ideal topological space (X, τ, I) is said to be an A_I^g-set if A = U∩ V where U is ⋆-open and V is a τ_g^*-closed set.

Example 1.48: Consider the ideal topological space (X, τ, I) where Let X= {a, b, c}, T= {φ, X, {a}, {a, b}} and I= {φ, {a}}. Then {b,c} is an A_I^g-set.

Remark 1.49: Let (X, τ, I) be an ideal topological space. Then

(i) Every ⋆-open set is an A_I^g-set.
(ii) Every τ_g^*-closed set is an A_I^g-set.

Proof:

(i) Let A be a ⋆-open set. Then A = A ∩ X where A is ⋆-open and X is τ_g^*-closed and hence A is an A_I^g-set.
(ii) Let A be an τ_g^*-closed set. Then A = X ∩ A where X is ⋆-open and A is τ_g^*-closed and hence A is an A_I^g-set.

Remark 1.50: The converses of the above theorem are not true as shown in the following example

Example 1.51: Let X= {a, b, c}, τ = {φ, X, {a}, {a, b}} and I= {φ, {a}}. Then {c} is an A_I^g-set but not a ⋆-open set.

Example 1.52: Let X= {a, b, c}, τ = {φ, X, {a}, {a, b}} and I= {φ, {a}}. Then {b} is an A_I^g-set but not a τ_g^* -closed set.

Theorem: 1.53: Every I-locally ⋆-closed set is an A_I^g-set.

Proof: Let A be an I-locally ⋆-closed set. Then A = U∩ V where U is open and V is a ⋆-closed set .Since every ⋆-closed set is a τ_g^*-closed set, V is τ_g^*-closed. Hence A is an A_I^g-set.

Remark 1.54: The converse of the above theorem is not true as shown in the following example.

Example 1.55: Let X= {a, b, c}, τ = {φ, X, {a}, {b, c}} and I= {φ, {a}, {b}, {a, b}}.

Then {a, c} is an A_I^g-set but not an I-locally ⋆-closed set.

Theorem 1.56: Every A_I^g-set is I-locally τ_g^*-closed.

Proof: Let A be an A_I^g-set. Then A=U ∩ V where U is ⋆-open and V is τ_g^*-closed.
Since every ⋆-open set is τ_g^*-open, A = U∩ V where U is τ_g^*-open and V is τ_g^*- closed. Hence A is an I-locally τ_g^*-closed set.

Remark 1.57: The converse of the above theorem is not true as seen from the following example

Example 1.58: Let X= {a, b, c}, τ = {φ, X, {a}, {a, b}} and I= {φ}. Then {a, c} is an I-locally τ_g^*-closed set but not an A_I^g-set.

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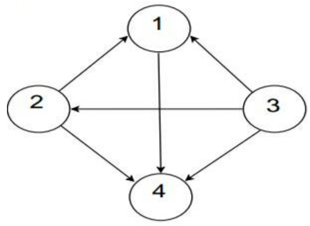
Theorem 1.59: Every I-locally closed set is an A_I^g-set.

Proof: Let A be an I-locally closed set. Then A = U∩ V where U is open and V is ⋆-perfect. Since every ⋆-perfect set is τ_g^*-closed. Hence A is an A_I^g-set.

Remark 1.60: From the above relations we get the following diagram where 1 → 2 represents 1 implies 2.



1: A_I^g-set

2: I-locally ⋆-closed

3: I-locally closed set

4: I-locally τ_g^*-closed

Theorem 1.61: Let (X, τ, I) be an ideal topological space, then the intersection of two A_I^g-sets is an A_I^g-set.

Proof: Let A and B be A_I^g-sets .Then $A = U_1 \cap V_1$ and $B = U_2 \cap V_2$ where U_1 and U_2 are ⋆-open sets and V_1 and V_2 are τ_g^*-closed sets.

Now, $A \cap B = (U_1 \cap V_1) \cap (U_2 \cap V_2) = (U_1 \cap U_2) \cap (V_1 \cap V_2)$ where $U_1 \cap U_2$ is ⋆-open and $V_1 \cap V_2$ is τ_g^*-closed which implies that A ∩ B is an A_I^g-set.

Theorem: 1.62: Let (X, τ, I) be an ideal topological space and A be an A_I^g-set of X, then the following hold

(a) If B is a τ_g^*-closed set, then A ∩ B is an A_I^g-set.

(b) If B is a ⋆-open set, then A ∩ B is an A_I^g-set..

Proof: Since A is an A_I^g-set, there exists a ⋆-open set U and a τ_g^*-closed set F such that A = U ∩ F.

(a) A ∩ B = (U ∩ F) ∩ B = U ∩ (F ∩ B) where F ∩ B is a τ_g^*-closed set and so A ∩B is an A_I^g-set.

(b) A ∩ B = (U ∩ F) ∩ B = (U ∩ B) ∩ F where U ∩ B is a ⋆-open set and so A ∩ B is an A_I^g-set.

Theorem: 1.63: Let A be a subset of an ideal topological space (X, τ, I). Then the following are equivalent.

(i) A is an A_I^g-set and an I_g**-closed set ,

(ii) A is a τ_g^*-closed set.

Proof:

(i) ⇒ (ii): Since A is an A_I^g-set, A = U ∩ V where U is a ⋆-open set and V is a τ_g^*-closed set. Therefore A ⊆ U and A ⊆ V. Since A is an I_g**-closed set, $A_g^* \subseteq U$. Also V is a τ_g^*-closed and $A_g^* \subseteq V_g^* \subseteq V$ which implies $A_g^* \subseteq V$. Consequently, $A_g^* \subseteq U \cap V = A$ and hence A is a τ_g^*-closed set.

(ii) ⇒ (i): A be a τ_g^*-closed set. By remark 1.49, A is an A_I^g-set and by theorem 1.4, A is an I_g**-closed set.

Theorem: 1.64. If A subset A of an ideal topological space (X, τ, I) is τ_g^*-closed Then A is I-locally τ_g^*-closed set and I_g**-closed set.

Proof: Since A is τ_g^*-closed, by theorem 1.4, A is an I_g**-closed. Also since every τ_g^*-closed set is I-locally τ_g^*-closed set, A is I-locally τ_g^*-closed set.

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Remark 1.65: The converse of the above theorem is not true as shown in the following example

Example 1.66: Let X= {a, b, c}, τ = {φ, X, {a}, {a, b}} and I= {φ, {a}}. Then {b} is an I-locally τ_g^*-closed set and also I_g**-closed set but not a τ_g^*-closed Set.

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