# International Journal of Mathematical Archive-9(8), 2018, 44-58 IMAAvailable online through www.ijma.info ISSN 2229-5046 

# VEDIC VINCULUM PROBLEMS WHEN THERE ARISE CARRY NUMBERS 

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(Received On: 02-07-18; Revised \& Accepted On: 05-08-18)


#### Abstract

Vinculum makes the digits over five to less than five. So during the process of addition and multiplication the probability of appearance of carry number is very less. Many authors have worked on the problems of vinculum but they have not touched the problem when carry number appears. In the present study, we have tried to solve all those problems when carry numbers appear during the solution of vinculum problems.


Keywords: Vinculum, Vedic mathematics, Carry number.
Mathematics Subject Classification: 01A32, 97A30, 97A80.

## 1. INTRODUCTION

In Vedic Mathematics 'Vinculum' is an ingenious device to reduce single digit larger than 5. A digit larger than five creates problem in calculations by creating carry number. If all the digits are less than five then calculation becomes easier and probability of appearance of carry number is very less. With this idea the concept of vinculum was entered in Vedic mathematical calculations of numbers.

Kenneth R William (2003), in his book "Astronomical Applications of Vedic Mathematics" has not used carry number in the case of multiplication of numbers with vinculum digits rather he has changed the vinculum digits into nonvinculum and then performed the multiplication. Similar case is found in the book "vertically and crosswise" written by Nicholas, Williams and Pickles (1999).

Kenneth R William (2005), in his book "Vedic Mathematics Teacher's Manual" has mentioned the subtraction of two numbers and then he has changed the digits above number five into vinculum. But he has not performed subtraction of number which contains vinculum in their digits.

Garain and Kumar (2018) studied "Algebraic representation of Vinculum and generalized rule for Vinculum Structure. They framed the generalized rule for removing and imposing vinculum on the digit of a number.

As the concept of vinculum make a digit over 5 to less than 5 . So during the process of addition and multiplication, the chance of appearance of carry number is very less.

Now, we shall try to solve all those problem of vinculum in which chance of appearance of carry number is possible.
Here, we'll try to find addition, subtraction and multiplication in which vinculum is involved. Also, we'll deal the same problems by removing vinculum.

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## 2. SOME OPERATIONS OF NUMBERS HAVING VINCULUM IN THEIR DIGITS

(A) Addition:

Type - I

## Calculation with Vinculum

(i) Evaluate:

$$
\begin{array}{r}
4 \quad \overline{3} \\
+25 \\
\hline 6 \quad 2
\end{array}
$$

$\because \overline{3}=-3$ and $5+\overline{3}=5-3=2$
(ii) Evaluate:

$$
\begin{array}{r}
3 \overline{4} \\
+1 \\
\hline 4 \overline{2}
\end{array}
$$

By removing the vinculum in $4 \overline{2}$.

$$
\begin{aligned}
\therefore \quad 4 \overline{2} & =4-1 / 10-2 \\
& =38
\end{aligned}
$$

## Calculation by removing Vinculum

$$
\begin{array}{r}
4 \overline{3} \\
+25 \\
\hline
\end{array}
$$

By removing the vinculum in $4 \overline{3}$.
$\therefore 4 \overline{3}=4-1 / 10-3=37$
Then,

$$
\begin{array}{r}
37 \\
+25 \\
\hline 6 \quad 2
\end{array}
$$

$$
\begin{array}{r}
3 \overline{4} \\
+12 \\
\hline
\end{array}
$$

By removing the vinculum in $3 \overline{4}$.
$\therefore 3 \overline{4}=3-1 / 10-4=26$
Then,

$$
\begin{array}{r}
26 \\
+12 \\
\hline 38
\end{array}
$$

(iii) Evaluate:

$$
\begin{array}{rrr}
3 & 4 & \overline{2} \\
4 & 0 & 1 \\
\hline 7 & 4 & \overline{1}
\end{array}
$$

By removing the vinculum in $74 \overline{1}$.

$$
\begin{aligned}
\therefore \quad 74 \overline{1} & =7 / 4-1 / 10-1 \\
& =739
\end{aligned}
$$

$$
\begin{array}{r}
34 \\
4 \\
4
\end{array} 018
$$

By removing the vinculum in $34 \overline{2}$.
$\therefore 34 \overline{2}=3 / 4-1 / 10-2=338$
Then,

$$
\begin{array}{r}
33 \\
430 \\
+73
\end{array}
$$

(iv) Evaluate:

$$
\begin{array}{r}
3 \overline{4} \\
\hline 4 \\
+3 \\
\hline 7 \\
\hline
\end{array}
$$

By removing the vinculum in $7 \overline{1} 6$.

$$
\begin{aligned}
\therefore \quad 7 \overline{1} 6 & =7-1 / 10-1 / 6 \\
& =696
\end{aligned}
$$

$$
\begin{array}{r}
3 \overline{4} 4 \\
+432 \\
\hline
\end{array}
$$

By removing the vinculum in $3 \overline{4} 4$.
$\therefore 3 \overline{4} 4=3-1 / 10-4 / 4=264$
Then,

$$
\begin{array}{r}
264 \\
+432 \\
\hline 6966
\end{array}
$$

(v) Evaluate:

$$
\begin{aligned}
& 4 \overline{2} \overline{3} \\
& +\begin{array}{lll}
2 & 5 & 1 \\
\hline 6 & 3 \quad \overline{2}
\end{array}
\end{aligned}
$$

$\because \overline{3}+1=-3+1=-2=\overline{2}$ and $\overline{2}+5=-2+5=3$
By removing the vinculum in $63 \overline{2}$.

$$
\begin{aligned}
\therefore \quad 63 \overline{2} & =6 / 3-1 / 10-2 \\
& =628
\end{aligned}
$$

$$
\begin{array}{rrr}
4 & \overline{2} & \overline{3} \\
2 & 5 & 1 \\
\hline
\end{array}
$$

By removing the vinculum in $4 \overline{2} \overline{3}$.
$\therefore 4 \overline{2} \overline{3}=4-1 / 10-2 / \overline{3}=3 / 8 / \overline{3}$

$$
=3 / 8-1 / 10-3=377
$$

Then,

$$
\begin{array}{rrr}
3 & 7 & 7 \\
2 & 5 & 1 \\
\hline 6 & 28
\end{array}
$$

## Type - II

## Calculation with Vinculum

## Calculation by removing Vinculum

(i) Evaluate:

$$
\begin{array}{r}
7 \quad 2 \\
+8 \quad \overline{3} \\
\hline 15 \quad \overline{1}
\end{array}
$$

$\because \overline{3}=-3$ and $2+\overline{3}=2+(-3)=2-3=-1=\overline{1}$
By removing the vinculum in $15 \overline{1}$.
$\therefore 15 \overline{1}=1 / 5-1 / 10-1$

$$
=149
$$

(ii) Evaluate:

$$
\begin{array}{r}
89 \\
+5 \quad \overline{4} \\
\hline 135
\end{array}
$$

(iii) Evaluate:

$$
\begin{array}{rrr}
2 & 4 & 3 \\
+5 & 2 & \overline{4} \\
\hline 7 & 6 & \overline{1}
\end{array}
$$

$\because \overline{4}=-4$ and $3+\overline{4}=3+(-4)=3-4=-1=\overline{1}$
By removing the vinculum in $76 \overline{1}$.
$\therefore 76 \overline{1}=7 / 6-1 / 10-1$

$$
\begin{array}{r}
89 \\
+5 \quad \overline{4} \\
\hline
\end{array}
$$

By removing the vinculum in $5 \overline{4}$.
$\therefore 5 \overline{4}=5-1 / 10-4=46$
Then,

$$
\begin{array}{r}
89 \\
+46 \\
\hline 135
\end{array}
$$

$$
\begin{array}{r}
72 \\
+\quad \overline{3} \\
\hline
\end{array}
$$

By removing the vinculum in $8 \overline{3}$.
$\therefore 8 \overline{3}=8-1 / 10-3=77$
Then,

$$
\begin{array}{r}
72 \\
+77 \\
\hline 149
\end{array}
$$

Then,

$$
\begin{array}{r}
24 \\
5
\end{array} \quad 2 \quad 4
$$

By removing the vinculum in $52 \overline{4}$.
$\therefore 52 \overline{4}=5 / 2-1 / 10-4=516$
Then,

$$
\begin{array}{r}
2 \\
4
\end{array} 3
$$

(iv) Evaluate:

$$
\begin{array}{r}
72 \\
7 \\
+5 \\
\hline 12 \\
\hline
\end{array}
$$

By removing the vinculum in $12 \overline{2} 3$.
$\therefore 12 \overline{2} 3=1 / 2-1 / 10-2 / 3$

$$
=1183
$$

(v) Evaluate:

$$
\begin{array}{rrr}
8 & 9 & 2 \\
+5 & \overline{3} & \overline{4} \\
\hline 13 & 6 & \overline{2}
\end{array}
$$

By removing the vinculum in $136 \overline{2}$.

$$
\begin{aligned}
\therefore \quad 136 \overline{2} & =1 / 3 / 6-1 / 10-2 \\
& =1358
\end{aligned}
$$

$$
\begin{array}{r}
72 \\
\hline 5 \\
\hline
\end{array}
$$

By removing the vinculum in $5 \overline{4} 3$.
$\therefore 5 \overline{4} 3=5-1 / 10-4 / 3=463$
Then,

$$
\begin{array}{ccc}
7 & 2 & 0 \\
4 & 6 & 3 \\
\hline 11 & 8 & 3
\end{array}
$$

$$
\begin{array}{r}
8 \\
\hline
\end{array}
$$

By removing the vinculum in $5 \overline{3} \overline{4}$.

$$
\begin{aligned}
\therefore \quad 5 \overline{3} \overline{4} & =5-1 / 10-3 / \overline{4}=4 / 7 / \overline{4} \\
& =4 / 7-1 / 10-4=466
\end{aligned}
$$

Then,

$$
\begin{array}{r}
892 \\
+466 \\
\hline 13588
\end{array}
$$

## Type - III

Calculation with Vinculum
(i) Evaluate:

$$
\begin{array}{r}
6 \overline{2} \\
+5 \overline{4} \\
\hline 11 \overline{6}
\end{array}
$$

$\because \overline{2}+\overline{4}=-2+(-4)=-2-4=-6=\overline{6}$.
By removing the vinculum in $11 \overline{6}$.

$$
\begin{aligned}
\therefore \quad 11 \overline{6} & =1 / 1-1 / 10-6 \\
& =104
\end{aligned}
$$

## Calculation by removing Vinculum

$$
\begin{array}{r}
6 \quad \overline{2} \\
+5 \overline{4} \\
\hline
\end{array}
$$

By removing the vinculum in $6 \overline{2}$ and $5 \overline{4}$
$\therefore 6 \overline{2}=6-1 / 10-2=58$
and $5 \overline{4}=5-1 / 10-4=46$
Then,
(ii) Evaluate:

$$
\begin{array}{r}
9 \quad \overline{3} \\
\hline
\end{array}
$$

$\because \overline{2}+6=-2+6=4$ and $\overline{3}+\overline{4}=-3+(-4)=-3-4=-7=\overline{7}$
By removing the vinculum in $13 \overline{7} 4$
$\begin{aligned} \therefore 13 \overline{7} 4 & =1 / 3-1 / 10-7 / 4 \\ & =1234\end{aligned}$

$$
=1234
$$

$$
\begin{array}{r}
58 \\
+46 \\
\hline 104
\end{array}
$$

Then,

$$
\begin{array}{r}
9 \quad \overline{3} \quad \overline{2} \\
+4 \overline{4} 6 \\
\hline
\end{array}
$$

By removing the vinculum in $9 \overline{3} \overline{2}$ and $4 \overline{4} 6$
$\therefore 9 \overline{3} \overline{2}=9-1 / 10-3 / \overline{2}=8 / 7 / \overline{2}$

$$
=8 / 7-1 / 10-2=868
$$

and $4 \overline{4} 6=4-1 / 10-4 / 6=366$
Then,

$$
\begin{array}{r}
868 \\
+3666 \\
\hline 12 \quad 34
\end{array}
$$

(iii) Evaluate:

$$
\begin{array}{rcc}
6 & 2 & \overline{3} \\
+8 & \overline{4} & \overline{1} \\
\hline 14 & \overline{2} & \overline{4}
\end{array}
$$

By removing the vinculum in $14 \overline{24}$.

$$
\begin{aligned}
\therefore \quad 14 \overline{24} & =1 / 4-1 / 10-2 / \overline{4} \\
& =1 / 3 / 8 / \overline{4} \\
& =1 / 3 / 8-1 / 10-4 \\
& =1376
\end{aligned}
$$

(iv) Evaluate:

$$
\begin{array}{r}
8 \\
3
\end{array} 0
$$

By removing the vinculum in $11 \overline{1} \overline{4}$.

$$
\begin{aligned}
\therefore 11 \overline{1} \overline{4} & =1 / 1-1 / 10-1 / \overline{4} \\
& =1 / 0 / 9 / \overline{4} \\
& =1 / 0 / 9-1 / 10-4 \\
& =1086
\end{aligned}
$$

$$
\begin{array}{rrr}
6 & 2 & \overline{3} \\
8 & \overline{4} & \overline{1} \\
\hline
\end{array}
$$

By removing the vinculum in $62 \overline{3}$ and $8 \overline{4} \overline{1}$.

$$
\begin{aligned}
\therefore \quad 62 \overline{3} & =6 / 2-1 / 10-3=617 \\
\text { and } 8 \overline{4} \overline{1} & =8-1 / 10-4 / \overline{1}=7 / 6 / \overline{1} \\
& =7 / 6-1 / 10-1=759
\end{aligned}
$$

Then,

$$
\begin{array}{r}
617 \\
+759 \\
\hline 13766
\end{array}
$$

$$
\begin{aligned}
& 8 \quad \overline{3} \quad 0 \\
& +\begin{array}{l}
3 \quad 2 \quad \overline{4} \\
\hline
\end{array}
\end{aligned}
$$

By removing the vinculum in $8 \overline{3} 0$ and $32 \overline{4}$.
$\therefore 8 \overline{3} 0=8-1 / 10-3 / 0=770$
and $32 \overline{4}=3 / 2-1 / 10-4=316$
Then,

$$
\begin{array}{r}
7760 \\
+3166 \\
\hline 10866
\end{array}
$$

(v) Evaluate:

$$
\begin{array}{rrr}
2 & 4 & \overline{1} \\
4 & \overline{3} & 0 \\
\hline 6 & 1 & \overline{1}
\end{array}
$$

By removing the vinculum in $61 \overline{1}$.

$$
\begin{aligned}
\therefore \quad 61 \overline{1} & =6 / 1-1 / 10-1 \\
& =609
\end{aligned}
$$

$$
\begin{array}{r}
24 \\
4 \\
4 \\
\hline
\end{array}
$$

By removing the vinculum in $24 \overline{1}$
and $4 \overline{3} 0$.
$\therefore 24 \overline{1}=2 / 4-1 / 10-1=239$
and $4 \overline{3} 0=4-1 / 10-3 / 0=370$
Then,

$$
\begin{array}{r}
239 \\
+370 \\
\hline 6009
\end{array}
$$

## Calculation by removing Vinculum

$$
\begin{array}{rcc}
7 & 5 & \overline{3} \\
2 & \overline{4} & \overline{1} \\
9 & \overline{2} & \overline{4} \\
\hline
\end{array}
$$

By removing the vinculum in $75 \overline{3}$, $2 \overline{4} \overline{1}$ and $9 \overline{2} \overline{4}$.

By removing the vinculum in $18 \overline{1} \overline{8}$,
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$\therefore 18 \overline{1} \overline{8}=1 / 8-1 / 10-1 / \overline{8}$

$$
\begin{aligned}
& =1 / 7 / 9 / \overline{8} \\
& =1 / 7 / 9-1 / 10-8 \\
& =1782
\end{aligned}
$$

(ii) Evaluate:

$$
\begin{array}{rcc}
3 & \overline{4} & \overline{3} \\
6 & \overline{3} & 5 \\
8 & \overline{1} & \overline{4} \\
\hline 17 & \overline{8} & \overline{2}
\end{array}
$$

By removing the vinculum in $17 \overline{8} \overline{2}$.
$\therefore 17 \overline{8} \overline{2}=1 / 7-1 / 10-8 / \overline{2}$
$=1 / 6 / 2 / \overline{2}$
$=1 / 6 / 2-1 / 10-2$

$$
=1618
$$

$\therefore 75 \overline{3}=7 / 5-1 / 10-3=747$,
$2 \overline{4} \overline{1}=2-1 / 10-4 / \overline{1}=1 / 6 / \overline{1}$

$$
=1 / 6-1 / 10-1=159
$$

and $9 \overline{2} \overline{4}=9-1 / 10-2 / \overline{4}=8 / 8 / \overline{4}$ $=8 / 8-1 / 10-4=876$
Then,

$$
\begin{aligned}
& \begin{array}{lll}
7 & 4 & 7
\end{array} \\
& 159 \\
& +\begin{array}{cc}
8 & 76 \\
\hline 17 \quad 8 \quad 2
\end{array}
\end{aligned}
$$

$$
\begin{array}{rcc}
3 & \overline{4} & \overline{3} \\
6 & \overline{3} & 5 \\
+ & \overline{1} & \overline{4} \\
\hline
\end{array}
$$

By removing the vinculum in $3 \overline{4} \overline{3}$,
$6 \overline{3} 5$ and $8 \overline{1} \overline{4}$.
$\therefore 3 \overline{4} \overline{3}=3-1 / 10-4 / \overline{3}=2 / 6-1 / 10-3=257$
$6 \overline{3} 5=6-1 / 10-3 / 5=575$
and $8 \overline{1} \overline{4}=8-1 / 10-1 / \overline{4}=7 / 9-1 / 10-4$

$$
=786
$$

Then,
(iii) Evaluate:

$$
\begin{array}{rcc}
3 & \overline{4} & \overline{4} \\
7 & \overline{1} & \overline{3} \\
8 & \overline{3} & 2 \\
+5 & \overline{4} & \overline{3} \\
\hline 22 & \overline{2} & \overline{8}
\end{array}
$$

$\because \overline{4}+\overline{1}+\overline{3}+\overline{4}=-4+(-1)+(-3)+(-4)=-4-1-3-4$

$$
=-12=\overline{12}=\overline{1} \overline{2}
$$

Here, carry number $=\overline{1}$
$\because 3+7+8+5+\overline{1}=23+(-1)=23-1=22$
By removing the vinculum in $22 \overline{2} \overline{8}$.

$$
\begin{aligned}
\therefore \quad 22 \overline{2} \overline{8} & =2 / 2-1 / 10-2 / \overline{8} \\
& =2 / 1 / 8 / \overline{8} \\
& =2 / 1 / 8-1 / 10-8 \\
& =2172
\end{aligned}
$$

$$
\begin{array}{rrr}
2 & 5 & 7 \\
5 & 7 & 5 \\
7 & 8 & 6 \\
\hline 16 & 1 & 8
\end{array}
$$

$$
\begin{array}{rcc}
3 & \overline{4} & \overline{4} \\
7 & \overline{1} & \overline{3} \\
8 & \overline{3} & 2 \\
+5 & \overline{4} & \overline{3} \\
\hline
\end{array}
$$

By removing the vinculum in $3 \overline{4} \overline{4}$, $7 \overline{1} \overline{3}, 8 \overline{3} 2$ and $5 \overline{4} \overline{3}$.

$$
\begin{aligned}
\therefore 3 \overline{44} & =3-1 / 10-4 / \overline{4}=2 / 6-1 / 10-4=256 \\
7 \overline{1} \overline{3} & =7-1 / 10-1 / \overline{3}=6 / 9-1 / 10-3=687
\end{aligned}
$$

$8 \overline{3} 2=8-1 / 10-3 / 2=772$
and $5 \overline{4} \overline{3}=5-1 / 10-4 / \overline{3}=4 / 6-1 / 10-3$

$$
=457
$$

Then,

$$
\begin{array}{rrr}
2 & 5 & 6 \\
6 & 8 & 7 \\
7 & 7 & 2 \\
4 & 5 & 7 \\
\hline 21 & 7 & 2
\end{array}
$$

(iv) Evaluate:

$$
\begin{array}{rcccc}
2 & \overline{3} & 8 & \overline{1} & \overline{4} \\
4 & \overline{4} & 0 & \overline{3} & 6 \\
3 & \overline{2} & \overline{3} & 4 & \overline{3} \\
7 & \overline{4} & \overline{4} & \overline{4} & \overline{2} \\
\hline 15 & \overline{3} & 1 & \overline{4} & \overline{3}
\end{array}
$$

By removing the vinculum in $15 \overline{3} 1 \overline{4} \overline{3}$.

$$
\begin{aligned}
\therefore 15 \overline{3} 1 \overline{4} \overline{3} & =1 / 5-1 / 10-3 / 1-1 / 10-4 / \overline{3} \\
& =1 / 4 / 7 / 0 / 6 / \overline{3} \\
& =1 / 4 / 7 / 0 / 6-1 / 10-3 \\
& =147057
\end{aligned}
$$

| 2 | $\overline{3}$ | 8 | $\overline{1}$ | $\overline{4}$ |
| ---: | :---: | :---: | :---: | :---: |
| 4 | $\overline{4}$ | 0 | $\overline{3}$ | 6 |
| 3 | $\overline{2}$ | $\overline{3}$ | 4 | $\overline{3}$ |
| 7 | $\overline{4}$ | $\overline{4}$ | $\overline{4}$ | $\overline{2}$ |

By removing the vinculum in $2 \overline{3} 8 \overline{1} \overline{4}$,
$4 \overline{4} 0 \overline{3} 6,3 \overline{2} \overline{3} 4 \overline{3}$ and $7 \overline{4} \overline{44} \overline{2}$.
$\therefore 2 \overline{3} 8 \overline{1} \overline{4}=2-1 / 10-3 / 8-1 / 10-1 / \overline{4}$

$$
\ldots \quad=1 / 7 / 7 / 9-1 / 10-4=17786
$$

$$
4 \overline{4} 0 \overline{3} 6=4-1 / 10-4 / 0-1 / 10-3 / 6
$$

$$
=3 / 6 / \overline{1} / 7 / 6=3 / 6-1 / 10-1 / 7 / 6
$$

$$
=35976
$$

$$
3 \overline{2} \overline{3} 4 \overline{3}=3-1 / 10-2 / \overline{3} / 4-1 / 10-3
$$

$$
=2 / 8-1 / 10-3 / 3 / 7=27737
$$

and $7 \overline{4442}=7-1 / 10-4 / \overline{4} / \overline{4} / \overline{2}$
$=6 / 6-1 / 10-4 / \overline{4} / \overline{2}$
$=6 / 5 / 6-1 / 10-4 / \overline{2}$
$=6 / 5 / 5 / 6-1 / 10-2=65558$
Then,

$$
\begin{array}{rcccc}
1 & 7 & 7 & 8 & 6 \\
3 & 5 & 9 & 7 & 6 \\
2 & 7 & 7 & 3 & 7 \\
6 & 5 & 5 & 5 & 8 \\
\hline 14 & 7 & 0 & 5 & 7
\end{array}
$$

## (B) Subtraction:

## Type - I

## Calculation with Vinculum

## Calculation by removing Vinculum

(i) Evaluate:

$$
\begin{array}{rr}
5 & \overline{4} \\
2 & 8 \\
-2 & \overline{2}
\end{array}
$$

$\because \overline{4}-8=-4-8=-12=\overline{12}=\overline{1} \overline{2}$
Here, carry number $=\overline{1}$ and $5-2+\overline{1}=5-2-1=2$
By removing the vinculum in $2 \overline{2}$.
$\therefore 2 \overline{2}=2-1 / 10-2$

$$
\begin{array}{r}
46 \\
-28 \\
\hline 18
\end{array}
$$

$\therefore 5 \overline{4}=5-1 / 10-4=46$
Then,
(ii) Evaluate:
$4 \overline{3}$
$-\begin{array}{r}2 \quad 9 \\ \hline 1 \quad \overline{2}\end{array}$
$4 \overline{3}$
29

By removing the vinculum in $1 \overline{2}$.
$\therefore 1 \overline{2}=1-1 / 10-2$

$$
=08
$$

By removing the vinculum in $4 \overline{3}$.
$\therefore 4 \overline{3}=4-1 / 10-3=37$
Then,

$$
\begin{array}{rr}
3 & 7 \\
2 & 9 \\
\hline 0 & 8
\end{array}
$$

(iii) Evaluate:

$$
\begin{array}{rrr}
5 & 7 & \overline{2} \\
-1 & 3 & 9 \\
\hline 4 & 3 & \overline{1}
\end{array}
$$

By removing the vinculum in $43 \overline{1}$.
$\therefore 43 \overline{1}=4 / 3-1 / 10-1$

$$
=429
$$

$$
\begin{array}{rrr}
5 & 7 & \overline{2} \\
1 & 3 & 9 \\
\hline
\end{array}
$$

By removing the vinculum in $57 \overline{2}$.
$\therefore 57 \overline{2}=5 / 7-1 / 10-2=568$
Then,

$$
\begin{array}{rrr}
5 & 6 & 8 \\
-1 & 3 & 9 \\
\hline 4 & 2 & 9
\end{array}
$$

(iv) Evaluate:

$$
\begin{array}{rrr}
5 & \overline{4} & 1 \\
3 & 9 & 4 \\
\hline 1 & \overline{3} & \overline{3}
\end{array}
$$

By removing the vinculum in $1 \overline{3} \overline{3}$.

$$
\begin{aligned}
\therefore \quad 1 \overline{3} \overline{3} & =1-1 / 10-3 / \overline{3}=0 / 7-1 / 10-3 \\
& =067
\end{aligned}
$$

$$
\begin{array}{rrr}
5 & \overline{4} & 1 \\
3 & 9 & 4 \\
\hline
\end{array}
$$

By removing the vinculum in $5 \overline{4} 1$.
$\therefore 5 \overline{4} 1=5-1 / 10-4 / 1=461$
Then,

$$
\begin{array}{rrr}
4 & 6 & 1 \\
3 & 9 & 4 \\
\hline 0 & 6 & 7
\end{array}
$$

(v) Evaluate:

$$
\begin{array}{rcc}
9 & \overline{3} & \overline{2} \\
-4 & 8 & 0 \\
\hline 4 & \overline{1} & \overline{2}
\end{array}
$$

$\because \overline{2}-0=-2-0=-2=\overline{2}$ and $\overline{3}-8=-3-8=-11=\overline{11}=\overline{1} \overline{1}$
By removing the vinculum in $4 \overline{1} \overline{2}$.
$\therefore 4 \overline{1} \overline{2}=4-1 / 10-1 / \overline{2}=3 / 9-1 / 10-2$

$$
=388
$$

$9 \quad \overline{3} \quad \overline{2}$
$4 \quad 8 \quad 0$

By removing the vinculum in $9 \overline{3} \overline{2}$.
$\therefore 9 \overline{3} \overline{2}=9-1 / 10-3 / \overline{2}=8 / 7 / \overline{2}$

$$
=8 / 7-1 / 10-2=868
$$

Then,

$$
\begin{array}{rrr}
8 & 6 & 8 \\
-4 & 8 & 0 \\
\hline 3 & 8 & 8
\end{array}
$$

Type - II

## Calculation with Vinculum

(i) Evaluate:

$$
\begin{array}{rr}
7 & 8 \\
-3 & 4 \\
\hline 5 & 2
\end{array}
$$

$\because \overline{4}=-4$ and $8-\overline{4}=8-(-4)=8+4=12$
(ii) Evaluate:

$$
\begin{array}{rrr}
7 & 8 & 9 \\
-3 & 5 & \overline{4} \\
\hline 4 & 4 & 3
\end{array}
$$

## Calculation by removing Vinculum

$$
\begin{array}{rr}
7 & 8 \\
3 & \overline{4} \\
\hline
\end{array}
$$

By removing the vinculum in $3 \overline{4}$.
$\therefore 3 \overline{4}=3-1 / 10-4=26$
Then,

$$
\begin{array}{rr}
7 & 8 \\
-2 & 6 \\
\hline 5 & 2
\end{array}
$$

By removing the vinculum in $35 \overline{4}$.
$\therefore 35 \overline{4}=3 / 5-1 / 10-4=346$
Then,

$$
\begin{array}{rrr}
7 & 8 & 9 \\
-3 & 4 & 6 \\
\hline 4 & 4 & 3
\end{array}
$$

(iii) Evaluate:

$$
\begin{array}{rrr}
5 & 8 & 2 \\
-1 & \overline{4} & 6 \\
\hline 5 & 2 & \overline{4}
\end{array}
$$

By removing the vinculum in $52 \overline{4}$.
$\therefore 52 \overline{4}=5 / 2-1 / 10-4$

$$
=516
$$

$$
\begin{array}{rrr}
5 & 8 & 2 \\
1 & \overline{4} & 6 \\
\hline
\end{array}
$$

By removing the vinculum in $1 \overline{4} 6$.
$\therefore 1 \overline{4} 6=1-1 / 10-4 / 6=066$
Then,

$$
\begin{array}{rrr}
5 & 8 & 2 \\
-0 & 6 & 6 \\
\hline 5 & 1 & 6
\end{array}
$$

(iv) Evaluate:

$$
\begin{array}{rrr}
8 & 9 & 2 \\
-5 & \overline{3} & \overline{4} \\
\hline 4 & 2 & 6
\end{array}
$$

By removing the vinculum in $5 \overline{3} \overline{4}$.

$$
\begin{aligned}
& \therefore 5 \overline{3} \overline{4}=5-1 / 10-3 / \overline{4}=4 / 7 / \overline{4} \\
& =4 / 7-1 / 10-4=466
\end{aligned}
$$

Then,

$$
\begin{array}{rrr}
8 & 9 & 2 \\
4 & 6 & 6 \\
\hline 4 & 2 & 6
\end{array}
$$

Type - III

## Calculation with Vinculum

(i) Evaluate:

$$
\begin{array}{rr}
9 & \overline{2} \\
-5 & \overline{4} \\
\hline 4 & 2
\end{array}
$$

$\because \overline{2}-\overline{4}=-2-(-4)=-2+4=2$.
(ii) Evaluate:

$$
\begin{array}{ccc}
9 & \overline{3} & \overline{2} \\
4 & \overline{4} & 6 \\
\hline 5 & 1 & \overline{8}
\end{array}
$$

$\because \overline{2}-6=-2-6=-8=\overline{8}$ and $\overline{3}-\overline{4}=-3-(-4)=-3+4=1$
By removing the vinculum in $51 \overline{8}$
$\therefore 51 \overline{8}=5 / 1-1 / 10-8$

$$
=502
$$

## Calculation by removing Vinculum

$$
\begin{array}{rr}
9 & \overline{2} \\
5 & \overline{4} \\
\hline
\end{array}
$$

By removing the vinculum in $9 \overline{2}$ and $5 \overline{4}$
$\therefore 9 \overline{2}=9-1 / 10-2=88$
and $5 \overline{4}=5-1 / 10-4=46$
Then,

$$
\begin{array}{rr}
8 & 8 \\
-4 & 6 \\
\hline 4 & 2
\end{array}
$$

$$
\begin{array}{rrr}
9 & \overline{3} & \overline{2} \\
4 & \overline{4} & 6 \\
\hline
\end{array}
$$

By removing the vinculum in $9 \overline{3} \overline{2}$ and $4 \overline{4} 6$
$\therefore 9 \overline{3} \overline{2}=9-1 / 10-3 / \overline{2}=8 / 7 / \overline{2}$

$$
=8 / 7-1 / 10-2=868
$$

and $4 \overline{4} 6=4-1 / 10-4 / 6=366$
Then,

$$
\begin{array}{rrr}
8 & 6 & 8 \\
3 & 6 & 6 \\
\hline 5 & 0 & 2
\end{array}
$$

(iii) Evaluate

$$
-\begin{array}{rcc}
6 & 2 & \overline{3} \\
3 & \overline{4} & \overline{1} \\
\hline 3 & 6 & \overline{2}
\end{array}
$$

By removing the vinculum in $36 \overline{2}$.
$\therefore 36 \overline{2}=3 / 6-1 / 10-2$

$$
=358
$$

$$
\begin{array}{ccc}
6 & 2 & \overline{3} \\
3 & \overline{4} & \overline{1} \\
\hline
\end{array}
$$

By removing the vinculum in $62 \overline{3}$
and $3 \overline{4} \overline{1}$.
$\therefore 62 \overline{3}=6 / 2-1 / 10-3=617$
and $3 \overline{4} \overline{1}=3-1 / 10-4 / \overline{1}=2 / 6 / \overline{1}$

$$
=2 / 6-1 / 10-1=259
$$

Then,

$$
\begin{array}{rrr}
6 & 1 & 7 \\
-2 & 5 & 9 \\
\hline 3 & 5 & 8
\end{array}
$$

(iv) Evaluate:

$$
\begin{array}{rrr}
9 & \overline{3} & 0 \\
-7 & 4 & \overline{4} \\
\hline 2 & \overline{7} & 4
\end{array}
$$

By removing the vinculum in $2 \overline{7} 4$.

$$
\begin{aligned}
\therefore 2 \overline{7} 4= & 2-1 / 10-7 / 4 \\
& =134
\end{aligned}
$$

and $74 \overline{4}=7 / 4-1 / 10-4=736$
Then,

$$
\begin{array}{rrr}
8 & 7 & 0 \\
7 & 3 & 6 \\
\hline 1 & 3 & 4
\end{array}
$$

(v) Evaluate:

$$
\begin{array}{rcc}
7 & \overline{4} & \overline{1} \\
-4 & \overline{3} & \overline{2} \\
\hline 3 & \overline{1} & 1
\end{array}
$$

By removing the vinculum in $3 \overline{1} 1$.

$$
\begin{aligned}
\therefore 3 \overline{1} 1 & =3-1 / 10-1 / 1 \\
& =291
\end{aligned}
$$

$\begin{array}{lll}7 & \overline{4} & \overline{1}\end{array}$
$4 \quad \overline{3} \quad \overline{2}$

By removing the vinculum in $7 \overline{4} \overline{1}$ and $4 \overline{3} \overline{2}$.
$\therefore 7 \overline{4} \overline{1}=7-1 / 10-4 / \overline{1}=6 / 6-1 / 10-1=659$
and $4 \overline{3} \overline{2}=4-1 / 10-3 / \overline{2}=3 / 7-1 / 10-2$

$$
=368
$$

Then,

$$
\begin{array}{rrr}
6 & 5 & 9 \\
-3 & 6 & 8 \\
\hline 2 & 9 & 1
\end{array}
$$

(ii) Evaluate:
$\because \overline{3} \times \overline{4}=-3 \times(-4)=12$
By removing the vinculum in
$\begin{aligned} \therefore 44 \overline{7} 2 & =4 / 4-1 / 10-7 / 2 \\ & =4332\end{aligned}$

$$
=4332
$$

## (C) Multiplication:

## Calculation with Vinculum

(i) Evaluate:

$$
\begin{array}{r}
3 \quad \overline{4} \\
\times \quad 3 \\
\hline 8 \quad \overline{2}
\end{array}
$$

$\because 3 \times \overline{4}=3 \times-4=-12=\overline{12}=\overline{1} \overline{2}$
and $3 \times 3+\overline{1}=9+(-1)=9-1=8$
By removing the vinculum in $8 \overline{2}$.
$\therefore 8 \overline{2}=8-1 / 10-2=78$

$$
\begin{aligned}
& 8 \overline{4} \\
& \begin{array}{r}
\times 6 \quad \overline{3} \\
\hline \overline{2} \overline{3} 2
\end{array} \\
& \begin{array}{cccc}
4 & 6 & \overline{4} & 0 \\
\hline 4 & 4 & \overline{7} & 2
\end{array}
\end{aligned}
$$

## Calculation by removing Vinculum

$$
\begin{array}{r}
34 \\
\times \quad 3 \\
\hline
\end{array}
$$

By removing the vinculum in $3 \overline{4}$
$\therefore 3 \overline{4}=3-1 / 10-4=26$
Then,

$$
\begin{array}{r}
26 \\
\times \quad 3 \\
\hline 78
\end{array}
$$

$$
\begin{array}{r}
8 \overline{4} \\
\times \quad 63 \\
\hline
\end{array}
$$

By removing the vinculum in $8 \overline{4}$ and $6 \overline{3}$
$\therefore 8 \overline{4}=8-1 / 10-4=76$
and $6 \overline{3}=6-1 / 10-3=57$
Then,

$$
\begin{array}{r}
76 \\
\\
\times \quad 5 \quad 7 \\
\hline 532 \\
3830 \\
\hline 4332
\end{array}
$$

(iii) Evaluate:

| 3 | 2 | $\overline{4}$ |
| :---: | :---: | :---: |
| $\times$ | 3 | $\overline{2}$ |
| $\overline{6}$ | $\overline{4}$ | 8 |

$$
\begin{array}{rll}
3 & 2 & \overline{4} \\
\times & 3 & \overline{2}
\end{array}
$$

By removing the vinculum in $32 \overline{4}$ and $3 \overline{2}$ 。
$\therefore 32 \overline{4}=3 / 2-1 / 10-4=316$
and $3 \overline{2}=3-1 / 10-2=28$ Then,

$$
\begin{aligned}
& 316 \\
& \begin{array}{r}
\times 28 \\
\hline 2528
\end{array} \\
& \begin{array}{llll}
6 & 3 & 2 & 0 \\
\hline 8 & 8 & 4 & 8
\end{array}
\end{aligned}
$$

(iv) Evaluate:

> |  |  | 5 | 4 | 3 |
| :--- | :--- | :--- | :--- | :--- |
|  |  | $\times$ | 5 | $\overline{4}$ |
|  | $\overline{1}$ | $\overline{9}$ | 5 | $\overline{2}$ |
| 2 | 4 | $\overline{9}$ | 5 | 0 |
| 2 | 2 | $\overline{7}$ | 0 | $\overline{2}$ |

By removing the vinculum in $22 \overline{7} 0 \overline{2}$.
$\begin{aligned} \therefore 22 \overline{7} 0 \overline{2} & =2 / 2-1 / 10-7 / 0-1 / 10-2 \\ & =2 / 1 / 3 / \overline{1} / 8 \\ & =2 / 1 / 3-1 / 10-1 / 8 \\ & =21298\end{aligned}$

$$
\begin{aligned}
& =2 / 1 / 3 / \overline{1} / 8 \\
& =2 / 1 / 3-1 / 10 \\
& =21298
\end{aligned}
$$

$$
\begin{array}{r}
54 \\
\times \quad 5 \\
\hline
\end{array}
$$

By removing the vinculum in $5 \overline{4} 3$ and $5 \overline{4}$
$\therefore 5 \overline{4} 3=5-1 / 10-4 / 3=463$
and $5 \overline{4}=5-1 / 10-4=46$
Then,

|  |  | 4 | 6 | 3 |
| :--- | :--- | :--- | :--- | :--- |
|  |  | $\times$ | 4 | 6 |
|  | 2 | 7 | 7 | 8 |
| 1 | 8 | 5 | 2 | 0 |
| 2 | 1 | 2 | 9 | 8 |

$$
\begin{array}{r}
34 \overline{2} \\
\times \quad 4 \overline{1}
\end{array}
$$

By removing the vinculum in $34 \overline{2}$ and $4 \overline{1} \overline{4}$
$\therefore 34 \overline{2}=3 / 4-1 / 10-2=338$
and $4 \overline{1} \overline{4}=4-1 / 10-1 / \overline{4}=3 / 9-1 / 10-4$
Then,
$\left.\begin{array}{lllll} & & 3 & 3 & 8 \\ & & 3 & 8 & 6 \\ \hline & 2 & 0 & 2 & 8 \\ & 2 & 7 & 0 & 4 \\ 1 & 0 & 1 & 4 & 0 \\ \hline 1 & 3 & 0 & 4 & 6\end{array}\right)$

## 3. VERTICALLY AND CROSSWISE VEDIC METHOD FOR MULTIPLICATION IN VINCULUM NUMBERS

For vertically and crosswise product of $m \bar{n}$ and $p \bar{q} \bar{r}$, we have to make number of digits of two number to be same. For this we include 0 (Zero) at hundreds place of $m \bar{n}$ and $m \bar{n}$ is written as $0 m \bar{n}$. Multiplication is written as -

$$
\begin{array}{r}
p \quad \bar{q} \quad \bar{r} \\
\times \quad 0 \quad m \quad \bar{n} \\
\hline
\end{array}
$$

Multiplication process is as follow
(i) Step-1. $\quad \begin{array}{lll} & p & \bar{q} \\ & 0 & m\end{array}\binom{\bar{r}}{\bar{n}}=\bar{r} \times \bar{n}=\mathrm{ab}$

$$
\mathrm{a}, \mathrm{~b} \in \mathrm{~N} \cup\{0\}, \mathrm{ab}=1 \times \mathrm{b}+10 \times \mathrm{a}
$$

Fig.-1
Here, $a$ be the carry number and $b$ is included in the result.
$\begin{aligned} & \text { (ii) Step - 2. } p\binom{\bar{q} \bar{r}}{m \bar{n}}=\bar{n} \times \bar{q}+\bar{r} \times \mathrm{m}=\mathrm{cd} \\ & \text { and } \mathrm{cd}+\mathrm{a}=\mathrm{ef}\end{aligned}$
Here, e be the carry number and f is included in the result left to b of step -1 .
(iii) Step - 3. $\quad\left(\begin{array}{ccc}p & \bar{q} & \bar{r} \\ 0 & m & \bar{n}\end{array}\right)=\bar{n} \times \mathrm{p}+\bar{r} \times 0+\bar{q} \times \bar{n}=\mathrm{gh}$
and $\mathrm{gh}+\mathrm{e}=\mathrm{ij}$
Here, i be the carry number and j is included in the result left to f of step -2 .
(iv) Step - 4. $\quad\left(\begin{array}{cc}p & \bar{q} \\ 0 & m\end{array}\right) \overline{\bar{r}}=\mathrm{m} \times \mathrm{p}+\bar{q} \times 0=\mathrm{kl}$

$$
\text { and } \mathrm{kl}+\mathrm{i}=\mathrm{st}
$$

Fig. -2


Fig. -3


Fig.-4
-

Fig.-5

Here, s is included in the result left to t of step -4 .
Hence,

$$
\therefore m \bar{n} \times p \bar{q} \bar{r}=\text { stifb }
$$

Example - Evaluate: $\quad 6 \overline{3} \times 32 \overline{4}$
We write

$$
\begin{array}{r}
3 \overline{2} \overline{4} \\
\times \quad 06 \overline{3} \\
\hline
\end{array}
$$

(i) Step -1 .

$$
\left.\begin{array}{ll}
3 & \overline{2}\left(\begin{array}{l}
\overline{4} \\
0
\end{array}\right. \\
6
\end{array}\right)=\overline{3} \times \overline{3}=12
$$

Here, 1 be the carry number and 2 is included in the result.
Fig. 6
(ii) Step - 2. $\begin{array}{ll}3 \\ 0\end{array}\left(\begin{array}{ll}\overline{2} & \overline{4} \\ 6 & \overline{3}\end{array}\right)=\overline{3} \times \overline{2}+\overline{4} \times 6=\overline{18}$

$$
\text { and } \overline{18}+1=\overline{17}
$$

Here, $\overline{1}$ be the carry number and $\overline{7}$ is included in the result left to 2 of step -1 .
(iii) Step - 3. $\quad\left(\begin{array}{lll}3 & \overline{2} & \overline{4} \\ 0 & 6 & \overline{3}\end{array}\right)=\overline{3} \times 3+\overline{4} \times 0+6 \times \overline{2}=\overline{21}$

$$
\text { and } \overline{21}+\overline{1}=\overline{22}
$$

Here, $\overline{2}$ be the carry number and $\overline{2}$ is included in the result left to $\overline{7}$ of step -2 .
(iv) Step - 4. $\quad\left(\begin{array}{ll}3 & \overline{2} \\ 0 & 6\end{array}\right) \frac{\overline{4}}{\overline{3}}=6 \times 3+\overline{2} \times 0=18$

$$
\text { and } 18+\overline{2}=16
$$

Here, 1 be the carry number and 6 is included in the result left to $\overline{2}$ of step -3 .
(v) Step - 5. $\quad\binom{3}{0} \overline{\overline{2}} \overline{4}=3 \times 0=0$ and $0+1=1$

Fig.-9
$\rightarrow$

Fig. -7


Fig.-8


Fig.-10

Here, 1 is included in the result left to 6 of step -4 .
Hence,

$$
\therefore 6 \overline{3} \times 32 \overline{4}=16 \overline{2} \overline{7} 2
$$

By removing the vinculum in $16 \overline{2} \overline{7} 2$
$\therefore 16 \overline{2} \overline{7} 2=1 / 6-1 / 10-2 / \overline{7} / 2=1 / 5 / 8 / \overline{7} / 2=1 / 5 / 8-1 / 10-7 / 2=15732$.
In short we may calculate this multiplication with the help of above process as follow step -
(i)

(iv)

(ii)

(iii)

(v)


Fig.-11

$$
\begin{array}{r}
3 \overline{2} \overline{4} \\
\times 06 \quad \overline{3} \\
\hline 16 \overline{2} 2
\end{array}
$$

$$
\therefore \text { Answer }=16 \overline{2} \overline{7} 2
$$

By removing the vinculum in $16 \overline{2} \overline{7} 2$
$\therefore 16 \overline{2} \overline{7} 2=1 / 6-1 / 10-2 / \overline{7} / 2=1 / 5 / 8 / \overline{7} / 2=1 / 5 / 8-1 / 10-7 / 2=15732$.

## Multiplication with vinculum Method:

|  |  | 3 | $\overline{2}$ | $\overline{4}$ |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $\times$ | 6 | $\overline{3}$ |
|  |  | $\overline{9}$ | 7 | 2 |
| 1 | 7 | $\overline{4}$ | $\overline{4}$ | 0 |
| 1 | 6 | $\overline{3}$ | 3 | 2 |

By removing the vinculum in $16 \overline{3} 32$.
$\therefore 16 \overline{3} 32=1 / 6-1 / 10-3 / 3 / 2=15732$
Example - Evaluate: $\quad 8 \overline{4} \times 6 \overline{3}$
Multiplication process is done by following steps -
(i)

(ii)

(iii)

-     - 

Fig.-12

$$
\begin{array}{r}
8 \quad \overline{4} \\
\times 6 \overline{3} \\
\hline 44 \overline{7} 2 \\
\therefore \text { Answer }=44 \overline{7} 2 .
\end{array}
$$

## 4. CONCLUSION

We have performed addition, subtraction and multiplication of numbers with involvement of vinculum in which carry number appears. During the process by vertically and crosswise Vedic method it becomes simpler.

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[^0]
[^0]:    Source of support: Nil, Conflict of interest: None Declared.
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