International Journal of Mathematical Archive-9(8), 2018, 75-80
Available online through www.ijma.info ISSN 2229 – 5046

TRIANGULAR DIVISOR CORDIAL LABELING FOR SOME STANDARD GRAPHS

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(Received On: 04-01-18; Revised & Accepted On: 13-08-18)

ABSTRACT

Let $G = (V, E)$ be a (p, q)- graph. A Triangular divisor cordial labeling of a graph G with vertex set V is a bijection $f : V \to \{T_1, T_2, T_3, \ldots, T_p\}$ where T_i is the i^{th} Triangular number such that if each edge uv is assigned the label 1 if f(u) divides f(v) or f(v) divides f(u) and 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. If a graph has a Triangular divisor cordial labeling, then it is called Triangular divisor cordial graph. In this paper, we proved the standard graphs such as Switching a pendant vertex in path P_n, wheel(W_n), Flower graph Fl_n, A book with rectangular pages, A book with pentagonal pages, Shell S_n, Umbrella $U(n, 3)$, The tensor product graph $(G_1(T_p)G_2)$ are Triangular divisor cordial graphs.

AMS subject classification: *05C78.*

Key words: *Cordial labeling, Divisor cordial labeling, Triangular divisor cordial labeling.*

1. INTRODUCTION

By a graph, we mean a finite, undirected graph without loops and multiples edges, for terms not defined here, we refer to Harary [6].

In a labeling of a particular type, the vertices are assigned values from a given set, the edges have a prescribed induced labeling must satisfy certain properties. An excellent reference on this subject is the survey by Gallian [3]. Two of the most important types of labeling are called graceful and harmonious, Graceful labeling were introduced independently by Rosa [7] in 1966 and Golombo [4] in 1972, while harmonious labeling were first studied by Graham and Sloane [5] in 1980.A third important type of labeling which contains aspects of both of the other two, is called cordial and was introduced by Cahit [1] in 1990. Whereas the label of an edge uv for graceful and harmonious labeling is given respectively by $|f(u) - f(v)|$ and $f(u) + f(v)(modulo q)$, cordial labeling use only labels 0 and 1 and the induced label $f(u) + f(v)(modulo 2)$, which is of course equals $|f(u) - f(v)|$. Because arithmetic modulo 2 is an integral part of computer science, cordial labeling has close connections with that field.

More precisely, cordial graphs are defined as follows.

Definition 1.1: Let $G = (V, E)$ be an (p, q)-graph, let $f: V \to \{0,1\}$ and for each edge uv, assign the label $|f(u) - f(v)|$. f is called a cordial labeling if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1 and the number of edges labeled 0 and the number of edges labeled 1 differ by at most 1. A graph is called cordial if it has a cordial labeling.

Definition 1.2: Let f be a function from the vertices of a graph $G to$ $\{0,1\}$ and for each edge uv assign the label $|f(u) - f(v)|$. The function f is called a cordial labeling of G if $\left|v_f(0) - v_f(1)\right|$.

Definition 1.3: Let $G = (V, E)$ be an (p, q)- graph. A mapping$f: V \to \{0,1\}$ is called binary vertex labeling of G and $f(v)$ is called the label of the vertex v of G under f.

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For an edge $e = uv$, the induced edge labeling $f^* : E(G) \to \{0,1\}$ is given by $f^*(e) = |f(u) - f(v)|$. Let $v_f(0), v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f and $e_f(0), e_f(1)$ be the number of edges having labels 0 and 1 respectively underf^*.

Graph labeling [3] is a strong communication between number theory [2] and structure of graphs [6]. By combining the triangular number and divisibility concept in Number Theory and cordial labeling concept in graph labeling, we introduce a new concept called Triangular divisor cordial labeling. In this paper, we proved the standard graphs such as Switching a pendant vertex in path P_n, wheel(W_n),Flower graph Fl_n, A book with rectangular pages, A book with pentagonal pages, Shell S_n, Umbrella $U(n,3)$, The tensor product graph $(G_1(T_p)G_2)$ are Triangular divisor cordial graphs. First we give the some concepts in Number Theory [6].

Definition 1.4: Let a and b be two integers. If a divides b means that there is a positive integer k such that b = ka. It is denoted by a/b.If a does not divide b, then we denote $a \nmid b$

Definition 1.5: The triangular number can be defined by

$$T_n = \binom{n+1}{2} n \geq 1$$

this generates the infinite sequence of integers beginning 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91,….

2. MAIN RESULTS

R.Varatharajan, S.Navaneethakrishnan and K.Nagarajan [8], introduced the notion of Divisor Cordial Labeling.

Definition 2.1: Let $G = (V, E)$ be a simple graph and$f : V \to \{1,2,3,\ldots,|V|\}$ be a bijection. For each edgeuv, assign the label 1 if either$f(u)/f(v)$ or$f(v)/f(u)$ and the label 0 if $f(u)\ f(v)$.$f$is called a divisor cordial labeling $|e_f(0) - e_f(1)| \leq 1$. A graph with a divisor cordial labeling is called a divisor cordial graph [8].

R.Sridevi, S.Navaneethakrishnan introduced the notion of Fibonacci Divisor Cordial Labeling.

Definition 2.2: Let $G = (V, E)$be a simple (p, q)- graph and $f : V \to \{F_1, F_2, F_3, \ldots, F_p\}$ where F_i is the i^{th} Fibonacci number, be a bijection.For each edge uv, assign the label 1 if either$f(u)/f(v)$ or$f(v)/f(u)$ and the label 0 if$f(u) \nmid f(v)$.$f$is called a Fibonacci divisor cordial labeling $|e_f(0) - e_f(1)| \leq 1$. A graph with a Fibonacci divisor cordial labeling is called a Fibonacci divisor cordial graph [9].

These definitions motivate us to define a new type of cordial labeling called Triangular divisor cordial labeling as follows.

Definition 2.3: Let $G = (V, E)$be a simple (p, q)- graph and $f : V \to \{T_1, T_2, T_3, \ldots, T_p\}$ where T_i is the i^{th} Triangular number, be a bijection. For each edge uv, assign the label 1 if either $f(u)/f(v)$ or$f(v)/f(u)$ and the label 0 otherwise.f is called a Triangular divisor cordial labeling $|e_f(0) - e_f(1)| \leq 1$. A graph with a Triangular divisor cordial labeling is called a Triangular divisor cordial graph.

Theorem 2.4: Switching a pendant vertex in path $P_n, n \geq 4$ is triangular divisor cordial graph.

Proof: Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of path P_n.

The graph G is obtained by switching of a pendant vertex in path P_n, v_1 and $v_n$are pendant vertex of path P_n.

Without loss of generality, let the switched vertex be v_1
Then $|V(G)| = n$ and $|E(G)| = 2n - 4$

Let $f: V(G) \to \{T_1, T_2, T_3, \ldots, T_n\}$ be defined as follows

$f(v_i) = T_i$, $i = 1,2$
$f(v_3) = T_4$
$f(v_4) = T_3$
$f(v_i) = T_i$, $5 \leq i \leq n$

Then $e_f(0) = e_f(1) = n - 2$

Therefore, $|e_f(0) - e_f(1)| \leq 1$

Hence the graph G is triangular divisor cordial graph.

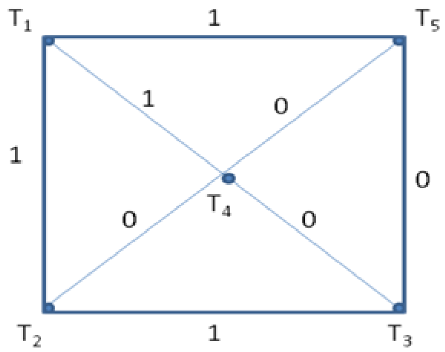
Theorem 2.5: Wheel $W_n, n \geq 4$ is triangular divisor cordial graph.

Proof: Let G be the graph Wheel W_n

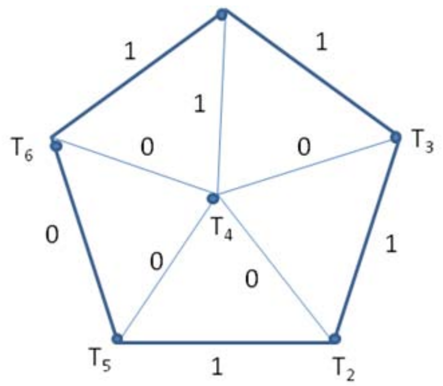
Let $V(G) = \{u_i \ : 0 \leq i \leq n\}$
and $E(G) = \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{u_1 u_n\} \cup \{u_0 u_i \ : 1 \leq i \leq n\}$
Then $|V(G)| = n + 1$ and $|E(G)| = 2n$

Let $f: V(G) \rightarrow \{T_1, T_2, T_3, \dots, T_{n+1}\}$ be defined as follows

Case-(i): $n = 4 \ and \ 5$



$e_f(0) = 4 \quad and \quad e_f(1) = 4$
$|e_f(0) - e_f(1)| = 0 < 1$



$e_f(0) = 5 \quad and \quad e_f(1) = 5$
$|e_f(0) - e_f(1)| = 0 < 1$

Case-(ii): $n \equiv 0 \ (mod \ 3)$

$f(u_0) = T_1$
$f(u_1) = T_2$
$f(u_2) = T_4$
$f(u_3) = T_3$
$f(u_i) = T_{i+1} \qquad 4 \leq i \leq n$

Then $e_f(0) = n \quad and \quad e_f(1) = n$

Therefore, $|e_f(0) - e_f(1)| \leq 1$

Case-(iii): $n \equiv 1 \ (mod \ 3)$

$f(u_0) = T_1$
$f(u_1) = T_2$
$f(u_2) = T_4$
$f(u_3) = T_3$
$f(u_i) = T_{i+1} \qquad 4 \leq i \leq n-2$
$f(u_{n-1}) = T_{n+1}$
$f(u_n) = T_n$

Then $e_f(0) = n \quad and \quad e_f(1) = n$
Therefore, $|e_f(0) - e_f(1)| \leq 1$

Case-(iv): $n \equiv 2\ (mod\ 3)$

$f(u_0) = T_1$
$f(u_1) = T_2$
$f(u_2) = T_4$
$f(u_3) = T_3$
$f(u_i) = T_{i+1} \quad 4 \leq i \leq n-3$
$f(u_{n-2}) = T_n$
$f(u_{n-1}) = T_{n+1}$
$f(u_n) = T_{n-1}$

Then $e_f(0) = n \quad and \quad e_f(1) = n$

Therefore, $|e_f(0) - e_f(1)| \leq 1$

Hence the graph Wheel $W_n, n \geq 4$ is triangular divisor cordial graph.

Theorem 2.5: Flower graph $Fl_n, n \geq 3$ is triangular divisor cordial graph.

Proof: Let G be the Flower graph Fl_n

Let v be the apex, $v_1, v_2, v_3, \ldots, v_n$ be the vertices of degree 4 and $u_1, u_2, u_3, \ldots, u_n$ be the vertices of degree 2 of Fl_n
Then $|V(G)| = 2n+1$ and $|E(G)| = 4n$

Let $f: V(G) \to \{T_1, T_2, T_3, \ldots, T_{2n+1}\}$ defined as follows

Case-(i): $n \equiv 0\ (mod\ 10)\ \ except\ \ 0\ \ and\ 2$

$f(v) = T_1$
$f(v_1) = T_4$
$f(v_2) = T_3$
$f(v_i) = T_{2i}, \quad 3 \leq i \leq n$
$f(u_1) = T_2$
$f(u_2) = T_5$
$f(u_i) = T_{2i+1}, \quad 3 \leq i \leq n$

Then $e_f(0) = 2n \quad and \quad e_f(1) = 2n$

Therefore, $|e_f(0) - e_f(1)| \leq 1$

Case-(ii): $n \equiv 0\ and\ 2\ (mod\ 10)$

$f(v) = T_1$
$f(v_1) = T_4$
$f(v_2) = T_3$
$f(v_i) = T_{2i}, \quad 3 \leq i \leq n-1$
$f(v_n) = T_{2n+1}$
$f(u_1) = T_2$
$f(u_2) = T_5$
$f(u_i) = T_{2i+1}, \quad 3 \leq i \leq n-1$
$f(u_n) = T_{2n}$

Then $e_f(0) = 2n \quad and \quad e_f(1) = 2n$

Therefore, $|e_f(0) - e_f(1)| \leq 1$

Hence the graph Flower $Fl_n, n \geq 3$ is triangular divisor cordial graph.

Theorem 2.6: A book with rectangular pages is triangular divisor cordial graph.

Proof: Let G be the graph of book with rectangular pages

Let $V(G) = \{u, u_i, v_i \ : 1 \leq i \leq n\}$
and $E(G) = \{uv, uu_i, vv_i, u_iv_i : 1 \leq i \leq n\}$
Then $|V(G)| = 2n+2$ and $|E(G)| = 3n+1$

Let $f: V(G) \to \{T_1, T_2, T_3, \dots, T_{2n+2}\}$ be defined as follows
$f(u) = T_1$
$f(v) = T_2$
$f(u_1) = T_3$
$f(v_1) = T_4$

Case-(i): $n \equiv 2 \ mod(3)$
$f(u_n) = T_{2n+1}$
$f(v_n) = T_{2n+2}$

Case-(ii): $n \equiv 0 \ mod(3)$
$f(u_n) = T_{2n+2}$
$f(v_n) = T_{2n+1}$

Case-(iii): $n \ is \ odd$

Subcase-(i): $n \equiv 1 \ mod(3)$, $n \neq 1$ and $n = 6i - 2$, $1 \leq i \leq \left\lceil \frac{n}{6} \right\rceil$
$f(u_n) = T_{2n+1}$
$f(v_n) = T_{2n+2}$

Subcase-(ii): $n \equiv 1 \ mod(3)$, $n \neq 1$ and $n = 6i + 1$, $1 \leq i \leq \left\lceil \frac{n}{6} \right\rceil$
$f(u_n) = T_{2n+2}$
$f(v_n) = T_{2n+1}$

Case-(iv): $n \ is \ even$

Subcase-(i): $n \equiv 1 \ mod(3)$, $n \neq 1$ and $n = 6i - 2$, $1 \leq i \leq \left\lceil \frac{n}{6} \right\rceil$
$f(u_n) = T_{2n+1}$
$f(v_n) = T_{2n+2}$
Then $e_f(0) = n + \frac{n+1}{2} \ and \ \ e_f(1) = n + \frac{n+1}{2} \quad if \ \ n \ is \ odd$
and $e_f(0) = n + \frac{n}{2} \ \ and \ \ e_f(1) = n + \frac{n}{2} + 1 \quad if \ \ n \ is \ even$

Therefore, $\left| e_f(0) - e_f(1) \right| \leq 1$

Hence a book with rectangular pages are triangular divisor cordial graph .

Theorem 2.6: A book with pentagonal pages is triangular divisor cordial graph.

Proof: Let G be the graph of book with pentagonal pages

Let $V(G) = \{u, v, u_i, v_i, w_i : 1 \leq i \leq n\}$
and $E(G) = \{uv, uu_i, vv_i, u_i w_i, v_i w_i, : 1 \leq i \leq n\}$
Then $|V(G)| = 3n + 2$ and $|E(G)| = 4n + 1$

Let $f: V(G) \to \{T_1, T_2, T_3, \dots, T_{3n+2}\}$ be defined as follows
$f(u) = T_1$
$f(v) = T_2$
$f(u_i) = T_{3i} \qquad 1 \leq i \leq n$
$f(v_i) = T_{3i+2} \qquad 1 \leq i \leq n$
$f(w_i) = T_{3i+1} \qquad 1 \leq i \leq n$
Then $e_f(0) = 2n \ \ and \ \ e_f(1) = 2n + 1$

Therefore, $\left| e_f(0) - e_f(1) \right| \leq 1$

Hence a book with pentagonal pages is triangular divisor cordial graph.

Theorem 2.7: The graph Shell S_n is triangular divisor cordial graph for $n \geq 4, n \in N$.

Proof: Let $u_1, u_2, u_3, \dots, u_n$ be the successive vertices of Shell S_n where u_1 is the apex vertex Shell S_n
Then $|V(G)| = n$ and $|E(G)| = 2n - 3$

Let $f: V(G) \to \{T_1, T_2, T_3, \dots., T_n\}$ be defined as follows

$f(u_i) = T_i, \quad 1 \leq i \leq 2 \; and \; 5 \leq i \leq n$

$f(u_3) = T_4$

$f(u_4) = T_3$

Then $e_f(0) = n - 2 \quad and \quad e_f(1) = n - 1$

Therefore, $\left|e_f(0) - e_f(1)\right| \leq 1$
Hence the graph Shell S_n is triangular divisor cordial graph for $n \geq 4, n \in N$.

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Source of support: Nil, Conflict of interest: None Declared.