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COMPARATIVE STUDY OF HA, MOA AND NAA METHODS FOR SOLVING ASSIGNMENT PROBLEMS

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ABSTRACT

The assignment problem is a special case of the transportation problem in which the objective is to assign n number of resources to n number of activities at a minimum cost or maximum profit. In this paper we attempt to introduce the algorithms and solution steps for HA, MOA and NAA methods used to solve assignment problems. Finally we compare with the optimal solutions among these three methods.

Keywords: *Assignment problem, Hungarian assignment method (HA-method), Matrix one's assignment method (MOA-method), New approach assignment method (NAA-method), Optimization.*

INTRODUCTION

Assignment problem is completely a degenerated form of a transportation problem. It appears in some decision-making situations such as to assign salesman to different regions, vehicles and drivers to different routes, products to factories, jobs to machines, requirements to suppliers etc. It is one of the fundamental combinatorial optimization problems in the branch of optimization or operations research in Mathematics.

Assignment problem is a special case of linear programming problem in which the objective is to assign a number of resources to the equal number of activities at a minimum cost (or maximum profit).

In this paper the given three methods have been formulated and numerical example has been considered to be illustrated. Finally we compare the optimal solutions among three methods.

MATHEMATICAL FORMULATION OF ASSIGNMENT PROBLEM

Considering the assignment problem of n resources (workers) to n activities (jobs) so as to minimize the overall cost or time in such a way that each resource can associate with one and only one job.

The cost (or effectiveness) matrix (c_{ij}) is given as below:

Activity

		A_1	A_2	…….	A_n	Available
Resource	R_1	C_{11}	C_{12}	…….	C_{1n}	1
	R_2	C_{21}	C_{22}	…….	C_{2n}	1
	⋮	⋮	⋮	⋮	⋮	⋮
	R_n	C_{n1}	C_{n2}	……..	C_{nn}	1
	Required	1	1	…….	1	

This cost matrix is same as that of a transportation problem except that availability at each of the resources and the requirement at each of the destinations is unity.

Let X_{ij} denote the assignment of i^{th} resource to j^{th} activity, such that

$$X_{ij} = \begin{cases} 1\ ; & if\ resource\ i\ is\ assigned\ to\ activity\ j \\ 0\ ; & otherwise \end{cases}$$

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Then, the mathematical formulation of the assignment problem is
Minimize z = $\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}X_{ij}$
Subject to the constraints
$\sum_{i=1}^{n} X_{ij}$ = 1 and $\sum_{j=1}^{n} X_{ij}$ =1 such that X_{ij} = 0 or 1 for all i = 1,2, ... ,n and j = 1,2,.......,n.

I. Hungarian Assignment (HA) Method

Hungarian method (also known as reduced matrix method) is for assigning jobs by a one-for-one matching to identify the lowest-cost solution. Each job must be assigned to only one machine. It is assumed that every machine is capable of handling every job, and that the costs or values associated with each assignment combination are known and fixed. The number of rows and columns must be the same.

HA-Algorithm
Step-1: Subtract the row minimum from each row.

Step-2: Subtract the column minimum from each column from the reduced matrix.

Step-3: Assign one “0” to each row and column.
(i) Mark (√) all unassigned rows.
(ii) If a row is marked (√) and has a “0”, then mark the corresponding column (if the column is not yet marked).
(iii) If a column is marked and has an assignment, then mark the corresponding row (if the row is not yet marked).
(iv) Repeat steps (ii) and (iii) till no more marking is possible.
(v) Draw straight lines through all unmarked rows and marked columns.
(vi) If the number of lines drawn is equal to the number of rows or columns, then the current solution is optimal solution. Otherwise go to next step.

Step-4: Find out the smallest number which does not have any line passing through it. Select the smallest number to subtract from all the numbers that do not have any lines passing through them and add to all those numbers that have two lines passing through them. Keep the rest of them the same.

Step 5: If we cannot get the optimal assignment in each row and column. Then repeat steps (3) and (4) successively till an optimum solution is obtained.

II. Matrix One’s Assignment (MOA) Method

The matrix one’s assignment is different from the preceding method, because of making assignment in terms of one’s. In this method is based on creating some ones in the assignment matrix and then try to find a complete assignment in terms of ones. By a complete assignment plan containing exactly n assigned independent ones, one in each row and one in each column.

MOA-Algorithm
Step-1: Find the minimum cost of each row and then divide each element of each row of the matrix by its minimum cost.

Step-2: Find the minimum cost of each column and divide each element of the column by its minimum cost.

Step 3: Assign one “1” to each row and column.
(i) Mark (√) all unassigned rows.
(ii) If a row is marked (√) and has a “1”, then mark the corresponding column (if the column is not yet marked).
(iii) If a column is marked and has an assignment, then mark the corresponding row (if the row is not yet marked).
(iv) Repeat steps (ii) and (iii) till no more marking is possible.
(v) Draw straight lines through all unmarked rows and marked columns.
(vi) If the number of lines drawn is equal to the number of rows or columns, then the current solution is optimal solution. Otherwise go to next step.

Step-4: Find out the smallest number which does not have any line passing through it. Select the smallest number to divide from all the numbers that do not have any lines passing through them and other numbers covered by lines remain unchanged.

Step-5: If we cannot get the optimal assignment in each row and column. Then repeat steps (3) and (4) successively till an optimum solution is obtained.

III. New Approach Assignment (NAA) Method

In this method we introduce a new approach for solving assignment problem with the help of HA-method and MOA-method but different from them. This new method is easy procedure to solve assignment problem.

NAA-Algorithm
Step-1: Find the smallest number (cost) of each row. Subtract this smallest number from every number in that row.

Step-2: Now add 1 to all element and we get at least one ones in each row. Then make assignment in terms of ones. If there are some rows and columns without assignment, then we cannot get the optimum solution. Then we go to the next step.

Step-3: Draw the minimum number of lines passing through all ones by using the following procedure:
(i) Mark (√) rows that do not have assignments.
(ii) Mark (√) columns that have crossed ones in that marked rows.
(iii) Mark (√) rows that have assignments in marked columns.
(iv) Repeat (b) and (c) till no more rows or columns can be marked.
(v) Draw straight lines through all unmarked rows and marked columns.
(vi) If the number of lines drawn is equal to the number of rows or columns, then the current solution is optimal solution. Otherwise go to next step.

Step-4: Select the smallest number of the reduced matrix not covered by the lines. Divide all uncovered numbers by this smallest number. Other numbers covered by lines remain unchanged. Then we get some new ones in row and column. Again make assignment in terms of ones.

Step-5: If we cannot get the optimal assignment in each row and column. Then repeat steps (3) and (4) successively till an optimum solution is obtained.

Numerical Comparison Of HA, MOA, NAA – Methods

Problem 1: Solve the assignment problem using
(i) Hungarian Assignment (HA) Method.
(ii) Matrix One's Assignment Method (MOA) Method.
(iii) New Approach Assignment (NAA) Method.

Consider the problem of assigning four jobs to four persons. The assignment cost are given below.

Person	Job 1	2	3	4
A	20	25	22	28
B	15	18	23	17
C	19	17	21	24
D	25	23	24	24

Determine the optimum assignment schedule and minimum assignment cost.

Solution:
(i) Hungarian Assignment (HA) Method.
Consider the problem of assigning four jobs to four persons. The assignment cost are given below.

Person	Job 1	2	3	4
A	20	25	22	28
B	15	18	23	17
C	19	17	21	24
D	25	23	24	24

Step-1: Select the minimum element in each row and subtract this element from every element in that row.

	1	2	3	4
A	0	5	2	8
B	0	3	8	2
C	2	0	4	7
D	2	0	1	1

Step-2: Select the minimum element in each column and subtract this element from every element in that column.

	1	2	3	4
A	0	5	1	7
B	0	3	7	1
C	2	0	3	6
D	2	0	0	0

Step-3: Make initial assignment.

	1	2	3	4
A	[0]	5	1	7
B	~~0~~	3	7	1
C	2	[0]	3	6
D	2	~~0~~	[0]	~~0~~

Here, 2nd row and 4th column do not have any assignment. Thus the solution is not optimum and we go to next.

Step-4: Find out the smallest number which does not have any line passing through it. Select the smallest number to subtract from all the numbers that do not have any lines passing through them and add to all those numbers that have two lines passing through them. Keep the rest of them the same.

	1	2	3	4
A	[0]	4	~~0~~	6
B	~~0~~	2	6	[0]
C	3	[0]	3	6
D	3	~~0~~	[0]	~~0~~

Here we see that all zeros are either assigned or crossed out. That is, the total assigned zero's is 4 which is equal to the number of rows or columns.

The optimum assignment is : $A \to 1,\ B \to 4,\ C \to 2,\ D \to 3.$

$\therefore$ The total minimum assignment cost = $20 + 17 + 17 + 24 = Rs. 78$

(ii) Matrix One's Assignment Method (MOA) Method.

Consider the problem of assigning four jobs to four persons. The assignment cost are given below.

Person	Job			
	1	2	3	4
A	20	25	22	28
B	15	18	23	17
C	19	17	21	24
D	25	23	24	24

Step-1: Find the minimum cost of each row and then divide each element of each row of the matrix by its minimum cost.

	1	2	3	4
A	1	1.25	1.1	1.4
B	1	1.2	1.53	1.13
C	1.12	1	1.24	1.41
D	1.08	1	1.04	1.04

Step-2: Find the minimum cost of each column and then divide each element of the column by its minimum cost.

	1	2	3	4
A	1	1.25	1.06	1.35
B	1	1.2	1.47	1.09
C	1.12	1	1.19	1.36
D	1.08	1	1	1

Step-3: Make initial assignment.

	1	2	3	4
A	1	1.25	1.06	1.35
B	1	1.2	1.47	1.09
C	1.12	1	1.19	1.36
D	1.08	1	1	1

Here, 2nd row and 4th column do not have any assignment. Thus the solution is not optimum and we go to next.

Step-4: Find out the smallest number which does not have any line passing through it. Select the smallest number to divide from all the numbers that do not have any lines passing through them and other numbers covered by lines remain unchanged.

	1	2	3	4
A	1	1.18	1	1.27
B	1	1.13	1.39	1.03
C	1.12	1	1.19	1.36
D	1.08	1	1	1

Here we see that all ones are either assigned or crossed out. That is, the total assigned one's is 4 which is equal to the number of rows or columns.

The optimum assignment is: $A \to 3,\ B \to 1,\ C \to 2,\ D \to 4.$

$\therefore$ The total minimum assignment cost = $22 + 15 + 17 + 24 = Rs. 78$

(iii) New Approach Assignment (NAA) Method.

Consider the problem of assigning four jobs to four persons. The assignment cost are given below.

Person	Job			
	1	2	3	4
A	20	25	22	28
B	15	18	23	17
C	19	17	21	24
D	25	23	24	24

Step-1: Find the minimum element of each row and subtract it from each element in that row. Then the reduced matrix is as follows:

	1	2	3	4
A	0	5	2	8
B	0	3	8	2
C	2	0	4	7
D	2	0	1	1

Step-2: Now add 1 to all element.

	1	2	3	4
A	1	6	3	9
B	1	4	9	3
C	3	1	5	8
D	3	1	2	2

Step-3: Now make initial assignment.

	1	2	3	4
A	1	6	3	9
B	1	4	9	3
C	3	1	5	8
D	3	1	2	2

Here, 3rd and 4th column do not have any assignment. Thus the solution is not optimum and we go to next.

Step-4: Select the smallest number of the reduced matrix not covered by the lines. Divide all uncovered number by this smallest number. Other numbers covered by lines remain unchanged. Then make assignment again.

	1	2	3	4
A	1	6	1.5	4.5
B	1	4	4.5	1.5
C	3	1	2.5	4
D	3	1	1	1

Here also 4th column do not have any assignment. Thus the solution is not optimum and we go to next.

Step-5: Repeat the steps (3) and (4).

	1	2	3	4
A	1	4	1	3
B	1	2.7	3	1
C	3	1	2.5	4
D	3	1	1	1

Here we see that all ones are either assigned or crossed out. That is, the total assigned one's is 4 which is equal to the number of rows or columns.

The optimum assignment is: $A \rightarrow 1,\ B \rightarrow 4,\ C \rightarrow 2,\ D \rightarrow 3.$

∴ The total minimum assignment cost = $20 + 17 + 17 + 24 = Rs.78$

Comparison of Optimal Values of three Methods

Example	HA- method	MOA-method	NAA- method	Optimum
1	78	78	78	78

Problem 2: Solve the assignment problem using
(i) Hungarian Assignment (HA) Method.
(ii) Matrix One's Assignment Method (MOA) Method.
(iii) New Approach Assignment (NAA) Method.

Consider the problem of assigning five jobs to five persons. The assignment cost are given below.

Persons	Jobs				
	1	2	3	4	5
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

Determine the optimum assignment schedule and minimum assignment cost.

Solution:
(i) Hungarian Assignment (HA) Method.
Consider the problem of assigning five jobs to five persons. The assignment cost are given below.

Persons	Jobs				
	1	2	3	4	5
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

Step-1: Select the minimum element in each row and subtract this element from every element in that row.

	1	2	3	4	5
A	7	3	1	5	0
B	0	9	5	5	4
C	1	6	7	0	3
D	4	3	1	0	3
E	4	0	3	4	0

Step-2: Select the minimum element in each column and subtract this element from every element in that column.

	1	2	3	4	5
A	7	3	0	5	0
B	0	9	4	5	4
C	1	6	6	0	3
D	4	3	0	0	3
E	4	0	2	4	0

Step-3: Make initial assignment.

	1	2	3	4	5
A	7	3	~~0~~	5	[0]
B	[0]	9	4	5	4
C	1	6	6	[0]	3
D	4	3	[0]	~~0~~	3
E	4	[0]	2	4	~~0~~

Here we see that all zeros are either assigned or crossed out. That is, the total assigned zero's is 5 which is equal to the number of rows or columns.

The optimum assignment is: $A \to 5,\ B \to 1,\ C \to 4,\ D \to 3, E \to 2.$

∴ The total minimum assignment cost = $1 + 0 + 2 + 1 + 5 = Rs. 9$

(ii) Matrix One's Assignment Method (MOA) Method.
Consider the problem of assigning five jobs to five persons. The assignment costs are given below.

Persons	Jobs				
	1	2	3	4	5
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

Step-1: Find the minimum cost of each row and then divide each element of each row of the matrix by its minimum cost.

	1	2	3	4	5
A	8	4	2	6	1
B	0	0	0	0	0
C	1.5	4	4.5	1	3
D	0	0	0	0	0
E	1.8	1	1.6	1.8	1

(Since 2nd & 4th rows minimum cost 0 we divide in that rows after we get indeterminate and infinite values, that is simply take it as 0).

Step-2: Find the minimum cost of each column and then divide each element of the column by its minimum cost.

	1	2	3	4	5
A	0	0	0	0	0
B	0	0	0	0	0
C	0	0	0	0	0
D	0	0	0	0	0
E	0	0	0	0	0

Here we see that there is no one's term to assign in the above matrix. So we can't solve this problem by MOA-method.

(iii) New Approach Assignment (NAA) Method.
Consider the problem of assigning five jobs to five persons. The assignment costs are given below.

Persons	Jobs				
	1	2	3	4	5
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

Step-1: Find the minimum element of each row and subtract it from each element in that row. Then the reduced matrix is as follows:

	1	2	3	4	5
A	7	3	1	5	0
B	0	9	5	5	4
C	1	6	7	0	4
D	4	3	1	0	3
E	4	0	3	4	0

Step-2: Now add 1 to all element.

	1	2	3	4	5
A	8	4	2	6	1
B	1	10	6	6	5
C	2	7	8	1	5
D	5	4	2	1	4
E	5	1	4	5	1

Step-3: Now make initial assignment.

	1	2	3	4	5
A	8	4	2	6	[1]
B	[1]	10	6	6	5
C	2	7	8	[1]	5
D	5	4	2	~~1~~	4
E	5	[1]	4	5	~~1~~

Here, 3rd row and 3rd column do not have any assignment. Thus the solution is not optimum and we go to next.

Step-4: Select the smallest number of the reduced matrix not covered by the lines. Divide all uncovered number by this smallest number. Other numbers covered by lines remain unchanged. Then make assignment again.

	1	2	3	4	5
A	8	4	2	6	[1]
B	[1]	10	6	6	5
C	~~1~~	3.5	4	[1]	2.5
D	2.5	2	[1]	~~1~~	2
E	5	[1]	4	5	~~1~~

Here we see that all ones are either assigned or crossed out. That is, the total assigned one's is 5 which is equal to the number of rows or columns.

The optimum assignment is : $A \to 5, B \to 1,\ C \to 4,\ D \to 3, E \to 2.$

∴ The total minimum assignment cost = $1 + 0 + 2 + 1 + 5 = Rs. 9$

Comparison of Optimal Values of three Methods

Example	HA- method	MOA-method	NAA- method	Optimum
2	9	-	9	9

Here new approach assignment method is introduced for solving assignment problem. Also an above examples using the new approach assignment method and two existing methods is examined and the optimal solutions are compared among three methods and the optimal solutions by three methods are same result.

Therefore we conclude that this New Approach Assignment Method is effective for solving assignment problem.

CONCLUSION

In this paper the algorithm for HA-method, MOA-method and NAA-method are used to solve the given assignment problem in three different ways. And we get the optimal solution which is same as the optimal solutions in our three assumption methods. Therefore this paper introduces a different approach (ie, NAA-method) which is easy to solve assignment problem.

REFERENCES

1. Humayra Dil Afroz, Dr.Mohammad Anwar Hossen, "New Proposed Method for Solving Assignment Problem and Comparative Study with the Existing Methods", Volume 13, Issue 2 Ver. IV (Mar.-Apr.2017), PP 84-88.
2. M.S. Bazarra, John J. Jarvis, Hanif D. Sherali, 2005, Linear programming and network flows.
3. D.F. Votaw, 1952, A. Orden, The perssonel assignment problem, Symposium on Linear Inequalities and Programming, SCOOP 10, US Air Force, pp. 155-163.
4. H.W. Kuhn, 1955, The Hungarian method for the assignment problem, Naval Research Logistics Quarterly 2 (1&2) 83-97 (original publication).
5. B.S. Goel, S.K. Mittal, 1982, Operations Research, Fifty Ed., 2405-2416.
6. Hadi Basirzadeh 2012, Applied Mathematical Sciences, 6(47), 2345-2355.
7. Hamdy A. Tsaha, 2007, Operations Research, an introduction, 8th Ed.
8. Singh S., Dubey G.C., Shrivastava R. (2012) ISRO Journal of Engineering, 2(8), 01-15.
9. Singh S. (2011) International Journal of Operation Research and Information System, 3(3), 87- 97.
10. Anshuman Sahu, Rudrajit Tapador, 2007, Solving the assignment problem using Genetic Algorithm And Simulated annealing, IJAM.
11. Shayle R. Searle, 2006, Matrix algebra useful for statistics, John Willey.
12. New proposed Method for Solving Assignment problem and Comparative Study with the Existing Methods.
13. Zimmermman H.J.(1996) Fuzzy Set Theory and its Applications, 3rd ed., Kluwer Academic, Boston.
14. Pentico D.W. (2007) European Journal of Operation Research, 176,774-793.
15. Turkensteen M., Ghosh D., Boris Goldengorin, Gerard Sierksma, 2008 European Journal of Operation Research, 189, 775-788.

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