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SOLVING TRANSPORTATION PROBLEMS IN UNCERTAIN ENVIRONMENT

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ABSTRACT

This paper discusses a method to solve the Fuzzy Transportation problem with the objective to minimize the fuzzy cost of fuzzy transportation and to find the optimal routes of fuzzy transportation from origins to destinations with minimum fuzzy cost. Here we use ranking method to find the big and small fuzzy number.

Keywords: *Transportation problem, minimization cost, Sources, Demand.*

1. INTRODUCTION

Transportation problems play an important role in logistics and supply chain management for reducing cost and improving service. In today's highly competitive market the pressure on organizations to find better ways to create and deliver products and services to customers becomes stronger. How and when to send the products to the customers in the qualities they want in a cost effective manner becomes more challenging. Transportation models provide a powerful framework to meet this challenge. They ensure efficient movement and timely availability of raw materials and finished goods.

In the 1920s A.N. Tolstoi was one of the first to study the transportation problem mathematically. In 1930, in the collection *Transportation Planning, Volume I* for the National Commissariat of Transportation of the Soviet Union, he published a paper "Methods of Finding the Minimal Kilometrage in Cargo-transportation in space". Major advances were made in the field during World War II by the Soviet/Russian mathematician and economist Leonid Kantorovich.

In 2012 Abdallah A. Hlayel *et al.* was introduced the best candidates method. A new approach to solve an initial basic feasible solution for transportation problems was introduced by Mollah Mesbahuddin Ahmed *et al.* in 2016. In 2016, Priya et al introduced ICMM method to solve the transportation problem. In 2012 Fegade *et al.* introduced zero suffix method to solve fuzzy transportation problem with Robust Ranking methodology. In 2014 Nareshkumar *et al.*, develop fuzzy version of VAM to solve transportation problem with symmetric triangular fuzzy number. In 2016, K. Jaikumar solved fully fuzzy transportation problem with new approach.

The concept of fuzzy set introduced by Zadeh in 1965 has rich potential for applications in several directions. Fuzzy set theory is now applied to problems in the fields of engineering, business, medical and related health sciences, and the natural sciences. The theory proposes a mathematical technique for handling imprecise concepts and problems that have many possible solutions. The concept of fuzzy mathematical programming in a general level was introduced by Tanaka *et al.* in the framework of the fuzzy decision of Bellman and Zadeh [3]. Relevant terms are defined in the following sections.

2. PRELIMINARIES

Fuzzy Set

A fuzzy set $\tilde{\mathbf{A}}$ is defined as, $\tilde{\mathbf{A}} = \{(x, \mu_A(x)): x \in A, \mu_A(x) \in [0,1]\}$. In the pair $(x, \mu_A(x))$, the first element x belongs to the classical set A, and the second element $\mu_A(x)$ belongs to the interval [0, 1], called *membership function*.

A fuzzy set can also be denoted by, $\tilde{\mathbf{A}} = \{\mu_A(x)/x: x \in A, \mu_A(x) \in [0,1]\}$. Here the symbol '/' does not represent the division sign. It indicates that the top number $\mu_A(x)$ is the membership value of the element x on the bottom.

Fuzzy Number

The notion of fuzzy numbers was introduced by Dubois. D and Prade.H [9]. A fuzzy subset $\tilde{A}$ of the real line R with membership function $\mu_{\tilde{A}}$: R → [0, 1] is called a fuzzy number if

i. $\tilde{\mathbf{A}}$ is normal, (i.e.) there exists an element x_0 such that $\boldsymbol{\mu_{\tilde{A}}(x_0) = 1}$
ii. $\tilde{\mathbf{A}}$ is fuzzy convex, (i.e.) $\boldsymbol{\mu_{\tilde{A}}[\lambda X_1 + (1-\lambda)X_2] \geq \mu_{\tilde{A}}(X_1)}$ $\boldsymbol{, \mu_{\tilde{A}}(X_2), X_1, X_2 \in R, \forall\lambda \in [0, 1]}$
iii. $\mu_{\tilde{A}}$ is upper continuous, and
iv. Support of $\tilde{A}$ is bounded, where supp $\tilde{A} = \{x \in R: \mu_{\tilde{A}}(x) > 0\}$

Triangular Fuzzy Number

The fuzzy set $\tilde{\boldsymbol{A}}$ = (a_1, a_2, a_3) where $a_1 \leq a_2 \leq a_3$ and defined on R, is called the triangular fuzzy number if membership function of $\tilde{\boldsymbol{A}}$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ \dfrac{a_3-x}{a_3-a_2} & a_2 \leq x \leq a_3 \\ 0 & Otherwise \end{cases}$$

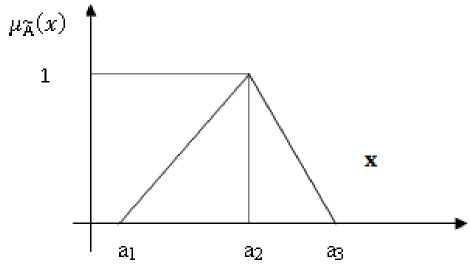

Figure :1.1 - Triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$

Operations on Fuzzy Numbers

Though different methods are available for the operation of fuzzy numbers, the function principle and the extension principle are used for the operation of fuzzy numbers in the present thesis.

Function Principle

The function principle was introduced by Chen to treat fuzzy arithmetical operations. This principle is used for the operation of addition, multiplication, subtraction and division of fuzzy numbers.

Suppose $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers.

Then

i. The **Addition** of $\tilde{A}$ and $\tilde{B}$ is
$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$ where $a_1, a_2, a_3, b_1, b_2, b_3$ are real numbers.

ii. The product of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}\times\tilde{B} = (c_1, c_2, c_3)$ where $T = \{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$
$c_1 = \min T$, $c_2 = a_2b_2$, $c_3 = \max T$
If $a_1, a_2, a_3, b_1, b_2, b_3$ are all non zero positive real numbers, then
$\tilde{A}\times\tilde{B} = (a_1b_1, a_2b_2, a_3b_3)$

iii. $-\tilde{B} = (-b_3, -b_2, -b_1)$ then the subtraction of $\tilde{B}$ from $\tilde{A}$ is
$\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$ where $a_1, a_2, a_3, b_1, b_2, b_3$ are real numbers.

iv. $\frac{1}{\tilde{B}} = \tilde{B}^{-1} = \left(\frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1}\right)$ where b_1, b_2, b_3 are all non zero real numbers then $\frac{\tilde{A}}{\tilde{B}} = \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right)$

v. Let $\alpha \in R$, then $\alpha\tilde{A} = (\alpha a_1, \alpha a_2, \alpha a_3)$ if $\alpha \geq 0$
$= (\alpha a_3, \alpha a_2, \alpha a_1)$ if $\alpha < 0$

Fuzzy Transportation Model

Suppose that there are m origins and n destinations

	D_1	D_2	⋯	D_n	
O_1	$\tilde{x}_{11}$ $\tilde{c}_{11}$	$\tilde{x}_{12}$ $\tilde{c}_{12}$	⋯	$\tilde{x}_{1n}$ $\tilde{c}_{1n}$	$\tilde{a}_1$
O_2	$\tilde{x}_{21}$ $\tilde{c}_{21}$	$\tilde{x}_{22}$ $\tilde{c}_{22}$	⋯	$\tilde{x}_{2n}$ $\tilde{c}_{2n}$	$\tilde{a}_2$
⋮	⋮	⋮	⋱	⋮	⋮
O_m	$\tilde{x}_{m1}$ $\tilde{c}_{m1}$	$\tilde{x}_{m2}$ $\tilde{c}_{m2}$	⋯	$\tilde{x}_{mn}$ $\tilde{c}_{mn}$	$\tilde{a}_m$
	$\tilde{b}_1$	$\tilde{b}_2$	⋯	$\tilde{b}_m$	

$O_1, O_2, O_3, \ldots, O_m$ are "m" origins and
$D_1, D_2, D_3, \ldots, D_n$ are 'n' destinations.
$\tilde{c}_{ij}$ = The fuzzy transportation cost per unit from i^{th} origin to j^{th} destination.
$\tilde{a}_i$ = Fuzzy availability in the i^{th} origin.
$\tilde{b}_j$ = Fuzzy requirement in the j^{th} destination.
$\tilde{x}_{ij}$= The fuzzy amount of allocation from i^{th} origin to j^{th} destination.

If $\sum \tilde{a}_i = \sum \tilde{b}_j$ then the given Transportation problem is balanced one.

If $\sum \tilde{a}_i \neq \sum \tilde{b}_j$ then the given Transportation problem is not balanced one.

We can solve the above the above Fuzzy Transportation problem using a Fuzzy Row minima method or Fuzzy Vogel's Approximation method using Ranking method.

3. PROPOSED METHOD

In this section new approach is proposed to solve initial fuzzy basic feasible solution for fully fuzzy transportation problem. Here cost, source destination and the variables are all taken as triangular fuzzy numbers.

Algorithm

Step-1: Examine whether the transportation problem is balanced or not. If it is balanced then go to next step.

Step-2: Find the smallest cost from each row and subtract the smallest cost from each element of the row

Step-3: Find the smallest cost from each column and subtract the smallest cost from each element of the column

Step-4: Find the difference between minimum and next minimum in each row or column which is called as row penalty and column penalty and write it in the side and bottom

Step-5: From that select the maximum value. From the selected row/column we need to allocate the minimum of supply/demand in the minimum element of the row or column. Eliminate by deleting the columns or rows corresponding to where the supply or demand is satisfied.

Step-6: Repeating the step 4 to step 5 until satisfaction of all the supply and demand is met.

Step-7:-Now total minimum cost is calculated as sum of the product of cost and corresponding allocate value of supply/demand.

4. NUMERICAL EXAMPLE

A company has three production facilities S_1, S_2 and S_3 and with production capacity of 4, 5, 6 units (in 100s) per week of a product respectively. These units are to be shipped to three warehousesD_1, D_2 and D_3 with requirement of 5, 6, and 4 units (in 100s) per week, respectively. The transportation costs (in rupees) per units between factories to warehouses are given in the table below

	D_1	D_2	D_3	Capacity
S_1	(4,5,6)	(3,4,5)	(6,7,8)	(3,4,5)
S_2	(1,2,3)	(5,6,7)	(4,5,6)	(5,6,7)
S_3	(3,4,5)	(7,8,9)	(2,3,4)	(4,5,6)
Demand	(4,5,6)	(5,6,7)	(3,4,5)	

Solving procedure:
Applying triangular ranking technique

$$A = \frac{a_1 + 4a_2 + a_3}{6}, \qquad A_{11} = \frac{4 + 4(5) + 6}{6} = 5$$

Table-1

	D_1	D_2	D_3	Capacity
S_1	(4,5,6)	(3,4,5)	(6,7,8)	(3,4,5)
S_2	(1,2,3)	(5,6,7)	(4,5,6)	(5,6,7)
S_3	(3,4,5)	(7,8,9)	(2,3,4)	(4,5,6)
Demand	(4,5,6)	(5,6,7)	(3,4,5)	

An effective methodology for solving transportation problem

Table-2

	D_1	D_2	D_3	Capacity
S_1	5	4	7	(3,4,5)
S_2	2	6	5	(5,6,7)
S_3	4	8	3	(4,5,6)
Demand	(4,5,6)	(5,6,7)	(3,4,5)	

Solution: Step 1 Since $\sum a_i = \sum b_j = 15$

Step-2: Find the smallest cost from each row and subtract the smallest cost from each element of the row

Table-3

	D_1	D_2	D_3	Capacity
S_1	1	0	3	(3,4,5)
S_2	0	4	3	(5,6,7)
S_3	1	5	0	(4,5,6)
Demand	(4,5,6)	(5,6,7)	(3,4,5)	

Step-3: Find the smallest cost from each column and subtract the smallest cost from each element of the column

Table-4

	D_1	D_2	D_3	Capacity
S_1	1	0	3	(3,4,5)
S_2	0	4	3	(5,6,7)
S_3	1	5	0	(4,5,6)
Demand	(4,5,6)	(5,6,7)	(3,4,5)	

Step-4: Find the difference between minimum and next minimum in each row or column

Table-5

	D_1	D_2	D_3	Capacity	Row Penalty
S_1	1	0 (3,4,5)	3	(3,4,5)	(1) - - -
S_2	0 (4,5,6)	4 (-3,1,5)	3	(5,6,7) (-1,1,3)	(3) (3) (1) (4)
S_3	1	5 (-1,1,3)	0 (3,4,5)	(4,5,6) (-1,1,3)	(1) (1) (5) (5)
Demand	(4,5,6)	(5,6,7) (0,2,4) (- 3,1,5)	(3,4,5)		
ColumnPenalty	(1) (1) - -	(4) (1) (1) (1)	(3) (3) (3) -		

Therefore, the allocation in the original TT is

Table-6

	D_1	D_2	D_3	Capacity
S_1	(4,5,6)	(3,4,5) (3,4,5)	(6,7,8)	(3,4,5)
S_2	(1,2,3) (4,5,6)	(5,6,7) (-3,1,5)	(4,5,6)	(5,6,7)
S_3	(3,4,5)	(7,8,9) (-1,1,3)	(2,3,4) (3,4,5)	(4,5,6)
Demand	(4,5,6)	(5,6,7)	(3,4,5)	

The Fuzzy transportation cost is

$$= (3,4,5)(3,4,5) + (4,5,6)\,(1,2,3) + (5,6,7)(-3,1,5) + (7,8,9)(-1,1,3) + (2,3,4)(3,4,5)$$
$$= (9,16,25) + (4,10,18) + (-21,6,25) + (-9,8,21) + (6,12,20)$$
$$= (-11,52,109)$$
$$= \frac{-11 + 4(52) + 109}{6} = 51$$

5. RESULT ANALYSIS

After obtaining an IBFS by the proposed method, the obtained result is compared with the results obtained by other existing methods in shown in below table

Method	Example
An effective Methodology	51
Average Penalty	53
N-W-C-M	69.6

6. CONCLUSION

The proposed method is an attractive method which is very simple, easy to understand and gives result exactly or even lessor to VAM method. All necessary qualities of being time efficient, easy applicability etc., forms the core of the being implemented successfully.

Also in this paper we have described the comparison between the transportation methods

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