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# MOMENT INEQUALITIES OF CERTAIN AGEING CLASSES

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***ABSTRACT***

*In this paper, the moment inequalities for ageing classes New Renewal Better than Used (NRBU), Renewal New Better than Used (RNBU), Decreasing Mean Residual Life (DMRL), New Better than Renewal Used in Laplace transform order (NBRUL) and New Better than Renewal Used in the Residual Probability order ($NBRU_{rp}$).*

***Keywords:*** *Ageing Classes, Moments inequalities, NRBU, RNBU, DMRL, NBRUL, $NBRU_{rp}$.*

## 1. INTRODUCTION

The purpose of this paper is to provide the moment inequalities of the classes NRBU, RNBU, DMRL, NBRUL and $NBRU_{rp}$ that will generally assert that if $\mu < \infty$ ,then the moments would exist for the above said classes. The rest of the paper is structured as follows, In section 2, we give the definition of the ageing classes that are required for the discussion. In section 3, we deduce the moment inequalities of these ageing classes. Finally, conclusion is given in section 4.

## 2. PRELIMINARIES

**Definition 2.1:** A life distribution F(·) or its survival function $\bar{F}(\cdot)$ with support s= {t:$\bar{F}$(t) >0 } and finite mean μ =$\int_0^\infty \bar{F}(t)dt$ is said to be INCREASING FAILURE RATE (IFR), if the condition survival function $\frac{\bar{F}(x+t)}{\bar{F}(t)}$ is decreasing in t, whenever x > 0 and t ϵ s.

**Definition 2.2:** A life distribution F(·) or its survival function $\bar{F}(\cdot)$ with support s = {t:$\bar{F}$(t) >0} and finite mean μ =$\int_0^\infty \bar{F}(t)dt$ is said to be INCREASING FAILURE RATE AVERAGE (IFRA), if $\frac{\ln \bar{F}(t)}{t}$ is increasing in s

**Definition 2.3:** A life distribution F(·) or its survival function $\bar{F}(\cdot)$ with support s = {t:$\bar{F}$(t) > 0} and finite mean μ =$\int_0^\infty \bar{F}(t)dt$ is said to be NEW BETTER THAN USED (NBU), if $\bar{F}$ (x + y) _ $\bar{F}$ (x) $\bar{F}$ (y), for all x, y ≥ 0.

**Definition 2.4:** A life distribution F(·) or its survival function $\bar{F}(\cdot)$ with support s= {t:$\bar{F}$(t) >0} and finite mean μ =$\int_0^\infty \bar{F}(t)dt$ is said to be NEW BETTER THAN USED IN EXPECTATION (NBUE), if

$\int_0^\infty \bar{F}$ (x+y)dy≤$\bar{F}$ (x)$\int_0^\infty \bar{F}$(y)dy, for all x ≥ 0

**Definition 2.5:** A random variable X or its distribution F is said to have New Renewal Better than Used, denoted by (NRBU) property, if $X_t \leq^{st} X_{WF}$, where $X_t$ is the conditional variable of X given t with distribution.

$$\bar{F}_y(t) = P(X \leq t | T \geq X) \quad (1)$$

This definition means that X is NRBU, if

$$\bar{F}_y(t) \leq \bar{W}_F(t)$$

When the renewal of the system is continued indefinitely, the (stationary) life distribution of a device in operation at time $x$ is $W_F(x)$. The corresponding renewal survival function is $\bar{W}_F(x)$, Where $\mu = \mu_F = \int_0^\infty \bar{F}(u)du < \infty$ is the mean life of the random variable

The inequality (1) can have the form

$$\bar{F}_y(y + t) \leq \bar{F}_y(y)\,\bar{W}_F(t) \quad \text{for all } y, t \geq 0 \quad (2)$$

Integrating both sides of inequality (2) w.r.to y over [x, ∞], gives

$$\bar{W}_F(x,t) \leq \bar{W}_F(x)\bar{W}_F(t) \tag{3}$$

That is, the renewal distribution is NBU and is denoted by RNBU.

**Definition 2.6:** The Mean Residual Life function is defined as

$$m(x) = E[X - x|X \geq x] = \int_0^\infty \bar{F}(u)du \,/\bar{F}(x)$$

If m(x) is nonincreasing in $x \geq 0$, then F is said to be a Decreasing Mean Residual Life (DMRL) distribution. The dual class, Increasing Mean Residual Life (IMRL) distribution, can be defined by replacing nonincreasing by nondecreasing in the definition of the DMRL class.

**Definition 2.7:** If X is a random variable with survival function F(x) ,then X is said to have New Better (Worse) than Renewal Used property, denoted by NBRU (NWRU), if

$$\bar{W}_F(x|t) \leq (\geq)\ \bar{F}\ (x|0)\ , \quad x \geq 0, t \geq 0$$

**Definition 2.8:** X is said to be New Better (Worse) than Renewal Used in Laplace Transform Order NBRUL (NWRUL) if

$$\int_0^\infty e^{-sx}\,\bar{W}_F(x+t)\,dx \leq (\geq)\bar{W}_F(t)\int_0^\infty e^{-sx}\bar{F}\,(x)dx \quad \text{for all } x, t, s \geq 0$$

**Remark:** It is obvious that NBRU ⇒ NBRUL ⇒ NBRUE.

**Definition 2.9:** The random variable X is said to be smaller than Y in the residual probability order (denoted by $X \leq_{rp} Y$) if,

$$\int_t^\infty [f(x)\bar{G}(x) - g(x)\bar{F}(x)]dx \geq 0.$$

**Definition 2.10:** A random life X is said to be New Better than Renewal Used in the RP order ($NBRU_{rp}$) if $X^* \leq_{rp} X$ , or equivalently,

$$\int_t^\infty \left[\bar{F}^2(x) - f(x)\int_0^\infty \bar{F}(u)du\right]dx \geq 0. \quad \forall\ t \geq 0$$

## 3. MOMENT INEQUALITY

### The NRBU and RNBU Classes

The moment inequalities of the ageing classes NRBU and RNBU are presented in this section.

**Theorem 3.1:** For all non-negative integer r ≥0, and F is NRBU, we get,

$$\frac{\mu_{(r+2)}}{(r+2)} \leq \frac{1}{\mu}\left[\sum_{i=0}^{r}\binom{r}{i}\frac{\mu_{(r-i+1)}}{(r-i+1)}\right]\frac{\mu_{(i+2)}}{(i+1)(i+2)}, \tag{4}$$

where, $\mu_r = E(X^r)$.

**Corollary 3.2:** If r = 1 then (4) reduces to the same form of the test of Mahmoud *et al.* (2003).

**Theorem 3.3:** For all non-negative integer r ≥0, and F is RNBU, we get,

$$\frac{\mu_{(r+3)}}{(r+2)(r+3)} \leq \frac{1}{\mu}\left[\sum_{i=0}^{r}\binom{r}{i}\frac{\mu_{(r-i+2)}\mu_{(i+2)}}{(r-i+1)(r-i+2)(i+1)(i+2)}\right], \tag{5}$$

**Corollary 3.4:** If r = 0 then (5) reduces to the same form of the test statistic of Mahmoud *et al.* (2003).

### The DMRL Class

The moment inequalities of the ageing class DMRL are presented in this section.

**Theorem 3.5:** If F is DMRL (IMRL), then

$$\mu_2 \geq (\leq)\frac{\mu^2}{2} \tag{6}$$

where $\mu(r) = E[\min(X1, X2)]^r$

### The NBRUL Class

The moment inequalities of the ageing class NBRUL are presented in this section.

 

**Theorem 3.6:** Let F be New Better than Used in Laplace transform order (NBRUL) life distribution such that all moments exist and finite then for integers r ≥0 ands ≥ 0. Then

$$\frac{\mu_{(r+2)}}{s(r+1)(r+2)}[1-\zeta(s)] \geq \frac{-(-1)^r r!}{s^{r+2}}[\mu_F - \frac{1}{s}(1-\zeta(s))] + \frac{r!}{s^{r+1}}\sum_{i=0}^{r}(-1)\frac{s^{r-i}}{(r-i+2)!}\mu_{(r-i+2)}\,, \quad (7)$$

$\mu_{(r)} = E(X^r), \zeta(s) = Ee^{-sX}$.

**Remark:** For r = 1, Eq. (7) will be reduced to

$$\frac{\mu_3}{6s}[1-\zeta(s)] \leq \frac{1}{s^3}[\mu - \frac{1}{s}(1-\zeta(s))] + \frac{1}{s^2}\left[\frac{s}{6}\mu_{(3)} - \frac{1}{2}\mu_{(2)}\right] \quad (8)$$

where $\mu_{(r)} = \int_0^\infty x^r\, dF(x)$.

**The $NBRUrp$ Class**

The moment inequalities of the ageing class $NBRUrp$ are presented in this section.

**Theorem 3.8:** If F is $NBRU_{rp}$, then for all integer r ≥ 0 ,

$$\int_0^\infty x^{r+2}\bar{F}(x)dF(x) \geq \frac{r+2}{r+4}\int_0^\infty x^{r+1}\left(\int_x^\infty t dF(t)\right) dF(x). \quad (9)$$

## 4. CONCLUSION

The moment inequalities for ageing classes New Renewal Better than Used (NRBU), Renewal New Better than Used (RNBU), Decreasing Mean Residual Life (DMRL), New Better than Renewal Used in Laplace transform order (NBRUL) and New Better than Renewal Used in the *R*esidual *P*robability order ($NBRUrp$).

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