

STUDY ON STRONG INTUITIONISTIC FUZZY GRAPHS  
OF SECOND TYPE AND THEIR PROPERTIES

S. SHEIK DHAVUDH<sup>1</sup> AND R. SRINIVASAN<sup>2</sup>

<sup>1</sup>Ph.D. Research Scholar, Department of Mathematics,  
Islamiah College (Autonomous), Vaniyambadi - 635 752, Tamil Nadu, India.

<sup>2</sup>Department of Mathematics,  
Islamiah College (Autonomous), Vaniyambadi - 635 752, Tamil Nadu, India.  
E-mail: [srinivasanmaths@yahoo.com](mailto:srinivasanmaths@yahoo.com)

(Received On: 17-08-18; Revised & Accepted On: 04-09-18)

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ABSTRACT

*In this paper, we define some new operations like direct product, semi-strong product, strong product, cartesian product and composition of intuitionistic fuzzy graphs of second type and also define strong intuitionistic fuzzy graphs of second type. Further we establish some of their properties.*

**Keywords:** Intuitionistic fuzzy graphs, Intuitionistic fuzzy graphs of second type, direct, semi-strong, cartesian, composition, strong.

**2010 AMS Subject Classification:** 05C72, 03E72.

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1. INTRODUCTION

Fuzzy sets were introduced by Lotfi. A. Zadeh [10] in 1965 as a generalisation of classical (crisp) sets. Further the fuzzy sets are generalised by Krassimir.T. Atanassov [1] in which he has taken non-membership values also into consideration. He introduced Intuitionistic fuzzy sets [IFS] and their extensions like Intuitionistic fuzzy sets of second type [IFSST], Intuitionistic L-fuzzy sets [ILFS] and Temporal Intuitionistic fuzzy sets [TIFS]. A. Shannon and K. T. Atanassov [6] discussed the theory of Intuitionistic fuzzy graphs. R. Parvathi and M. G. Karunambigai [3,4] introduced Intuitionistic Fuzzy Graphs [IFG] elaborately and analyzed its components also defined strong intuitionistic fuzzy graphs. After that they introduced and studied the operations cartesian product, composition on IFG. S. Ismail Mohideen, A. NagoorGani, B. Fathima Kani and C.Yasmin [2] discussed the properties of operations on regular IFG. Further Sankar Sahoo and Madhumangal Pal [5] defined and studied various operations like direct product, semi-strong product and strong product on IFG. The present authors [7,8,9] introduced the extension of IFG namely Intuitionistic Fuzzy Graphs of Second Type [IFGST] and defined some basic operations like union and join on IFGST. In section 2, we give some basic definitions and in section 3, we define some new operations direct product, semi-strong product, strong product, cartesian product and composition of IFGST. In section 4, we define the concept of strong intuitionistic fuzzy graphs of second type and establish some of their properties. The paper is concluded in section 5.

2. PRELIMINARIES

In this section, we give some basic definitions.

**Definition 2.1:** [3] An Intuitionistic Fuzzy Graph [IFG] is of the form  $G = [V, E]$  where

- (i)  $V = \{v_1, v_2, \dots, v_n\}$  such that  $\mu_1: V \rightarrow [0,1]$  and  $\nu_1: V \rightarrow [0,1]$  denote the degree of membership and nonmembership of the element  $v_i \in V$ , respectively, and  $0 \leq \mu_1(v_i) + \nu_1(v_i) \leq 1$  for every  $v_i \in V$ , ( $i = 1, 2, \dots, n$ ),
- (ii)  $E \subseteq V \times V$  where  $\mu_2: V \times V \rightarrow [0,1]$  and  $\nu_2: V \times V \rightarrow [0,1]$  are such that  
 $\mu_2(v_i, v_j) \leq \min[\mu_1(v_i), \mu_1(v_j)]$ ,  
 $\nu_2(v_i, v_j) \leq \max[\nu_1(v_i), \nu_1(v_j)]$   
and  $0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1$   
for every  $(v_i, v_j) \in E$ , ( $i, j = 1, 2, \dots, n$ ).

**Definition 2.2:** [3] An IFG,  $G = [V, E]$  is said to be a strong IFG if  $\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$  and  $\nu_{2ij} = \max(\nu_{1i}, \nu_{1j})$  for every  $(v_i, v_j) \in E$ .

**Definition 2.3:** [7] An Intuitionistic Fuzzy Graphs of Second Type [IFGST] is of the form  $G = [V, E]$  where

- (i)  $V = \{v_1, v_2, \dots, v_n\}$  such that  $\mu_1: V \rightarrow [0,1]$  and  $\nu_1: V \rightarrow [0,1]$  denote the degree of membership and nonmembership of the element  $v_i \in V$ , respectively, and  $0 \leq \mu_1(v_i)^2 + \nu_1(v_i)^2 \leq 1$  for every  $v_i \in V$ , ( $i = 1, 2, \dots, n$ ),
- (ii)  $E \subseteq V \times V$  where  $\mu_2: V \times V \rightarrow [0,1]$  and  $\nu_2: V \times V \rightarrow [0,1]$  are such that
  - $\mu_2(v_i, v_j) \leq \min[\mu_1(v_i)^2, \mu_1(v_j)^2]$ ,
  - $\nu_2(v_i, v_j) \leq \max[\nu_1(v_i)^2, \nu_1(v_j)^2]$
  - and  $0 \leq \mu_2(v_i, v_j)^2 + \nu_2(v_i, v_j)^2 \leq 1$
  - for every  $(v_i, v_j) \in E$ , ( $i, j = 1, 2, \dots, n$ ).

### 3. OPERATIONS ON INTUITIONISTIC FUZZY GRAPHS OF SECOND TYPE

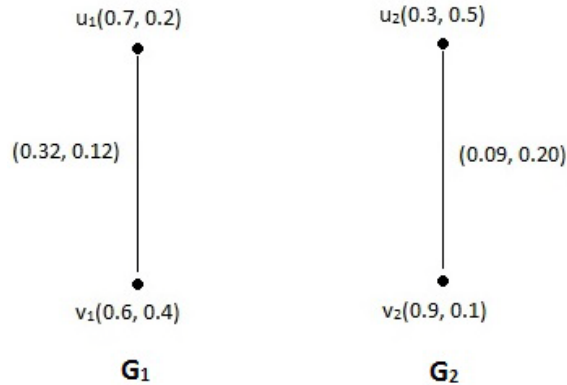
In this section, we define some new operations like direct product, semi-strong product, strong product, cartesian product and composition of IFGST with suitable examples.

**Definition 3.1:** Let  $G_1 = [V_1, E_1]$  and  $G_2 = [V_2, E_2]$  be two IFGST then the direct product  $G = G_1 \square G_2$  of  $G_1$  and  $G_2$  is also an IFGST defined by,

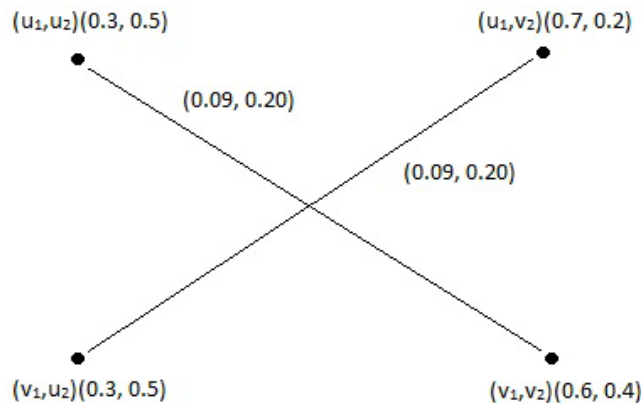
- (i)  $(\mu_1 \square \mu'_1)(u_1, u_2) = \min(\mu_1(u_1), \mu'_1(u_2))$  for every  $(u_1, u_2 \in V)$   
 $(\nu_1 \square \nu'_1)(u_1, u_2) = \max(\nu_1(u_1), \nu'_1(u_2))$  for every  $(u_1, u_2 \in V)$
- (ii)  $(\mu_2 \square \mu'_2)(u_1, u_2)(v_1, v_2) = \min(\mu_2(u_1v_1), \mu'_2(u_2v_2))$  for every  $u_1v_1 \in E_1, u_2v_2 \in E_2$   
 $(\nu_2 \square \nu'_2)(u_1, u_2)(v_1, v_2) = \max(\nu_2(u_1v_1), \nu'_2(u_2v_2))$  for every  $u_1v_1 \in E_1, u_2v_2 \in E_2$

where  $V = V_1 \times V_2$  and  $E = \{(u_1, u_2)(v_1, v_2) : u_1v_1 \in E_1, u_2v_2 \in E_2\}$ . Also  $(\mu_1, \nu_1)$ ,  $(\mu'_1, \nu'_1)$  are the vertex degree of membership and non-membership of the elements of  $V_1$  in  $G_1$  and  $V_2$  in  $G_2$  respectively and  $(\mu_2, \nu_2)$  and  $(\mu'_2, \nu'_2)$  are the edge degree of membership and non-membership of the elements of  $E_1$  in  $G_1$  and  $E_2$  in  $G_2$  respectively.

**Example 3.1:**



**Figure-1:**  $G_1$  and  $G_2$

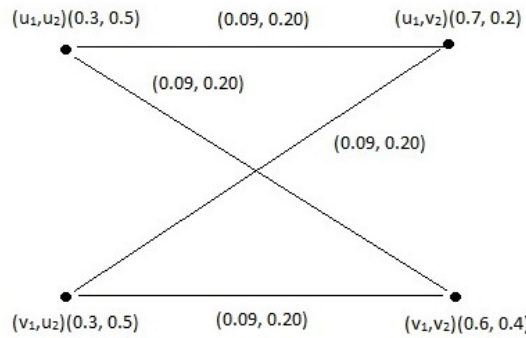


**Figure-2:** direct Product of  $G_1$  and  $G_2$  ( $G_1 \square G_2$ )

**Definition 3.2:** Let  $G_1 = [V_1, E_1]$  and  $G_2 = [V_2, E_2]$  be two IFGST then the semi-strong product  $G = G_1 \bullet G_2$  of  $G_1$  and  $G_2$  is also an IFGST defined by,

- (i)  $(\mu_1 \bullet \mu'_1)(u_1, u_2) = \min(\mu_1(u_1), \mu'_1(u_2))$  for every  $(u_1, u_2 \in V)$   
 $(\nu_1 \bullet \nu'_1)(u_1, u_2) = \max(\nu_1(u_1), \nu'_1(u_2))$  for every  $(u_1, u_2 \in V)$
  - (ii)  $(\mu_2 \bullet \mu'_2)(u, u_2)(u, v_2) = \min(\mu_1^2(u), \mu'_2(u_2v_2))$  for every  $u \in V_1, u_2v_2 \in E_2$   
 $(\nu_2 \bullet \nu'_2)(u, u_2)(u, v_2) = \max(\nu_1^2(u), \nu'_2(u_2v_2))$  for every  $u \in V_1, u_2v_2 \in E_2$
  - (iii)  $(\mu_2 \bullet \mu'_2)(u_1, u_2)(v_1, v_2) = \min(\mu_2(u_1v_1), \mu'_2(u_2v_2))$  for every  $u_1v_1 \in E_1, u_2v_2 \in E_2$   
 $(\nu_2 \bullet \nu'_2)(u_1, u_2)(v_1, v_2) = \max(\nu_2(u_1v_1), \nu'_2(u_2v_2))$  for every  $u_1v_1 \in E_1, u_2v_2 \in E_2$
- where  $V = V_1 \times V_2$  and  $E = \{(u, u_2)(u, v_2): u \in V_1, u_2v_2 \in E_2\} \cup \{(u_1, u_2)(v_1, v_2): u_1v_1 \in E_1, u_2v_2 \in E_2\}$ . Also  $(\mu_1, \nu_1), (\mu'_1, \nu'_1)$  are the vertex degree of membership and non-membership of the elements of  $V_1$  in  $G_1$  and  $V_2$  in  $G_2$  respectively and  $(\mu_2, \nu_2)$  and  $(\mu'_2, \nu'_2)$  are the edge degree of membership and non-membership of the elements of  $E_1$  in  $G_1$  and  $E_2$  in  $G_2$  respectively.

**Example 3.2:**

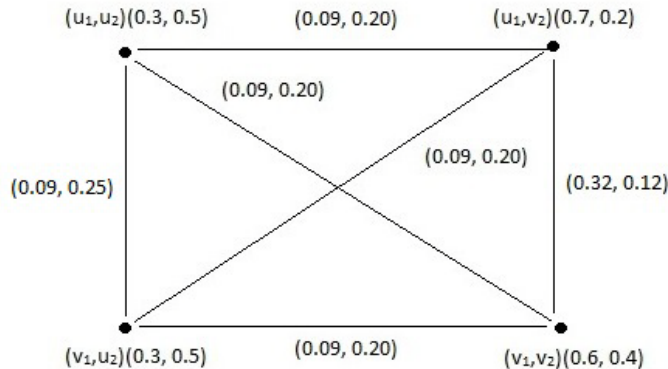


**Figure-3:** semi-strong product of  $G_1$  and  $G_2$  ( $G_1 \bullet G_2$ )

**Definition 3.3:** Let  $G_1 = [V_1, E_1]$  and  $G_2 = [V_2, E_2]$  be two IFGST then the strong product  $G = G_1 \otimes G_2$  of  $G_1$  and  $G_2$  is also an IFGST defined by,

- (i)  $(\mu_1 \otimes \mu'_1)(u_1, u_2) = \min(\mu_1(u_1), \mu'_1(u_2))$  for every  $(u_1, u_2 \in V)$   
 $(\nu_1 \otimes \nu'_1)(u_1, u_2) = \max(\nu_1(u_1), \nu'_1(u_2))$  for every  $(u_1, u_2 \in V)$
  - (ii)  $(\mu_2 \otimes \mu'_2)(u, u_2)(u, v_2) = \min(\mu_1^2(u), \mu'_2(u_2v_2))$  for every  $u \in V_1, u_2v_2 \in E_2$   
 $(\nu_2 \otimes \nu'_2)(u, u_2)(u, v_2) = \max(\nu_1^2(u), \nu'_2(u_2v_2))$  for every  $u \in V_1, u_2v_2 \in E_2$
  - (iii)  $(\mu_2 \otimes \mu'_2)(u_1, w)(v_1, w) = \min(\mu_2(u_1v_1), (\mu'_1)^2(w))$  for every  $w \in V_2, u_1v_1 \in E_1$   
 $(\nu_2 \otimes \nu'_2)(u_1, w)(v_1, w) = \max(\nu_2(u_1v_1), (\nu'_1)^2(w))$  for every  $w \in V_2, u_1v_1 \in E_1$
  - (iv)  $(\mu_2 \otimes \mu'_2)(u_1, u_2)(v_1, v_2) = \min(\mu_2(u_1v_1), \mu'_2(u_2v_2))$  for every  $u_1v_1 \in E_1, u_2v_2 \in E_2$   
 $(\nu_2 \otimes \nu'_2)(u_1, u_2)(v_1, v_2) = \max(\nu_2(u_1v_1), \nu'_2(u_2v_2))$  for every  $u_1v_1 \in E_1, u_2v_2 \in E_2$
- where  $V = V_1 \times V_2$  and  $E = \{(u, u_2)(u, v_2): u \in V_1, u_2v_2 \in E_2\} \cup \{(u_1, w)(v_1, w): w \in V_2, u_1v_1 \in E_1\} \cup \{(u_1, u_2)(v_1, v_2): u_1v_1 \in E_1, u_2v_2 \in E_2\}$ . Also  $(\mu_1, \nu_1), (\mu'_1, \nu'_1)$  are the vertex degree of membership and non-membership of the elements of  $V_1$  in  $G_1$  and  $V_2$  in  $G_2$  respectively and  $(\mu_2, \nu_2)$  and  $(\mu'_2, \nu'_2)$  are the edge degree of membership and non-membership of the elements of  $E_1$  in  $G_1$  and  $E_2$  in  $G_2$  respectively.

**Example 3.3:**



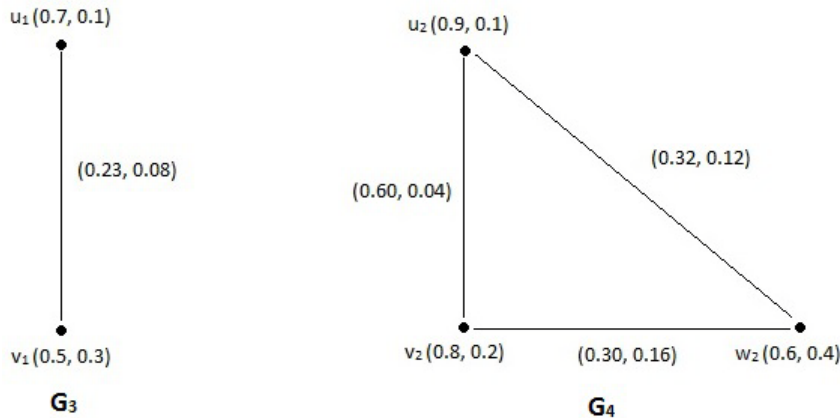
**Figure-4:** strong product of  $G_1$  and  $G_2$  ( $G_1 \otimes G_2$ )

**Definition 3.4:** Let  $G_1 = [V_1, E_1]$  and  $G_2 = [V_2, E_2]$  be two IFGST then the cartesian product  $G = G_1 \times G_2$  of  $G_1$  and  $G_2$  is also an IFGST defined by,

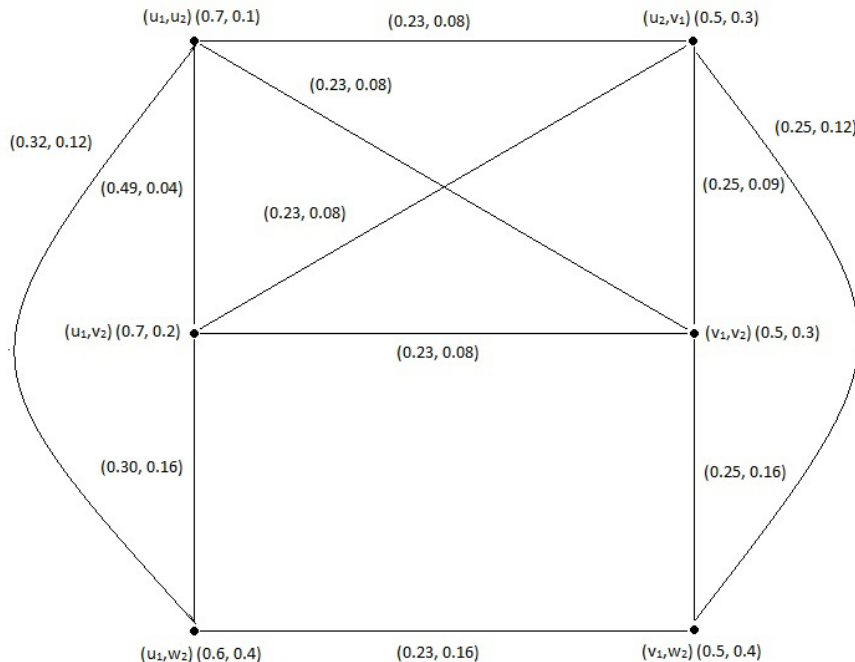
- (i)  $(\mu_1 \times \mu'_1)(u_1, u_2) = \min(\mu_1(u_1), \mu'_1(u_2))$  for every  $(u_1, u_2 \in V)$   
 $(\nu_1 \times \nu'_1)(u_1, u_2) = \max(\nu_1(u_1), \nu'_1(u_2))$  for every  $(u_1, u_2 \in V)$
- (ii)  $(\mu_2 \times \mu'_2)(u, u_2)(u, v_2) = \min(\mu_1^2(u), \mu'_2(u_2 v_2))$  for every  $u \in V_1, u_2 v_2 \in E_2$   
 $(\nu_2 \times \nu'_2)(u, u_2)(u, v_2) = \max(\nu_1^2(u), \nu'_2(u_2 v_2))$  for every  $u \in V_1, u_2 v_2 \in E_2$
- (iii)  $(\mu_2 \times \mu'_2)(u_1, w)(v_1, w) = \min(\mu_2(u_1 v_1), (\mu'_1)^2(w))$  for every  $u_1 v_1 \in E_1, w \in V_2$   
 $(\nu_2 \times \nu'_2)(u_1, w)(v_1, w) = \max(\nu_2(u_1 v_1), (\nu'_1)^2(w))$  for every  $u_1 v_1 \in E_1, w \in V_2$

where  $V = V_1 \times V_2$  and  $E = \{(u, u_2)(u, v_2): u \in V_1, u_2 v_2 \in E_2\} \cup \{(u_1, w)(v_1, w): u_1 v_1 \in E_1, w \in V_2\}$ . Also  $(\mu_1, \nu_1), (\mu'_1, \nu'_1)$  are the vertex degree of membership and non-membership of the elements of  $V_1$  in  $G_1$  and  $V_2$  in  $G_2$  respectively and  $(\mu_2, \nu_2)$  and  $(\mu'_2, \nu'_2)$  are the edge degree of membership and non-membership of the elements of  $E_1$  in  $G_1$  and  $E_2$  in  $G_2$  respectively.

**Example 3.4:**



**Figure-5:**  $G_3$  and  $G_4$



**Figure-6:** cartesian product of  $G_3$  and  $G_4$  ( $G_3 \times G_4$ )

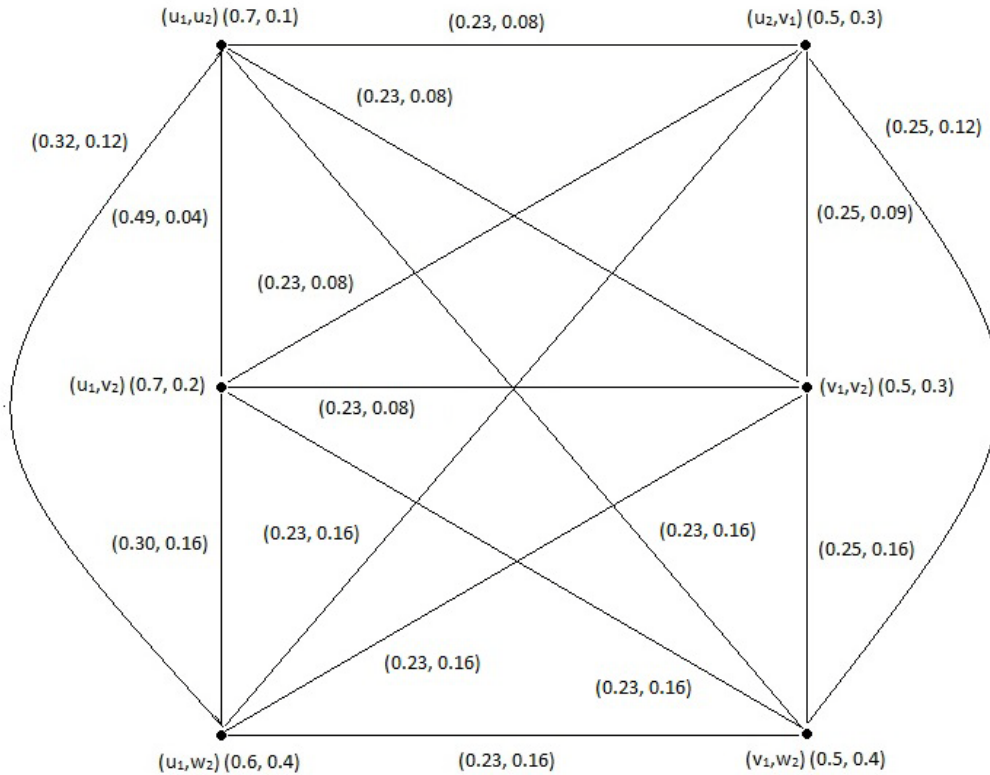
**Definition 3.5:** Let  $G_1 = [V_1, E_1]$  and  $G_2 = [V_2, E_2]$  be two IFGST then the composition  $G = G_1 \circ G_2$  of  $G_1$  and  $G_2$  is also an IFGST defined by,

- (i)  $(\mu_1 \circ \mu'_1)(u_1, u_2) = \min(\mu_1(u_1), \mu'_1(u_2))$  for every  $(u_1, u_2 \in V)$   
 $(\nu_1 \circ \nu'_1)(u_1, u_2) = \max(\nu_1(u_1), \nu'_1(u_2))$  for every  $(u_1, u_2 \in V)$
- (ii)  $(\mu_2 \circ \mu'_2)(u, u_2)(u, v_2) = \min(\mu_1^2(u), \mu'_2(u_2 v_2))$  for every  $u \in V_1, u_2 v_2 \in E_2$   
 $(\nu_2 \circ \nu'_2)(u, u_2)(u, v_2) = \max(\nu_1^2(u), \nu'_2(u_2 v_2))$  for every  $u \in V_1, u_2 v_2 \in E_2$
- (iii)  $(\mu_2 \circ \mu'_2)(u_1, w)(v_1, w) = \min(\mu_2(u_1 v_1), (\mu'_1)^2(w))$  for every  $u_1 v_1 \in E_1, w \in V_2$   
 $(\nu_2 \circ \nu'_2)(u_1, w)(v_1, w) = \max(\nu_2(u_1 v_1), (\nu'_1)^2(w))$  for every  $u_1 v_1 \in E_1, w \in V_2$

(iv)  $(\mu_2 \circ \mu'_2)(u_1, u_2)(v_1, v_2) = \min(\mu_2(u_1 v_1), (\mu'_1)^2(u_2), (\mu'_1)^2(v_2))$  for every  $u_1 v_1 \in E_1, u_2, v_2 \in V_2, u_2 \neq v_2$

$(v_2 \circ v'_2)(u_1, u_2)(v_1, v_2) = \max(v_2(u_1 v_1), (v'_1)^2(u_2), (v'_1)^2(v_2))$  for every  $u_1 v_1 \in E_1, u_2, v_2 \in V_2, u_2 \neq v_2$  where  $V = V_1 \times V_2$  and  $E = \{(u, u_2)(u, v_2): u \in V_1, u_2 v_2 \in E_2\} \cup \{(u_1, w)(v_1, w): u_1 v_1 \in E_1, w \in V_2\} \cup \{(u_1, u_2)(v_1, v_2): u_1 v_1 \in E_1, u_2, v_2 \in V_2, u_2 \neq v_2\}$ . Also  $(\mu_1, \nu_1), (\mu'_1, \nu'_1)$  are the vertex degree of membership and non-membership of the elements of  $V_1$  in  $G_1$  and  $V_2$  in  $G_2$  respectively and  $(\mu_2, \nu_2)$  and  $(\mu'_2, \nu'_2)$  are the edge degree of membership and non-membership of the elements of  $E_1$  in  $G_1$  and  $E_2$  in  $G_2$  respectively.

**Example 3.5:**



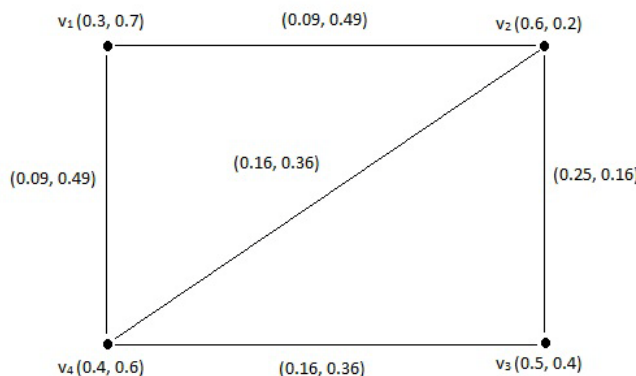
**Figure-7:** composition of  $G_3$  and  $G_4$  ( $G_3 \circ G_4$ )

**4. PROPERTIES OF STRONG INTUITIONISTIC FUZZY GRAPHS OF SECOND TYPE**

In this section, we define strong intuitionistic fuzzy graphs of second type and establish some of their properties based on newly defined operations.

**Definition 4.1:** An IFGST,  $G = [V, E]$  is said to be a strong IFGST if  $\mu_{2ij} = \min(\mu_{1i}^2, \mu_{1j}^2)$  and  $\nu_{2ij} = \max(\nu_{1i}^2, \nu_{1j}^2)$  for every  $(v_i, v_j) \in E$

**Example 4.1:**



**Figure-8:** strong IFGST

**Theorem 4.1:** Let  $G_1 = [V_1, E_1]$  and  $G_2 = [V_2, E_2]$  be any two IFGSTs. If  $G_1$  and  $G_2$  are strong IFGST, then  $G_1 \sqcap G_2$  is also strong IFGST.

**Proof:** Let  $G_1$  and  $G_2$  are strong IFGST then for all  $(u_1, v_1) \in E_1$  and  $(u_2, v_2) \in E_2$  we have,

$$\begin{aligned} \mu_2(u_1, v_1) &= \min(\mu_1^2(u_1), \mu_1^2(v_1)) \\ \nu_2(u_1, v_1) &= \max(\nu_1^2(u_1), \nu_1^2(v_1)) \\ \mu'_2(u_2, v_2) &= \min((\mu'_1)^2(u_2), (\mu'_1)^2(v_2)) \\ \nu'_2(u_2, v_2) &= \max((\nu'_1)^2(u_2), (\nu'_1)^2(v_2)) \end{aligned}$$

Now,

$$\begin{aligned} (\mu_2 \sqcap \mu'_2)(u_1, u_2)(v_1, v_2) &= \min(\mu_2(u_1 v_1), \mu'_2(u_2 v_2)) \\ &= \min(\min(\mu_1^2(u_1), \mu_1^2(v_1)), \min((\mu'_1)^2(u_2), (\mu'_1)^2(v_2))) \\ &= \min(\min(\mu_1^2(u_1), (\mu'_1)^2(u_2)), \min(\mu_1^2(v_1), (\mu'_1)^2(v_2))) \\ &= \min((\mu_1 \sqcap \mu'_1)^2(u_1, u_2), (\mu_1 \sqcap \mu'_1)^2(v_1, v_2)) \end{aligned} \tag{4.1}$$

Also,

$$\begin{aligned} (\nu_2 \sqcap \nu'_2)(u_1, u_2)(v_1, v_2) &= \max(\nu_2(u_1 v_1), \nu'_2(u_2 v_2)) \\ &= \max(\max(\nu_1^2(u_1), \nu_1^2(v_1)), \max((\nu'_1)^2(u_2), (\nu'_1)^2(v_2))) \\ &= \max(\max(\nu_1^2(u_1), (\nu'_1)^2(u_2)), \max(\nu_1^2(v_1), (\nu'_1)^2(v_2))) \\ &= \max((\nu_1 \sqcap \nu'_1)^2(u_1, u_2), (\nu_1 \sqcap \nu'_1)^2(v_1, v_2)) \end{aligned} \tag{4.2}$$

From equation (4.1) and (4.2)  $G_1 \sqcap G_2$  is also strong IFGST.

This completes the proof.

**Theorem 4.2:** Let  $G_1 = [V_1, E_1]$  and  $G_2 = [V_2, E_2]$  be any two IFGSTs. If  $G_1$  and  $G_2$  are strong IFGST, then  $G_1 \bullet G_2$  is also strong IFGST.

**Proof:** Let  $G_1$  and  $G_2$  are strong IFGST then for all  $(u_1, v_1) \in E_1$  and  $(u_2, v_2) \in E_2$  we have,

$$\begin{aligned} \mu_2(u_1, v_1) &= \min(\mu_1^2(u_1), \mu_1^2(v_1)) \\ \nu_2(u_1, v_1) &= \max(\nu_1^2(u_1), \nu_1^2(v_1)) \\ \mu'_2(u_2, v_2) &= \min((\mu'_1)^2(u_2), (\mu'_1)^2(v_2)) \\ \nu'_2(u_2, v_2) &= \max((\nu'_1)^2(u_2), (\nu'_1)^2(v_2)) \end{aligned}$$

If  $u \in V_1$  and  $(u_2, v_2) \in E_2$  then,

$$\begin{aligned} (\mu_2 \bullet \mu'_2)(u, u_2)(u, v_2) &= \min(\mu_1^2(u), \mu'_2(u_2 v_2)) \\ &= \min(\mu_1^2(u), \min((\mu'_1)^2(u_2), (\mu'_1)^2(v_2))) \\ &= \min(\min(\mu_1^2(u), (\mu'_1)^2(u_2)), \min(\mu_1^2(u), (\mu'_1)^2(v_2))) \\ &= \min((\mu_1 \bullet \mu'_1)^2(u, u_2), (\mu_1 \bullet \mu'_1)^2(u, v_2)) \end{aligned} \tag{4.3}$$

Similarly we have

$$(\nu_2 \bullet \nu'_2)(u, u_2)(u, v_2) = \max((\nu_1 \bullet \nu'_1)^2(u, u_2), (\nu_1 \bullet \nu'_1)^2(u, v_2)) \tag{4.4}$$

Again if  $(u_1, v_1) \in E_1$  and  $(u_2, v_2) \in E_2$  then,

$$\begin{aligned} (\mu_2 \bullet \mu'_2)(u_1, u_2)(v_1, v_2) &= \min(\mu_2(u_1 v_1), \mu'_2(u_2 v_2)) \\ &= \min(\min(\mu_1^2(u_1), \mu_1^2(v_1)), \min((\mu'_1)^2(u_2), (\mu'_1)^2(v_2))) \\ &= \min(\min(\mu_1^2(u_1), (\mu'_1)^2(u_2)), \min(\mu_1^2(v_1), (\mu'_1)^2(v_2))) \\ &= \min((\mu_1 \bullet \mu'_1)^2(u_1, u_2), (\mu_1 \bullet \mu'_1)^2(v_1, v_2)) \end{aligned} \tag{4.5}$$

Similarly we have

$$(\nu_2 \bullet \nu'_2)(u_1, u_2)(v_1, v_2) = \max((\nu_1 \bullet \nu'_1)^2(u_1, u_2), (\nu_1 \bullet \nu'_1)^2(v_1, v_2)) \tag{4.6}$$

From the equations (4.3),(4.4),(4.5) and (4.6),  $G_1 \bullet G_2$  is also strong IFGST.

This completes the proof.

**Theorem 4.3:** Let  $G_1 = [V_1, E_1]$  and  $G_2 = [V_2, E_2]$  be any two IFGSTs. If  $G_1$  and  $G_2$  are strong IFGST, then  $G_1 \otimes G_2$  is also strong IFGST.

**Proof:** Let  $G_1$  and  $G_2$  are strong IFGST then for all  $(u_1, v_1) \in E_1$  and  $(u_2, v_2) \in E_2$  we have,

$$\begin{aligned} \mu_2(u_1, v_1) &= \min(\mu_1^2(u_1), \mu_1^2(v_1)) \\ \nu_2(u_1, v_1) &= \max(\nu_1^2(u_1), \nu_1^2(v_1)) \\ \mu'_2(u_2, v_2) &= \min((\mu'_1)^2(u_2), (\mu'_1)^2(v_2)) \\ \nu'_2(u_2, v_2) &= \max((\nu'_1)^2(u_2), (\nu'_1)^2(v_2)) \end{aligned}$$

If  $u \in V_1$  and  $(u_2, v_2) \in E_2$  then,

$$\begin{aligned} (\mu_2 \otimes \mu'_2)(u, u_2)(u, v_2) &= \min(\mu_1^2(u), \mu'_2(u_2 v_2)) \\ &= \min(\mu_1^2(u), \min((\mu'_1)^2(u_2), (\mu'_1)^2(v_2))) \\ &= \min(\min(\mu_1^2(u), (\mu'_1)^2(u_2)), \min(\mu_1^2(u), (\mu'_1)^2(v_2))) \\ &= \min((\mu_1 \otimes \mu'_1)^2(u, u_2), (\mu_1 \otimes \mu'_1)^2(u, v_2)) \end{aligned} \quad (4.7)$$

Similarly we have

$$(\nu_2 \otimes \nu'_2)(u, u_2)(u, v_2) = \max((\nu_1 \otimes \nu'_1)^2(u, u_2), (\nu_1 \otimes \nu'_1)^2(u, v_2)) \quad (4.8)$$

If  $(u_1, v_1) \in E_1$  and  $w \in V_2$  then,

$$\begin{aligned} (\mu_2 \otimes \mu'_2)(u_1, w)(v_1, w) &= \min(\mu_2(u_1 v_1), (\mu'_1)^2(w)) \\ &= \min(\min(\mu_1^2(u_1), \mu_1^2(v_1)), (\mu'_1)^2(w)) \\ &= \min(\min(\mu_1^2(u_1), (\mu'_1)^2(w)), \min(\mu_1^2(v_1), (\mu'_1)^2(w))) \\ &= \min((\mu_1 \otimes \mu'_1)^2(u_1, w), (\mu_1 \otimes \mu'_1)^2(v_1, w)) \end{aligned} \quad (4.9)$$

Similarly we have

$$(\nu_2 \otimes \nu'_2)(u_1, w)(v_1, w) = \max((\nu_1 \otimes \nu'_1)^2(u_1, w), (\nu_1 \otimes \nu'_1)^2(v_1, w)) \quad (4.10)$$

Again if  $(u_1, v_1) \in E_1$  and  $(u_2, v_2) \in E_2$  then,

$$\begin{aligned} (\mu_2 \otimes \mu'_2)(u_1, u_2)(v_1, v_2) &= \min(\mu_2(u_1 v_1), \mu'_2(u_2 v_2)) \\ &= \min(\min(\mu_1^2(u_1), \mu_1^2(v_1)), \min((\mu'_1)^2(u_2), (\mu'_1)^2(v_2))) \\ &= \min(\min(\mu_1^2(u_1), (\mu'_1)^2(u_2)), \min(\mu_1^2(v_1), (\mu'_1)^2(v_2))) \\ &= \min((\mu_1 \otimes \mu'_1)^2(u_1, u_2), (\mu_1 \otimes \mu'_1)^2(v_1, v_2)) \end{aligned} \quad (4.11)$$

Similarly we have

$$(\nu_2 \otimes \nu'_2)(u_1, u_2)(v_1, v_2) = \max((\nu_1 \otimes \nu'_1)^2(u_1, u_2), (\nu_1 \otimes \nu'_1)^2(v_1, v_2)) \quad (4.12)$$

From the equations (4.7), (4.8), (4.9), (4.10), (4.11) and (4.12) it follows that  $G_1 \otimes G_2$  is also strong IFGST.

This completes the proof.

**Theorem 4.4:** Let  $G_1 = [V_1, E_1]$  and  $G_2 = [V_2, E_2]$  be any two IFGSTs. If  $G_1$  and  $G_2$  are strong IFGST, then  $G_1 \times G_2$  is also strong IFGST.

**Proof:** Let  $G_1$  and  $G_2$  are strong IFGST then for all  $(u_1, v_1) \in E_1$  and  $(u_2, v_2) \in E_2$  we have,

$$\begin{aligned} \mu_2(u_1, v_1) &= \min(\mu_1^2(u_1), \mu_1^2(v_1)) \\ \nu_2(u_1, v_1) &= \max(\nu_1^2(u_1), \nu_1^2(v_1)) \\ \mu'_2(u_2, v_2) &= \min((\mu'_1)^2(u_2), (\mu'_1)^2(v_2)) \\ \nu'_2(u_2, v_2) &= \max((\nu'_1)^2(u_2), (\nu'_1)^2(v_2)) \end{aligned}$$

If  $u \in V_1$  and  $(u_2, v_2) \in E_2$  then,

$$\begin{aligned} (\mu_2 \times \mu'_2)(u, u_2)(u, v_2) &= \min(\mu_1^2(u), \mu'_2(u_2 v_2)) \\ &= \min(\mu_1^2(u), \min((\mu'_1)^2(u_2), (\mu'_1)^2(v_2))) \\ &= \min(\min(\mu_1^2(u), (\mu'_1)^2(u_2)), \min(\mu_1^2(u), (\mu'_1)^2(v_2))) \\ &= \min((\mu_1 \times \mu'_1)^2(u, u_2), (\mu_1 \times \mu'_1)^2(u, v_2)) \end{aligned} \quad (4.13)$$

Similarly we have

$$(\nu_2 \times \nu'_2)(u, u_2)(u, v_2) = \max((\nu_1 \times \nu'_1)^2(u, u_2), (\nu_1 \times \nu'_1)^2(u, v_2)) \quad (4.14)$$

Again if  $(u_1, v_1) \in E_1$  and  $w \in V_2$  then,

$$\begin{aligned} (\mu_2 \times \mu'_2)(u_1, w)(v_1, w) &= \min(\mu_2(u_1 v_1), (\mu'_1)^2(w)) \\ &= \min(\min(\mu_1^2(u_1), \mu_1^2(v_1)), (\mu'_1)^2(w)) \\ &= \min(\min(\mu_1^2(u_1), (\mu'_1)^2(w)), \min(\mu_1^2(v_1), (\mu'_1)^2(w))) \\ &= \min((\mu_1 \times \mu'_1)^2(u_1, w), (\mu_1 \times \mu'_1)^2(v_1, w)) \end{aligned} \quad (4.15)$$

Similarly we have

$$(\nu_2 \times \nu'_2)(u_1, w)(v_1, w) = \max((\nu_1 \times \nu'_1)^2(u_1, w), (\nu_1 \times \nu'_1)^2(v_1, w)) \quad (4.16)$$

From the equations (4.13),(4.14),(4.15) and (4.16),  $G_1 \times G_2$  is also strong IFGST.

This completes the proof.

**Theorem 4.5:** Let  $G_1 = [V_1, E_1]$  and  $G_2 = [V_2, E_2]$  be any two IFGSTs. If  $G_1$  and  $G_2$  are strong IFGST, then  $G_1 \circ G_2$  is also strong IFGST.

**Proof:** Let  $G_1$  and  $G_2$  are strong IFGST then for all  $(u_1, v_1) \in E_1$  and  $(u_2, v_2) \in E_2$  we have,

$$\begin{aligned} \mu_2(u_1, v_1) &= \min(\mu_1^2(u_1), \mu_1^2(v_1)) \\ \nu_2(u_1, v_1) &= \max(\nu_1^2(u_1), \nu_1^2(v_1)) \\ \mu'_2(u_2, v_2) &= \min((\mu'_1)^2(u_2), (\mu'_1)^2(v_2)) \\ \nu'_2(u_2, v_2) &= \max((\nu'_1)^2(u_2), (\nu'_1)^2(v_2)) \end{aligned}$$

If  $u \in V_1$  and  $(u_2, v_2) \in E_2$  then,

$$\begin{aligned} (\mu_2 \circ \mu'_2)(u, u_2)(u, v_2) &= \min(\mu_1^2(u), \mu'_2(u_2 v_2)) \\ &= \min(\mu_1^2(u), \min((\mu'_1)^2(u_2), (\mu'_1)^2(v_2))) \\ &= \min(\min(\mu_1^2(u), (\mu'_1)^2(u_2)), \min(\mu_1^2(u), (\mu'_1)^2(v_2))) \\ &= \min((\mu_1 \circ \mu'_1)^2(u, u_2), (\mu_1 \circ \mu'_1)^2(u, v_2)) \end{aligned} \tag{4.17}$$

Similarly we have

$$(\nu_2 \circ \nu'_2)(u, u_2)(u, v_2) = \max((\nu_1 \circ \nu'_1)^2(u, u_2), (\nu_1 \circ \nu'_1)^2(u, v_2)) \tag{4.18}$$

If  $(u_1, v_1) \in E_1$  and  $w \in V_2$  then,

$$\begin{aligned} (\mu_2 \circ \mu'_2)(u_1, w)(v_1, w) &= \min(\mu_2(u_1 v_1), (\mu'_1)^2(w)) \\ &= \min(\min(\mu_1^2(u_1), \mu_1^2(v_1)), (\mu'_1)^2(w)) \\ &= \min(\min(\mu_1^2(u_1), (\mu'_1)^2(w)), \min(\mu_1^2(v_1), (\mu'_1)^2(w))) \\ &= \min((\mu_1 \circ \mu'_1)^2(u_1, w), (\mu_1 \circ \mu'_1)^2(v_1, w)) \end{aligned} \tag{4.19}$$

Similarly we have

$$(\nu_2 \circ \nu'_2)(u_1, w)(v_1, w) = \max((\nu_1 \circ \nu'_1)^2(u_1, w), (\nu_1 \circ \nu'_1)^2(v_1, w)) \tag{4.20}$$

Again if  $(u_1, v_1) \in E_1$ ,  $(u_2, v_2) \in E_2$  and  $u_2 \neq v_2$  then,

$$\begin{aligned} (\mu_2 \circ \mu'_2)(u_1, u_2)(v_1, v_2) &= \min(\mu_2(u_1 v_1), (\mu'_1)^2(u_2), (\mu'_1)^2(v_2)) \\ &= \min(\min(\mu_1^2(u_1), \mu_1^2(v_1)), (\mu'_1)^2(u_2), (\mu'_1)^2(v_2)) \\ &= \min(\min(\mu_1^2(u_1), (\mu'_1)^2(u_2)), \min(\mu_1^2(v_1), (\mu'_1)^2(v_2))) \\ &= \min((\mu_1 \circ \mu'_1)^2(u_1, u_2), (\mu_1 \circ \mu'_1)^2(v_1, v_2)) \end{aligned} \tag{4.21}$$

Similarly we have

$$(\nu_2 \circ \nu'_2)(u_1, u_2)(v_1, v_2) = \max((\nu_1 \circ \nu'_1)^2(u_1, u_2), (\nu_1 \circ \nu'_1)^2(v_1, v_2)) \tag{4.22}$$

From the equations (4.17),(4.18),(4.19),(4.20),(4.21) and (4.22),  $G_1 \circ G_2$  is also strong IFGST.

This completes the proof.

## 5. CONCLUSION

In this paper, we have defined some new operations direct product, semi-strong product, strong product, cartesian product and composition on IFGST. Also we have defined the concept of strong intuitionistic fuzzy graphs of second type and established some of their properties. In future we will study some more properties and applications of IFGST.

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**Source of support: Nil, Conflict of interest: None Declared.**

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