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A STUDY ON INTUITIONISTIC FUZZY MULTISETS OF TYPE II

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ABSTRACT

In this paper, the concept of Intuitionistic Fuzzy Multiset (IFMS) has been further extended namely to the INTUITIONISTIC FUZZY MULTISETS OF SECOND TYPE (IFMS ST) and various basic operations such as union, intersection, addition, multiplication etc. are defined.

Keywords: *Fuzzy set,Intuitionistic fuzzy set, Fuzzy multi set, Intuitionistic Fuzzy multi set, Intuitionistic Fuzzy multi set of second type.*

1. INTRODUCTION

Modern set theory formulated by the German mathematician GEORGE CANTOR is fundamental for the whole of mathematics. In fact, set theory is the language of mathematics, science, logic and philosophy. One issue associated with the notion of set is the concept of vagueness. This vagueness or the representation of imperfect knowledge has been a problem for a long time. However recently it became a crucial issue for computer scientists particularly in the area of artificial intelligence. To handle situations like this many tools were suggested which includes Fuzzy sets, Soft sets, Rough sets, Multi sets and many more.

Considering the unpredictable factor in decision making Lofti A Zadeh introduced the idea of fuzzy set which has a membership function that assigns to each element of the universe of discourse, a member from the unit interval [0, 1] to indicate the degree of belongingness to the set under consideration.

Atanassov subsequently proposed the concept of INTUITIONISTIC FUZZY SET by bringing a non-membership function together with the membership function of the fuzzy set introduced earlier by Zadeh. Among the various notions of higher order fuzzy sets, IFS proposed by Atanassov provides a flexible framework to elaborate uncertainty and vagueness.

As a generalization of fuzzy sets, Yager introduced the concept of Fuzzy Multiset. An element of a Fuzzy Multiset can occur more than once with possibly the same or different membership values then years after, SHINOJ and SUNIL made an attempt to combine the concepts called Intuitionistic fuzzy multi set. In this paper IFMS has been further extended to IFMS of second type.

This paper proceeds as follows. In section 2; we give some basic definitions related to fuzzy set, Intuitionistic fuzzy set, Fuzzy multi set, Intuitionistic fuzzy multi set. In section 3; we define intuitionistic fuzzy multisets of second type and their basic operations such as union, intersection, addition and multiplication. The paper is concluded in section 4.

2. PRELIMINARIES

Definition 2.1: Let X be a non-empty set. A fuzzy set 'A' drawn from X is defined as $A = \{(x, \mu_{A(x)}) | x \in X\}$ where $\mu_{A(x)}: X \to [0,1]$ is the membership function of the fuzzy set A.

Definition 2.2: Let X be a non-empty set. A fuzzy multiset (FMS) 'A' drawn from X is characterized by a function COUNT MEMBERSHIP of A denoted by CM_A such that $CM_A: X \to Q$ where Q is the set of all crisp multisets drawn from the unit interval [0,1] Then for any $x \in X$, the value $CM_A(x)$ is a crisp multiset drawn from [0,1]. For each $x \in X$, the membership sequence is defined as the decreasingly ordered sequence of elements in $CM_A(x)$. It is denoted by $\mu_A^1 \ (x)$ where $\mu_A^1(x) \geq \mu_A^2(x) \geq, \ldots, \geq \mu_A^p(x)$.

Definition 2.3: Let X be a nonempty set. An *Intuitionistic Fuzzy Set* (IFS) A is an object having the form $A = \{< x: \mu_A(x), \nu_A(x) > : x \in X\}$, where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define respectively the degree of membership and the degree of non-membership of the element $x \in X$ to the set A with $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Remark: Every Fuzzy Set A on a nonempty set X is obviously an IFS having the form $A = \{< x: \mu_A(x), 1 - \mu_A(x) >: x \in X\}$

Definition 2.4: Let X be a nonempty set. An *Intuitionistic Fuzzy Multiset A* denoted by IFMS drawn from X is characterized by two functions 'Count membership' of A (CM_A) and 'count non membership' of A (CN_A) given respectively by $CM_A : \boldsymbol{X} \to \boldsymbol{Q}$ and $CN_A : \boldsymbol{X} \to \boldsymbol{Q}$ where Q is the set of all crisp multisets drawn from the unit interval [0,1] such that for each $x \in X$, the membership sequence is defined as a decreasingly ordered sequence of elements in $CM_A(x)$ which is denoted by $(\mu_A^1(x), \mu_A^2(x), \ldots, \mu_A^p(x))$ where $\mu_A^1(x) \geq \mu_A^2(x) \geq \ldots \geq \mu_A^p(x)$ and the corresponding non membership sequence will be denoted by $(\nu_A^1(x), \nu_A^2(x), \ldots, \nu_A^p(x))$ such that $0 \leq \mu_A^i(x) + \nu_A^i(x) \leq 1$ for every $x \in X$ and $i = 1,2,\ldots,p$. An IFMS is denoted by

$$A = \{< x : (\mu^1{}_A(x), \mu^2{}_A(x), \ldots, \mu^p{}_A(x)), (\nu_A{}^1(x), \nu_A{}^2(x), \ldots, \nu_A{}^p(x)) > : x \in X\}$$

Remark: Since we arrange the membership sequence in decreasing order the corresponding non membership sequence may not be in increasing or decreasing order.

3. INTUITIONISTIC FUZZY MULTISET (SECOND TYPE*)*

Definition 3.1: Let X be a non-empty set. An Intuitionistic fuzzy multiset of type two 'A' denoted by IFMS ST drawn from X is characterized by two functions count membership of A ($\mathrm{CM_A}$) and count non-membership of A ($\mathrm{CN_A}$) given by $\mathrm{CM_A}$:X→ Q and $\mathrm{CN_A}$:X→ Q where Q is the set of all crisp multisets drawn from the unit interval [0,1] such that for each $x \in X$; the membership sequence is defined as a decreasingly ordered sequence of elements in CM_A(x) which is denoted by where $\mu_A^1(x) \geq \mu_A^2(x) \geq \ldots \geq \mu_A^p(x$ and the corresponding non membership sequence will be denoted by $(\nu_A{}^1(x), \nu_A{}^2(x), \ldots, \nu_A{}^p(x))$ such that $0 \leq \mu_A{}^i(x)^2 + \nu_A{}^i(x)^2 \leq 1$ for every $x \in X$ and $i = 1,2,\ldots,p$.

An IFMS of second type is denoted by

$$\boldsymbol{A = \{< x: (\mu^1{}_A(x), \mu^2{}_A(x), \ldots, \mu^p{}_A(x)), (\nu_A{}^1(x), \nu_A{}^2(x), \ldots, \nu_A{}^p(x)) > : x \in X\}}$$

Definition 3.2: Length of an element x in an IFMS (ST) A is defined as the Cardinality of $CM_A(x)$ or $CN_A(x)$ for which $0 \leq \mu_A{}^i(x)^2 + \nu_A{}^i(x)^2 \leq 1$ and it is denoted by $L(x : A)$. That is $\mathbf{L(x: A) = |CM_A(x)| = |CN_A(x)|}$

Definition 3.3: If and B are IFMS(ST) drawn from X then $\boldsymbol{L(x: A,B) = Max\{L(x : A), L(x : B)\}}$ We can use the notation $L(x)$ for $L(x : A,B)$.

Example: Consider $X = \{x, y, z, w\}$
A= {< x : (0.4),(0.2) > ,< y : (1,0.3,0.2),(0,0.4,0.5) > ,< z: (0.2,0.1),(0.7,0.8) >}
B ={< x : (0.3,0.2),(0.4,0.5) > < y : (1,0.5,0.5),(0,0.5,0.2) > < w : (0.5,0.4,0.3,0.2),(0.4,0.6,0.4,0.7) >}
Here,
$L(x : A) = 1$ $L(x : B) = 2$ $L(x)=L(x : A,B) = 2$
$L(y : A) = 3$ $L(y : B) = 3$ $L(y)=L(y : A,B) = 3$
$L(z : A) = 2$ $L(z : B) = 0$ $L(z)=L(z : A,B) = 2$
$L(w : A) = 0$ $L(w : B) = 4$ $L(w)=L(w : A,B) = 4$

Definition 3.4: For any two IFMS ST A and B drawn from a set X Such that

$A = \{< x : (\mu^1{}_A(x),\mu^2{}_A(x),...,\mu^p{}_A(x)),(\nu_A{}^1(x),\nu_A{}^2(x),...,\nu_A{}^p(x)) > : x \in X\}$ and

$B = \{< x : (\mu^1{}_B(x),\mu^2{}_B(x),...,\mu^p{}_B(x)),(\nu_B{}^1(x),\nu_B{}^2(x),...,\nu_B{}^p(x)) > : x \in X\}$ then

Define the following operations on A and B

(i) $A \subset B \iff \mu_A{}^i(x) \leq \mu_B{}^i(x) \;\; \nu_A{}^i(x) \geq \nu_B{}^i(x); \quad i = 1,2,..., L(x), \quad x \in X$

(ii) $A = B \iff A \subset B$ and $B \subset A$

(iii) $\sim A = \{x: \nu_A^1(x), \nu_A^2(x), .., \nu_A^p(x), \mu_A^1(x), \mu_A^2(x), ..., \mu_A^p(x)\}$

(iv) $(A \cup B)=\left\{x; \max\{\mu_A^i(x), \mu_B^i(x)\}\min\{\nu_A^i(x), \nu_B^i(x)\}\right\}$

(v) $(A \cap B)=\{\text{x}, \min\{\mu_A^i(x), \mu_B^i(x)\}\max\{\nu_A^i(x), \nu_B^i(x)\}, x \in X.\}$

(vi) $\text{A@B}=\left\{x; \frac{\mu_A^i(x)+\mu_B^i(x)}{2}; \frac{\nu_A^i(x)+\nu_B^i(x)}{2}; x \in X.\right\}$

(vii) $\text{A\$B}=\left\{x; \sqrt{\mu_A^i(x).\mu_B^i(x)}, \sqrt{\nu_A^i(x).\nu_B^i(x)}; x \in X.\right\}$

(viii) $(\text{A*B})=\left\{x, \frac{\mu_A^i(x)+\mu_B^i(x)}{2(\mu_A^i(x).\mu_B^i(x)+1)}, \frac{\nu_A^i(x)+\nu_B^i(x)}{2(\nu_A^i(x).\nu_B^i(x)+1)}, x \in X.\right\}$

Example:

Let A={<x:(0.5,0.2),(0.3,0.6)>,<y:(0.6,0.5,0.4,0.2),(0.1,0.3,0.2,0.5)>,<z:(0.5,0.3,0.1),(0.30.4,0.2)>,<w:(0,0),(1,1)}
B={<x:(0.4,0.3),(0.5,0.4)> ,< y : (0,0,0,0),(1,1,1,1) >,<z:(0.5,0.4,0.2),(0.3,0.2,0.4)> ,< w : (0.6,0.2),(0.1,0.4) >}

$(\boldsymbol{A \cup B})=\left\{\boldsymbol{x}; \textbf{max}\{\boldsymbol{\mu_A^i(x), \mu_B^i(x)}\}\textbf{min}\{\boldsymbol{\nu_A^i(x), \nu_B^i(x)}\}\right\}$

$A \cup B$ ={< x : (0.5,0.3),(0.3,0.4) > ,< y : (0.6,0.5,0.4,0.2),(0.1,0.3,0.2,0.5) > ,< z : (0.5,0.4,0.2),(0.3,0.2,0.2) > ,< w : (0.6,0.2),(0.1,0.4) >}

$(\boldsymbol{A \cap B})=\{\textbf{x}, \textbf{min}\{\boldsymbol{\mu_A^i(x), \mu_B^i(x)}\}\textbf{max}\{\boldsymbol{\nu_A^i(x), \nu_B^i(x)}\}, \boldsymbol{x \in X}.\}$

$(A \cap B)$ ={< x : (0.4,0.2),(0.5,0.6) > ,< y : (0,0,0,0),(1,1,1,1) > ,< z : (0.5,0.3,0.1,),(0.3,0.4,0.4) > ,< w : (0,0),(1,1) >}
$(A + B)$ ={< x : (0.7,0.44),(0.15,0.24) > ,< y : (0.6,0.5,0.4,0.2),(0.1,0.3,0.2,0.5) > ,< z : (0.75,0.58,0.28,),(0.09,0.08,0.08) > ,< w : (0.6,0.2),(0.1,0.4) >}
$(A.B)$ ={< x : (0.2.0.06),(0.65,0.76) > ,< y : (0,0,0,0),(1,1,1,1) > ,< z : (0.25,0.12,0.02,),(0.54,0.52,0.52) > ,< w : (0,0),(1,1) >}

(A@B)={< x : (0.45,0.25),(0.4,0.5) > ,< y : (0.3,0.25,0.2,0.1),(0.55,0.65,0.6,0.75) > ,< z : (0.5,0.35,0.15),(0.3,0.3,0.3) > ,< w : (0.3,0.1),(0.55,0.7) >}

(A$B)={< x : (0.45,0.24),(0.39,0.49) > ,< y : (0,0,0,0),(0.32,0.55,0.45,0.71) > ,< z : (0.5,0.35,0.14),(0.3,0.28,0.28) > ,< w : (0,0),(0.32,0.63) >}

(A*B)={< x : (0.38,0.24),(0.35,0.4) > ,< y : (0.3,0.25,0.2,0.1),(0.5,0.5,0.5,0.5) > ,< z : (0.4,0.31,0.15),(0.28,0.28,0.28) > ,< w : (0.3,0.1),(0.5,0.5) >}
are all intuitionistic fuzzy multi sets of second type.

4. CONCLUSION

In this paper, we have defined the Intuitionistic Fuzzy Multiset of second type and their basic operations with suitable examples. There is an excellent opportunity for further research in IFMSST and its applications.

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