International Journal of Mathematical Archive-9(9), 2018, 23-25
Available online through www.ijma.info ISSN 2229 – 5046

OPTIMAL REPLACEMENT POLICY FOR A DETERIORATING SYSTEM OF INCREASING REPAIR TIMES WITH VARYING COST STRUCTURES

GOVINDARAJU.P AND SURESH KUMAR.N

Department of Mathematics, Islamiah College, Vaniyambadi, India.
E-mail: govindrajmaths69@gmail.com

(Received On: 17-08-18; Revised & Accepted On: 04-09-18)

ABSTRACT

In this paper, the maintenance problem of a repairable deteriorating System subject to random shocks with varying cost structures is studied. An Explicit expression for the long run average cost per unit time under policy N is derived and an optimal policy N for minimizing the long run average cost per unit time is determined analytically.*

Key Words: *GEOMETRIC PROCESS, REPLACEMENT POLICY, RENEWAL REWARD PROCESS, STOCHASTIC ORDERS.*

AMS Subject Classification: *[2000] 60K10, 90B25.*

1. INTRODUCTION

In most of the real life situations, owing to the ageing effect and accumulated wearing, systems are degenerative in the sense that the successive operating times between failures will be shorter and shorter, while the consecutive repair times after failures are getting longer and longer. In other words, the successive operating times are stochastically decreasing and finally dying out, while the consecutive repair times are stochastically increasing and finally tend to infinity. Barlow and Proschan (1983) introduced an imperfect repair model. Other studies along this line include Block.et.al (1985), Kijima (1989), Wang and Zhang (2009), Zhang and Wang (2011) and Zong, Chai and Zhang (2013). Most of existing shock models were based on the accumulated or extreme damage causing a system failure. Li (1984) first introduced the $\delta -$shock model to avoid measuring the amount of damage which may not be easy in many situations. The $\delta -$ shock model focuses mainly on the frequency of shocks rather than the magnitude of shocks. In this paper, we adopt a general $\delta -$ shock model by letting δ to be an exponentially distributed random variable with parameter varying with number of repairs. Lam and Zang (2004) studied a general $\delta -$shock model for both repairable improving and deteriorating systems and derived the N^* replacement policy.

2. PRELIMINARIES

To formulate the $\delta -$shock model, we need the geometric process first introduced by Lam [1998].

Definition 2.1: A random variable X is said to be stochastically smaller than another random variable Y, if $P(X > \alpha) \leq P(Y > \alpha),\ for\ all\ real\ \alpha$. It is denoted by $X \leq_{st} Y$. Further, a stochastic process $\{X_n, n = 1,2, \dots\}$ is said to be stochastically increasing, if $X_n \leq_{st} X_{n+1}, for\ n = 1,2, \dots$.

Definition 2.2: A Stochastic process $\{X_n, n = 1,2, \dots\}$ is a geometric process (GP), if there exists a real constant $a > 0$ such that $\{a^{n-1}X_n, n = 1,2, \dots\}$ forms a renewal process. The number a is called the ratio of the geometric process.
If $0 < a < 1$, the GP is stochastically increasing; if $a > 1$, the GP is stochastically decreasing and if $a = 1$, the GP will reduce to a renewal process.

Definition 2.3: An integer valued random variable N is said to be a *stopping time* for the sequence of independent random variables $X_1, X_2, \dots$, if the event $\{N = n\}$ is independent of $X_{n-1}, X_{n-2}, \dots$, for all $n = 1,2, \dots$.

Theorem 2.4: Wald's equation. If $X_1, X_2, \dots$,independent and identically distributed random variables are having finite expectations and if N the stopping is time for $X_1, X_2, \dots$, such that $E[N] < \infty$, then

$$E\left[\sum_{n=1}^{N} X_n\right] \quad E(N)E(X_1)$$

Definition 2.5: The $N-policy$. It is a policy under which the system will be replaced upon the $N-th$ failure of the system, since the last replacement.

3. THE REPLACEMENT POLICY *N*

In this section, we study the maintenance problem of a repairable system and we use the *N* policy with various cost structures. Under the replacement policy N , the problem is to determine an optimal N^* such that the long run average cost per unit time is minimized. We make the following assumptions:

A1. At $t=0$, a new system is installed. Whenever the system fails, it is either repaired or replaced with a new identical one.

A2. Let $\{X_{ni}\ , i=1,2,\ldots\}$ are the intervals between the $(i-1)st$ and the ith shock after the $(n-1)st$ repair. $\{Y_{ni}, i=1,2,\ldots\}$ is the sequence of the amount of shock damage produced by the ith shock after the $(n-1)st$ repair. Let $E[X_{11}=\lambda], E[Y_{11}=\mu]$.

A3. Assume that $\{X_{ni}\ , i=1,2,\ldots\}$ and $\{Y_{ni}\ , i=1,2,\ldots\}$ are independent identically distributed sequences for all n . After the $(n-1)st$ repair the system will fail if the amount of a shock damage first exceeds a^{n-1} where $0\leq a\leq 1$.

A4. Let Z_n be the repair time after nth repair. $\{Z_{ni}\ , i=1,2,\ldots\}$ Constitutes a Geometric Process with $E[Z_1]=\delta$ and a ratio$b$, such that $0<b\leq 1$. $N_n(t)$ is the counting process of the number of s $\{X_{ni}\ , i=1,2,\ldots\}$ and $\{Y_{ni}\ , i=1,2,\ldots\}$ hoks after the $(n-1)st$ repair and $E[Z_n]=\left(\frac{1}{b^{n-1}}\right)\delta$.

A5. Let Z be the replacement time and $E[Z]=r$.

A6. The process $\{X_{ni}\ , i=1,2,\ldots\}$, $\{Y_{ni}\ , i=1,2,\ldots\}$ and Z are independent

A7. The operating reward rate is r_i for the ith reward and the repair cost rate is c_i for the ith repair and the replacement cost is c.
Let T_1 be the first replacement time for $n\geq 2$. Let T_n be the time between the $(n-1)st$ and the nth replacement. Then $\{T_n\ , n=1,2,\ldots\}$ forms a renewal process.

By the renewal reward theorem, the long-run average cost per unit time under the replacement policy *N* is given by time

$$C(N)=\frac{the\ expected\ cost\ incurred\ in\ a\ cycle}{the\ expected\ length\ of\ a\ cycle}$$

Let $L_n=\min\{l: Y_{nl}>a^{n-1}M\}$ then $W_n=\sum_{i=1}^{L_n}X_{ni}$.
L_n is the number of shocks until the first deadly shock occurred following the $(n-1)st$ failure. Then L_n has a geometric distribution $G(p_n)$, with $P\{L_n=k\}=p_nQ_n^{k-1}, k=1,2,\ldots$, where $P\{Y_{nl}>a^{n-1}M\}$ and $q_n=1-p_n$.
The $E[L_n]=\frac{1}{p_n}$. We have $\lambda_n=E[W_n]=E[L_n]E[X_{n1}]=\frac{1}{p_n}\lambda$.

Under the *N* policy the average cost rate is

$$\begin{aligned}C(N)&=\frac{E\left(\sum_{n=1}^{N-1}c_nZ_n\right)-E\left(\sum_{n=1}^{N}r_nW_n\right)+c}{E\left(\sum_{n=1}^{N-1}Z_n\right)+E\left(\sum_{n=1}^{N}W_n\right)+E(Z)}\\&=\frac{\sum_{n=1}^{N-1}c_nE(Z_n)-\sum_{n=1}^{N}r_nE(W_n)+c}{E\left(\sum_{n=1}^{N-1}Z_n\right)+E\left(\sum_{n=1}^{N}W_n\right)+E(Z)}\\&=\frac{\delta\sum_{n=1}^{N-1}\left(\frac{c_n}{b^{n-1}}\right)-\sum_{n=1}^{N}r_n\lambda_n+c}{\delta\sum_{n=1}^{N-1}\left(\frac{1}{b^{n-1}}\right)+\sum_{n=1}^{N}\lambda_n+r}\end{aligned}$$

We shall determine an optimal replacement policy for minimizing $C(N)$. Since $a\leq 1, \lambda_n$ is decreasing in n. The above equation becomes

$$C(N)=\frac{\delta\sum_{n=1}^{N-1}\left(\frac{c_n+r_n}{b^{n-1}}\right)+r\tau+c}{\delta\sum_{n=1}^{N-1}\left(\frac{1}{b^{n-1}}\right)+\sum_{n=1}^{N}\lambda_n+\tau}-r_n$$

To find the optimal policyN^* , we study the difference between $C(N+1)$ and $C(N)$.

Let $h(N)=\delta\sum_{n=1}^{N-1}\left(\frac{1}{b^{n-1}}\right)+\sum_{n=1}^{N}\lambda_n+\tau$ then

$$C(N+1)-C(N)=\frac{\delta\sum_{n=1}^{N-1}(c_n+r_n\lambda_n)-\lambda_{N+1}\sum_{n=1}^{N-1}b^{N-n}+\tau}{b^{N-1}h(N+1)h(N)}-\frac{(r\tau+c)(\lambda_{N+1}b^{N-1}+\delta)}{b^{N-1}h(N+1)h(N)}$$

To compare the difference between $\mathcal{C}(N+1)$ and $\mathcal{C}(N)$, we introduce the auxiliary function, $B(N)$ as follows

$$B(N) = \frac{\delta \sum_{n=1}^{N-1}(c_n + r_n\lambda_n) - \lambda_{N+1}\sum_{n=1}^{N-1} b^{N-n} + \tau}{(r\tau + c)(\lambda_{N+1}b^{N-1} + \delta)}$$

Since the denominator of $\mathcal{C}(N+1) - \mathcal{C}(N)$ is positive, the sign of $\mathcal{C}(N+1) - \mathcal{C}(N)$ is the same as the sign of its numerator. We have

$$\mathcal{C}(N+1) > \mathcal{C}(N) \Leftrightarrow B(N) > 1,$$
$$\mathcal{C}(N+1) = \mathcal{C}(N) \Leftrightarrow B(N) = 1,$$
$$\mathcal{C}(N+1) < \mathcal{C}(N) \Leftrightarrow B(N) < 1,$$

Note here that $\mathcal{B}(N+1) - \mathcal{B}(N) \geq 0$, that is $\mathcal{B}(N)$ is non-decreasing in N if and only if which on simplification yields

The optimal replacement policy N^* can be determined by

$$N^* = \min\{N|\mathcal{B}(N) \geq 1\}$$

If $B(N^*) > 1$, that the optimal policy N^* is unique. Because $B(N)$ is non decreasing in N , there exists an integer N^* such that

$$B(N) \geq \Leftrightarrow N \geq N^* \text{ and } B(N) < \Leftrightarrow N < N^*$$

CONCLUSION

In this paper, we studied the optimal replacement policy for a repairable and deteriorating system. Using a δ shock model with varying cost structures. We obtain the optimal replacement policy N^* by minimizing the average cost rate $\mathcal{C}(N)$.

REFERENCES

1. Barlow, R.E. and F. Proschan, (1965) *Mathematical Theory of Reliability*, John Wiley and Sons, New York.
2. Barlow,R.E and Proschan,F. *Imperfect repair*, J. Appl. Probab. 20 (1983) 851-859
3. Block,H.W, Borges,W.S and Savits,T.H.(1985)*Age-dependent minimal repair*, J. Appl. Probab. 22, 370-385.
4. Chen and Li (2008) *An extended extreme shock maintenance model for a deteriorating system Reliability Engg and system safety*, 1123-1129.
5. Feller, W. (1965) *An Introduction to Probability Theory and its Applications*, John Wiley and Sons, New York.
6. Lam,Y. (1988) *Geometric processes and replacement problem*, Acta Math. Appl. Sin. 4, 366-377.
7. Lam,Y and Zhang,Y.L. (2004) *A shock model for the maintenance problem of a repairable system*, Comput. Oper. Res. 31, 1807-1820.
8. Li,L. (1984) *Some probability distributions on Poisson shocks and its application in city trafic*, J. Lanzhou Univ. 20 (1984) 127-136.
9. Kijima,M.(1989) *Some results for repairable systems with general repair*, J. Appl. Probab. 26, 89-102.
10. Ross,S.M (1983) *Stochastic Processes*, 2nd ed., Wiley, New York.
11. Thangaraj, V. and Revathy Sundarajan (1997) *Multivariate Optimal Replacement Policies for a System subject to Shocks*, Optimization, 41,173 - 195.
12. Wang,G.J. and Zhang,Y.L(2009) *A geometric process repair model for a repairable cold standby system with priority in use and repair*, Reliab. Eng. Syst. Saf. 94, 1782-1787.
13. Zhang,Y.L and Wang,G.L.(2011) *An extended replacement policy for a deteriorating system with multi-failure modes*, Appl. Math. Comput. 218 (2011) 1820-1830.
14. Zong, Chai, and Zhang, (2013) *optimal replacement policy for a deteriorating system with increasing repair times*, Applied Mathematical Modelling, 37, 9768-9775.

Source of support: Nil, Conflict of interest: None Declared.