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SOME COMMON FIXED POINT THEOREMS IN FUZZY METRIC SPACES FOR MAPPINGS WITHOUT OWC

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ABSTRACT

Recently most of the results on fixed point theorems in fuzzy metric spaces deal either with occasionally weakly compatible (OWC) introduced by Al-Thagafi and Shahzad [3] or E.A. property introduced by Aamir and Moutawakil [1] and by Pant and Pant [7]. Our objective is to prove some common fixed point theorems by removing the occasionally weakly compatibility and semi weakly compatibility of mappings by using CLR property in Fuzzy metric space in the light of results of Bisht and Pant[4] and Sintunavarat and Kumam [10] which generalizes the result of Gupta, Deep and Tripathi [5].

Keywords: *Fuzzy metric space, weakly compatible mappings, CLR property.*

Mathematics Subject Classification*: 52H25, 47H10.*

INTRODUCTION

A number of interesting results have been obtained by various authors during the study of common fixed points of mapping, satisfying some contractive type condition. In 2009, Abbas, Altun and Gopal [2] proved some fixed point theorems using non-compatible mappings. In 2002, Aamri and Moutawakil [1] defined the idea of property E.A, which is the generalization of the concept of non compatible maps. Further, Pant and Pant [7] studied the common fixed points of a pair of property E.A in fuzzy metric space. It is observed that property (E.A) and common property (E.A) require the closedness of the subspaces for the existence of fixed point. In 2008, Al-Thagafi and Shahzad [3] introduced the concept of occasionally weakly compatible (owc). Recently, Gupta, Deep and Tripathi [5] proved fixed point theorems via notion of property E.A., semi-compatible mappings and occasionally weakly compatible mappings (owc) in fuzzy metric spaces satisfying contractive type condition under closedness of subsets.

Recently, in 2011, Sintunavarat and Kumam [10] coined the idea of “common limit range property” which never requires the closedness of the subspaces for the existence of fixed point.

On the other hand, Bisht and Pant [4] criticize the concept of owc as follows “Under contractive conditions the existence of a common fixed point and occasional weak compatibility are equivalent conditions, and consequently, proving existence of fixed points by assuming owc is equivalent to proving the existence of fixed points by assuming the existence of fixed points”.

Our objective is to prove some common fixed point theorems by removing the occasionally weakly compatibility and semi weakly compatibility of mappings by using CLR property in Fuzzy metric space in the light of results of Bisht and Pant[4] and [10]. Our result generalizes the result of Gupta, Deep and Tripathi [5].

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PRELIMINARIES

Definition 2.1 [11]: A binary operation $*$: $[0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norm if $*$ satisfies the following conditions:

(1) $*$ is commutative and associative,

(2) $*$ is continuous,

(3) $a * 1 = a$ for all $a \in [0, 1]$,

(4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ and $a, b, c, d \in [0, 1]$.

Example of t-norm are $a * b = ab$ and $a * b = \min\{a, b\}$.

Definition 2.2 [6]: The 3-tuple $(X, M, *)$ is said to be Fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a Fuzzy set in $X^2 \times [0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$ and $s, t > 0$.

(1) $M(x, y, 0) = 0$
(2) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$
(3) $M(x, y, t) = M(y, x, t)$
(4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$
(5) $M(x, y, .) : [0, \infty) \to [0, 1]$ is left continuous
(6) $\lim_{t\to\infty} M(x, y, t) = 1$

Definition 2.3 [8]: Two self mappings A and S of a Fuzzy metric space $(X, M, *)$ are said to be semi-compatible if and only if $M(ASx_n, Sp, t) \to 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $Sx_n, Ax_n \to p$ for some point in X, as $n \to \infty$.

Definition 2.4 [9]: Two self mappings A and S of a Fuzzy metric space $(X, M, *)$ are said to be weakly-compatible if they commute at their coincidence points, i.e

$$Ax = Sx \text{ implies } ASx=SAx.$$

Definition 2.5 [2]: Two self mappings A and S of a Fuzzy metric space $(X, M, *)$ are said to satisfy the property E.A if there exist sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z \text{ for some } z \in X.$$

Definition 2.6 [11]: A pair (f, g) of self-mappings of a FMS $(X, M,*)$ is said to satisfy the common limit in the range property with respect to mapping g (briey, (CLR_g) property), if there exists a sequence x_n in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gu, \text{ for some } u \in X.$$

Recently, Gupta, Deep and Tripathi [5] proved the following:

Theorem 2.7: Let A, B, S and T be self mappings on a Fuzzy metric space $(X, M, *)$ satisfying the following condition:
$M(Ax, By, t) \geq r\,(\min\{M(Sx, Ty, t), M(Sx, Ax, t), M(Sx, By, t), M(Ty, Ax, t)\})$

for all $x, y \in X$, where $r : [0, 1] \mapsto [0, 1]$ is a continuous function such that

$$r(t) > t \text{ for each } t < 1 \text{ and } r(t) = 1 \text{ for } t = 1.$$

Also, suppose the pair (A, S) and (B, T) share the common property (E.A), and S (X) and T (X) are closed subsets of X, then the pair (A, S) as well as (B, T) have a coincidence point. Further A, B, S, T have a unique common fixed point provided the pair (A, S) is semi-compatible and (B, T) is occasionally weakly compatible.

MAIN RESULTS

Theorem 3.1: Let $A,\ B,\ S$ and T be self-mappings of a FMS $(X, M,*)$ satisfying:
(3.1.1) the pairs (A, S) and (B, T) are weakly compatible and enjoy (CLR_S) and (CLR_T) property, respectively,
(3.1.2) for all $x, y \in X$ and $t > 0$,

$$M(Ax, By, t) \geq r\left\{min\begin{pmatrix}M(Sx, Ty, t), M(Sx, Ax, t),\\ M(Sx, By, t), M(Ty, Ax, t)\end{pmatrix}\right\}$$

where $r: [0,1] \to [0,1]$ is continuous function such that

$$r(p) > p, \text{ for each } p < 1 \text{ and } r(p) = 1, \text{ for } p = 1.$$

Then A, B, S and T have a unique common fixed point in X.

Proof: Since the pairs (A, S) and (B, T) enjoy (CLR_S) and (CLR_T) property, respectively, therefore there exist sequences x_n and y_n in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = Sx,$$

and

$$\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = Ty,$$

for some $x, y \in X$.

We claim that $Sx = Ty$. If not, then by using condition (3.1.2) with $x = x_n$ and $y = y_n$ we have,

$$M(Ax_n, By_n, t) \geq r\left\{min\begin{pmatrix} M(Sx_n, Ty_n, t), M(Sx_n, Ax_n, t), \\ M(Sx_n, By_n, t), M(Ty_n, Ax_n, t) \end{pmatrix}\right\};$$

Take the limit as $n \to \infty$, we get,

$$M(Sx, Ty, t) \geq r\left\{min\begin{pmatrix} M(Sx, Ty, t), M(Sx, Sx, t), \\ M(Sx, Ty, t), M(Ty, Sx, t) \end{pmatrix}\right\};$$

$$M(Sx, Ty, t) \geq r\left\{min\begin{pmatrix} M(Sx, Ty, t), 1, \\ M(Sx, Ty, t), M(Ty, Sx, t) \end{pmatrix}\right\};$$

$$M(Sx, Ty, t) \geq r\{M(Sx, Ty, t)\} > M(Sx, Ty, t);$$

which is a contradiction. Hence, we have, $Ax = Sx$.

We claim that $Sx = Ty$. If not, then by using condition (3.1.2) with $y = y_n$ we have,

$$M(Ax, By_n, t) \geq r\left\{min\begin{pmatrix} M(Sx, Ty_n, t), M(Sx, Ax, t), \\ M(Sx, By_n, t), M(Ty_n, Ax, t) \end{pmatrix}\right\};$$

Take the limit as $n \to \infty$, we get,

$$M(Ax, Ty, t) \geq r\left\{min\begin{pmatrix} M(Sx, Ty, t), M(Sx, Ax, t), \\ M(Sx, Ty, t), M(Ty, Ax, t) \end{pmatrix}\right\};$$

$$M(Ax, Sx, t) \geq r\left\{min\begin{pmatrix} M(Sx, Sx, t), M(Sx, Ax, t), \\ M(Sx, Sx, t), M(Sx, Ax, t) \end{pmatrix}\right\};$$

$$M(Ax, Sx, t) \geq r\left\{min\begin{pmatrix} 1, M(Sx, Ax, t), \\ 1, M(Sx, Ax, t) \end{pmatrix}\right\};$$

$$M(Ax, Sx, t) \geq r\{M(Ax, Sx, t)\} > M(Ax, Sx, t);$$

which is a contradiction. Hence, we have, $Ax = Sx$. Similarly, we can show that $Ty = By$.

Thus,

$$Ax = Sx = Ty = By.$$

Since, the pairs (A, S) and (B, T) are weakly compatible therefore

$$Ax = Sx, \text{ implies that } ASx = SAx,$$

and

$$Ty = By, \text{ implies that } BTy = TBy.$$

Thus we have

$$AAx = ASx = SAx = SSx$$

and

$$BBy = BAx = BTy = TBy = TAx = TTy.$$

We claim that $AAx = Ax$.. If not, then by using condition (3.1.2) with $x = Ax$, we have,

$$M(AAx, By, t) \geq r\left\{min\begin{pmatrix} M(SAx, Ty, t), M(SAx, AAx, t), \\ M(SAx, By, t), M(Ty, AAx, t) \end{pmatrix}\right\};$$

$$M(AAx, Ax, t) \geq r\left\{min\begin{pmatrix} M(AAx, Ax, t), M(AAx, AAx, t), \\ M(AAx, Ax, t), M(Ax, AAx, t) \end{pmatrix}\right\};$$

$$M(AAx, Ax, t) \geq r\left\{min\begin{pmatrix} M(AAx, Ax, t), 1, \\ M(AAx, Ax, t), M(Ax, AAx, t) \end{pmatrix}\right\};$$

$$M(AAx, Ax, t) \geq r\{M(AAx, Ax, t)\} > M(AAx, Ax, t);$$

which is a contradiction. Hence, we have, $AAx = Ax$. Similarly, we can show that $BBy = By$.

Now we have,

$$SAx = AAx = Ax = By = BBy = BAx = TAx.$$

Let $m = Ax$, then

$$Am = Sm = Bm = Tm = m,$$

i.e., m is a common fixed point of A, B, S and T.

The uniqueness of fixed point follows from (3.1.2).

Theorem 3.2: Let A, B, S and T be self-mappings of a FMS $(X, M,*)$ satisfying:
(3.1.1) the pairs (A, S) and (B, T) are weakly compatible and enjoy ($\mathrm{CLR_S}$) and ($\mathrm{CLR_T}$) property, respectively,
(3.1.2) for all $x, y \in X$ and $t > 0$,

$$\int_0^{M(Ax,By,t)} \phi(t)dt \geq \int_0^{r\{m(x,y,t)\}} \phi(t)dt;$$

where $\phi : R^+ \to R^+$ is a Lebesque-integrable mapping which is summable, non-negative and such that $\int_0^{\epsilon} \phi(t)dt > 0$, for each $\epsilon > 0$, and

$$m(x,y,t) \geq r\left\{min\begin{pmatrix} M(Sx,Ty,t), M(Sx,Ax,t), \\ M(Sx,By,t), M(Ty,Ax,t) \end{pmatrix}\right\}$$

where r: [0,1] → [0,1] is continuous function such that

$r(p) > p$, for each $p < 1$ and $r(p) = 1$, for $p = 1$.

Then A, B, S and T have a unique common fixed point in X.

Proof: Proof is an easy consequence of Theorem 3.1.

CONCLUSION

Closeness of the subspaces have been removed as well as E.A. property and owc has been replaced by CLR property to improve the existing results.

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