VEDIC VINCULUM PROBLEMS WHEN THERE ARISE CARRY NUMBERS

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ABSTRACT

Vinculum makes the digits over five to less than five. So during the process of addition and multiplication the probability of appearance of carry number is very less. Many authors have worked on the problems of vinculum but they have not touched the problem when carry number appears. In the present study, we have tried to solve all those problems when carry numbers appear during the solution of vinculum problems.

Keywords: Vinculum, Vedic mathematics, Carry number.

Mathematics Subject Classification: 01A32, 97A30, 97A80.

1. INTRODUCTION

In Vedic Mathematics 'Vinculum' is an ingenious device to reduce single digit larger than 5. A digit larger than five creates problem in calculations by creating carry number. If all the digits are less than five then calculation becomes easier and probability of appearance of carry number is very less. With this idea the concept of vinculum was entered in Vedic mathematical calculations of numbers.

Kenneth R William (2003), in his book "Astronomical Applications of Vedic Mathematics" has not used carry number in the case of multiplication of numbers with vinculum digits rather he has changed the vinculum digits into non-vinculum and then performed the multiplication. Similar case is found in the book "vertically and crosswise" written by Nicholas, Williams and Pickles (1999).

Kenneth R William (2005), in his book "Vedic Mathematics Teacher's Manual" has mentioned the subtraction of two numbers and then he has changed the digits above number five into vinculum. But he has not performed subtraction of number which contains vinculum in their digits.

Garain and Kumar (2018) studied "Algebraic representation of Vinculum and generalized rule for Vinculum Structure. They framed the generalized rule for removing and imposing vinculum on the digit of a number.

As the concept of vinculum make a digit over 5 to less than 5. So during the process of addition and multiplication, the chance of appearance of carry number is very less.

Now, we shall try to solve all those problem of vinculum in which chance of appearance of carry number is possible.

Here, we'll try to find addition, subtraction and multiplication in which vinculum is involved. Also, we'll deal the same problems by removing vinculum.

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2. SOME OPERATIONS OF NUMBERS HAVING VINCULUM IN THEIR DIGITS

(A) Addition:

Type – I

Calculation with Vinculum

(i) Evaluate:

$$
\begin{array}{cc}
4 & \bar{3} \\
+\,2 & 5 \\
\hline
6 & 2
\end{array}
$$

$\because \ \bar{3} = -3$ and $5 + \bar{3} = 5 - 3 = 2$

(ii) Evaluate:

$$
\begin{array}{cc}
3 & \bar{4} \\
+\,1 & 2 \\
\hline
4 & \bar{2}
\end{array}
$$

By removing the vinculum in $4\bar{2}$.

$\therefore \ 4\bar{2} = 4\text{-}1/10\text{-}2$

$\quad\ = 38$

(iii) Evaluate:

$$
\begin{array}{ccc}
3 & 4 & \bar{2} \\
+\,4 & 0 & 1 \\
\hline
7 & 4 & \bar{1}
\end{array}
$$

By removing the vinculum in $74\bar{1}$.

$\therefore \ 74\bar{1} = 7/4\text{-}1/10\text{-}1$

$\quad\ = 739$

(iv) Evaluate:

$$
\begin{array}{ccc}
3 & \bar{4} & 4 \\
+\,4 & 3 & 2 \\
\hline
7 & \bar{1} & 6
\end{array}
$$

By removing the vinculum in $7\bar{1}6$.

$\therefore \ 7\bar{1}6 = 7\text{-}1/10\text{-}1/6$

$\quad\ = 696$

Calculation by removing Vinculum

$$
\begin{array}{cc}
4 & \bar{3} \\
+\,2 & 5 \\
\hline
\end{array}
$$

By removing the vinculum in $4\bar{3}$.

$\therefore \ 4\bar{3} = 4\text{-}1/10\text{-}3 = 37$

Then,

$$
\begin{array}{cc}
3 & 7 \\
+\,2 & 5 \\
\hline
6 & 2
\end{array}
$$

$$
\begin{array}{cc}
3 & \bar{4} \\
+\,1 & 2 \\
\hline
\end{array}
$$

By removing the vinculum in $3\bar{4}$.

$\therefore \ 3\bar{4} = 3\text{-}1/10\text{-}4 = 26$

Then,

$$
\begin{array}{cc}
2 & 6 \\
+\,1 & 2 \\
\hline
3 & 8
\end{array}
$$

$$
\begin{array}{ccc}
3 & 4 & \bar{2} \\
+\,4 & 0 & 1 \\
\hline
\end{array}
$$

By removing the vinculum in $34\bar{2}$.

$\therefore \ 34\bar{2} = 3/4\text{-}1/10\text{-}2 = 338$

Then,

$$
\begin{array}{ccc}
3 & 3 & 8 \\
+\,4 & 0 & 1 \\
\hline
7 & 3 & 9
\end{array}
$$

$$
\begin{array}{ccc}
3 & \bar{4} & 4 \\
+\,4 & 3 & 2 \\
\hline
\end{array}
$$

By removing the vinculum in $3\bar{4}4$.

$\therefore \ 3\bar{4}4 = 3\text{-}1/10\text{-}4/4 = 264$

Then,

$$
\begin{array}{ccc}
2 & 6 & 4 \\
+\,4 & 3 & 2 \\
\hline
6 & 9 & 6
\end{array}
$$

(v) Evaluate:

$$
\begin{array}{ccc}
4 & \overline{2} & \overline{3} \\
2 & 5 & 1 \\
\hline
6 & 3 & \overline{2} \\
\end{array} +
$$

∵ $\overline{3} + 1 = -3 + 1 = -2 = \overline{2}$ and $\overline{2} + 5 = -2 + 5 = 3$

By removing the vinculum in $63\overline{2}$.

∴ $63\overline{2} = 6/3-1/10-2$
     $= 628$

$$
\begin{array}{ccc}
4 & \overline{2} & \overline{3} \\
2 & 5 & 1 \\
\hline
\end{array} +
$$

By removing the vinculum in $4\overline{2}\overline{3}$.

∴ $4\overline{2}\overline{3} = 4\text{-}1/10\text{-}2/\overline{3} = 3/8/\overline{3}$
     $= 3/8\text{-}1/10\text{-}3 = 377$

Then,

$$
\begin{array}{ccc}
3 & 7 & 7 \\
2 & 5 & 1 \\
\hline
6 & 2 & 8 \\
\end{array} +
$$

Type – II

| **Calculation with Vinculum** | **Calculation by removing Vinculum** |

(i) Evaluate:

$$
\begin{array}{cc}
7 & 2 \\
8 & \overline{3} \\
\hline
15 & \overline{1} \\
\end{array} +
$$

∵ $\overline{3} = -3$ and $2 + \overline{3} = 2 + (-3) = 2 - 3 = -1 = \overline{1}$

By removing the vinculum in $15\overline{1}$.

∴ $15\overline{1} = 1/5-1/10-1$

     $= 149$

$$
\begin{array}{cc}
7 & 2 \\
8 & \overline{3} \\
\hline
\end{array} +
$$

By removing the vinculum in $8\overline{3}$.

∴ $8\overline{3} = 8\text{-}1/10\text{-}3 = 77$

Then,

$$
\begin{array}{cc}
7 & 2 \\
7 & 7 \\
\hline
14 & 9 \\
\end{array} +
$$

(ii) Evaluate:

$$
\begin{array}{cc}
8 & 9 \\
5 & \overline{4} \\
\hline
13 & 5 \\
\end{array} +
$$

$$
\begin{array}{cc}
8 & 9 \\
5 & \overline{4} \\
\hline
\end{array} +
$$

By removing the vinculum in $5\overline{4}$.

∴ $5\overline{4} = 5\text{-}1/10\text{-}4 = 46$

Then,

$$
\begin{array}{cc}
8 & 9 \\
4 & 6 \\
\hline
13 & 5 \\
\end{array} +
$$

(iii) Evaluate:

$$
\begin{array}{ccc}
2 & 4 & 3 \\
5 & 2 & \overline{4} \\
\hline
7 & 6 & \overline{1} \\
\end{array} +
$$

∵ $\overline{4} = -4$ and $3 + \overline{4} = 3 + (-4) = 3 - 4 = -1 = \overline{1}$

By removing the vinculum in $76\overline{1}$.

∴ $76\overline{1} = 7/6-1/10-1$

     $= 759$

$$
\begin{array}{ccc}
2 & 4 & 3 \\
5 & 2 & \overline{4} \\
\hline
\end{array} +
$$

By removing the vinculum in $52\overline{4}$.

∴ $52\overline{4} = 5/2-1/10-4 = 516$

Then,

$$
\begin{array}{ccc}
2 & 4 & 3 \\
5 & 1 & 6 \\
\hline
7 & 5 & 9 \\
\end{array} +
$$

(iv) Evaluate:

$$
\begin{array}{ccc}
7 & 2 & 0 \\
5 & \bar{4} & 3 \\
\hline
12 & \bar{2} & 3
\end{array}
$$

By removing the vinculum in $12\bar{2}3$.

$\therefore 12\bar{2}3 = 1/2\text{-}1/10\text{-}2/3$

$= 1183$

$$
\begin{array}{ccc}
7 & 2 & 0 \\
5 & \bar{4} & 3 \\
\hline
\end{array}
$$
+

By removing the vinculum in $5\bar{4}3$.

$\therefore 5\bar{4}3 = 5\text{-}1/10\text{-}4/3 = 463$

Then,

$$
\begin{array}{ccc}
7 & 2 & 0 \\
4 & 6 & 3 \\
\hline
11 & 8 & 3
\end{array}
$$

(v) Evaluate:

$$
\begin{array}{ccc}
8 & 9 & 2 \\
5 & \bar{3} & \bar{4} \\
\hline
13 & 6 & \bar{2}
\end{array}
$$

By removing the vinculum in $136\bar{2}$.

$\therefore 136\bar{2} = 1/3/6\text{-}1/10\text{-}2$

$=1358$

$$
\begin{array}{ccc}
8 & 9 & 2 \\
5 & \bar{3} & \bar{4} \\
\hline
\end{array}
$$
+

By removing the vinculum in $5\bar{3}\bar{4}$.

$\therefore 5\bar{3}\bar{4} = 5\text{-}1/10\text{-}3/\bar{4} = 4/7/\bar{4}$

$= 4/7\text{-}1/10\text{-}4 = 466$

Then,

$$
\begin{array}{ccc}
8 & 9 & 2 \\
4 & 6 & 6 \\
\hline
13 & 5 & 8
\end{array}
$$

Type – III
Calculation with Vinculum

(i) Evaluate:

$$
\begin{array}{cc}
6 & \bar{2} \\
5 & \bar{4} \\
\hline
11 & \bar{6}
\end{array}
$$

$\because \bar{2} + \bar{4} = \text{-}2 + (\text{-}4) = \text{-}2 - 4 = \text{-}6 = \bar{6}$.

By removing the vinculum in $11\bar{6}$.

$\therefore 11\bar{6} = 1/1\text{-}1/10\text{-}6$

$= 104$

(ii) Evaluate:

$$
\begin{array}{ccc}
9 & \bar{3} & \bar{2} \\
4 & \bar{4} & 6 \\
\hline
13 & \bar{7} & 4
\end{array}
$$

$\because \bar{2} + 6 = \text{-}2 + 6 = 4$ and $\bar{3} + \bar{4} = \text{-}3 + (\text{-}4) = \text{-}3 - 4 = \text{-}7 = \bar{7}$

By removing the vinculum in $13\bar{7}4$

$\therefore 13\bar{7}4 = 1/3\text{-}1/10\text{-}7/4$

$= 1234$

Calculation by removing Vinculum

$$
\begin{array}{cc}
6 & \bar{2} \\
5 & \bar{4} \\
\hline
\end{array}
$$
+

By removing the vinculum in $6\bar{2}$ and $5\bar{4}$

$\therefore 6\bar{2} = 6\text{-}1/10\text{-}2 = 58$

and $5\bar{4} = 5\text{-}1/10\text{-}4 = 46$

Then,

$$
\begin{array}{cc}
5 & 8 \\
4 & 6 \\
\hline
10 & 4
\end{array}
$$

$$
\begin{array}{ccc}
9 & \bar{3} & \bar{2} \\
4 & \bar{4} & 6 \\
\hline
\end{array}
$$
+

By removing the vinculum in $9\bar{3}\bar{2}$ and $4\bar{4}6$

$\therefore 9\bar{3}\bar{2} = 9\text{-}1/10\text{-}3/\bar{2} = 8/7/\bar{2}$

$= 8/7\text{-}1/10\text{-}2 = 868$

and $4\bar{4}6 = 4\text{-}1/10\text{-}4/6 = 366$

Then,

$$
\begin{array}{ccc}
8 & 6 & 8 \\
3 & 6 & 6 \\
\hline
12 & 3 & 4
\end{array}
$$

(iii) Evaluate:

$$
\begin{array}{r}
6 \quad 2 \quad \overline{3} \\
8 \quad \overline{4} \quad \overline{1} \\
\hline
+14 \quad \overline{2} \quad \overline{4}
\end{array}
$$

By removing the vinculum in $14\overline{2}\overline{4}$.

$\therefore \;\; 14\overline{2}\overline{4} = 1/4\text{-}1/10\text{-}2/\overline{4}$

$\qquad = 1/3/8/\overline{4}$

$\qquad = 1/3/8\text{-}1/10\text{-}4$

$\qquad = 1376$

(iv) Evaluate:

$$
\begin{array}{r}
8 \quad \overline{3} \quad 0 \\
3 \quad 2 \quad \overline{4} \\
\hline
+11 \quad \overline{1} \quad \overline{4}
\end{array}
$$

By removing the vinculum in $11\,\overline{1}\,\overline{4}$.

$\therefore \;\; 11\,\overline{1}\,\overline{4} = 1/1\text{-}1/10\text{-}1/\overline{4}$

$\qquad = 1/0/9/\overline{4}$

$\qquad = 1/0/9\text{-}1/10\text{-}4$

$\qquad = 1086$

(v) Evaluate:

$$
\begin{array}{r}
2 \quad 4 \quad \overline{1} \\
4 \quad \overline{3} \quad 0 \\
\hline
+6 \quad 1 \quad \overline{1}
\end{array}
$$

By removing the vinculum in $61\,\overline{1}$.

$\therefore \;\; 61\,\overline{1} = 6/1\text{-}1/10\text{-}1$

$\qquad = 609$

$$
\begin{array}{r}
6 \quad 2 \quad \overline{3} \\
8 \quad \overline{4} \quad \overline{1} \\
\hline
+\rule{2cm}{0.4pt}
\end{array}
$$

By removing the vinculum in $62\overline{3}$ and $8\overline{4}\,\overline{1}$.

$\therefore \;\; 62\overline{3} = 6/2\text{-}1/10\text{-}3 = 617$

and $8\overline{4}\,\overline{1} = 8\text{-}1/10\text{-}4/\,\overline{1} = 7/6/\,\overline{1}$

$\qquad = 7/6\text{-}1/10\text{-}1 = 759$

Then,

$$
\begin{array}{r}
6 \quad 1 \quad 7 \\
7 \quad 5 \quad 9 \\
\hline
+13 \quad 7 \quad 6
\end{array}
$$

$$
\begin{array}{r}
8 \quad \overline{3} \quad 0 \\
3 \quad 2 \quad \overline{4} \\
\hline
+\rule{2cm}{0.4pt}
\end{array}
$$

By removing the vinculum in $8\overline{3}0$ and $32\overline{4}$.

$\therefore \;\; 8\overline{3}0 = 8\text{-}1/10\text{-}3/0 = 770$

and $32\overline{4} = 3/2\text{-}1/10\text{-}4 = 316$

Then,

$$
\begin{array}{r}
7 \quad 7 \quad 0 \\
3 \quad 1 \quad 6 \\
\hline
+10 \quad 8 \quad 6
\end{array}
$$

$$
\begin{array}{r}
2 \quad 4 \quad \overline{1} \\
4 \quad \overline{3} \quad 0 \\
\hline
+\rule{2cm}{0.4pt}
\end{array}
$$

By removing the vinculum in $24\overline{1}$ and $4\overline{3}0$.

$\therefore \;\; 24\overline{1} = 2/4\text{-}1/10\text{-}1 = 239$

and $4\overline{3}0 = 4\text{-}1/10\text{-}3/0 = 370$

Then,

$$
\begin{array}{r}
2 \quad 3 \quad 9 \\
3 \quad 7 \quad 0 \\
\hline
+6 \quad 0 \quad 9
\end{array}
$$

Type – IV

Calculation with Vinculum	**Calculation by removing Vinculum**

(i) Evaluate:

$$
\begin{array}{r}
7 \quad 5 \quad \overline{3} \\
2 \quad \overline{4} \quad \overline{1} \\
9 \quad \overline{2} \quad \overline{4} \\
\hline
+18 \quad \overline{1} \quad \overline{8}
\end{array}
$$

$\because \;\; \overline{3}+\overline{1}+\overline{4} = \text{-}3 + (\text{-}1) + (\text{-}4) = \text{-}3 - 1 - 4 = \text{-}8 = \overline{8}$

By removing the vinculum in $18\,\overline{1}\,\overline{8}$,

$$
\begin{array}{r}
7 \quad 5 \quad \overline{3} \\
2 \quad \overline{4} \quad \overline{1} \\
9 \quad \overline{2} \quad \overline{4} \\
\hline
+\rule{2cm}{0.4pt}
\end{array}
$$

By removing the vinculum in $75\overline{3}$, $2\overline{4}\,\overline{1}$ and $9\overline{2}\overline{4}$.

∴ $18\bar{1}8$ = 1/8-1/10-1/$\bar{8}$

= 1/7/9/$\bar{8}$

= 1/7/9-1/10-8

= 1782

∴ $75\bar{3}$ = 7/5-1/10-3 = 747,

$24\bar{1}$ = 2-1/10-4/$\bar{1}$ = 1/6/$\bar{1}$

= 1/6-1/10-1 = 159

and $9\bar{2}4$ = 9-1/10-2/$\bar{4}$ = 8/8/$\bar{4}$

= 8/8-1/10-4 = 876

Then,

	7	4	7
	1	5	9
+	8	7	6
	17	8	2

(ii) Evaluate:

	3	$\bar{4}$	$\bar{3}$
	6	$\bar{3}$	5
	8	$\bar{1}$	$\bar{4}$
+	17	$\bar{8}$	$\bar{2}$

By removing the vinculum in $17\bar{8}\bar{2}$.

∴ $17\bar{8}\bar{2}$ = 1/7-1/10-8/$\bar{2}$

= 1/6/2/$\bar{2}$

= 1/6/2-1/10-2

= 1618

	3	$\bar{4}$	$\bar{3}$
	6	$\bar{3}$	5
	8	$\bar{1}$	$\bar{4}$
+			

By removing the vinculum in $3\bar{4}\bar{3}$,

$6\bar{3}5$ and $8\bar{1}\bar{4}$.

∴ $3\bar{4}\bar{3}$ = 3-1/10-4/$\bar{3}$ = 2/6-1/10-3 = 257

$6\bar{3}5$ = 6-1/10-3/5 = 575

and $8\bar{1}\bar{4}$ = 8-1/10-1/$\bar{4}$ = 7/9-1/10-4

= 786

Then,

	2	5	7
	5	7	5
+	7	8	6
	16	1	8

(iii) Evaluate:

	3	$\bar{4}$	$\bar{4}$
	7	$\bar{1}$	$\bar{3}$
	8	$\bar{3}$	2
+	5	$\bar{4}$	$\bar{3}$
	22	$\bar{2}$	$\bar{8}$

∵ $\bar{4}+\bar{1}+\bar{3}+\bar{4}$ = -4 + (-1) + (-3) + (-4) = -4 - 1 - 3 − 4

= -12 = $\bar{1}\bar{2}$ = $\bar{1}\bar{2}$

Here, carry number = $\bar{1}$

∵ 3 +7 + 8 +5 + $\bar{1}$ = 23 + (-1) = 23 − 1 = 22

By removing the vinculum in $22\bar{2}\bar{8}$.

∴ $22\bar{2}\bar{8}$ = 2/2-1/10-2/$\bar{8}$

= 2/1/8/$\bar{8}$

= 2/1/8-1/10-8

= 2172

	3	$\bar{4}$	$\bar{4}$
	7	$\bar{1}$	$\bar{3}$
	8	$\bar{3}$	2
+	5	$\bar{4}$	$\bar{3}$

By removing the vinculum in $3\bar{4}\bar{4}$,

$7\bar{1}\bar{3}$, $8\bar{3}2$ and $5\bar{4}\bar{3}$.

∴ $3\bar{4}\bar{4}$ = 3-1/10-4/$\bar{4}$ = 2/6-1/10-4 = 256

$7\bar{1}\bar{3}$ = 7-1/10-1/$\bar{3}$ = 6/9-1/10-3 = 687

$8\bar{3}2$ = 8-1/10-3/2 = 772

and $5\bar{4}\bar{3}$ = 5-1/10-4/$\bar{3}$ = 4/6-1/10-3

= 457

Then,

$$
\begin{array}{ccc}
2 & 5 & 6 \\
6 & 8 & 7 \\
7 & 7 & 2 \\
+\,4 & 5 & 7 \\
\hline
21 & 7 & 2
\end{array}
$$

(iv) Evaluate:

$$
\begin{array}{ccccc}
2 & \bar{3} & 8 & \bar{1} & \bar{4} \\
4 & \bar{4} & 0 & \bar{3} & 6 \\
3 & \bar{2} & \bar{3} & 4 & \bar{3} \\
+\,7 & \bar{4} & \bar{4} & \bar{4} & \bar{2} \\
\hline
15 & \bar{3} & 1 & \bar{4} & \bar{3}
\end{array}
$$

By removing the vinculum in $15\bar{3}1\bar{4}\bar{3}$.

$\therefore \ 15\bar{3}1\bar{4}\bar{3} = 1/5\text{-}1/10\text{-}3/1\text{-}1/10\text{-}4/\bar{3}$

$\qquad = 1/4/7/0/6/\bar{3}$

$\qquad = 1/4/7/0/6\text{-}1/10\text{-}3$

$\qquad = 147057$

$$
\begin{array}{ccccc}
2 & \bar{3} & 8 & \bar{1} & \bar{4} \\
4 & \bar{4} & 0 & \bar{3} & 6 \\
3 & \bar{2} & \bar{3} & 4 & \bar{3} \\
+\,7 & \bar{4} & \bar{4} & \bar{4} & \bar{2} \\
\hline
\end{array}
$$

By removing the vinculum in $2\bar{3}8\bar{1}\bar{4}$, $4\bar{4}0\bar{3}6$, $3\bar{2}\bar{3}4\bar{3}$ and $7\bar{4}\bar{4}\bar{4}\bar{2}$.

$\therefore \ 2\bar{3}8\bar{1}\bar{4} = 2\text{-}1/10\text{-}3/8\text{-}1/10\text{-}1/\bar{4}$

$\qquad = 1/7/7/9\text{-}1/10\text{-}4 = 17786$

$4\bar{4}0\bar{3}6 = 4\text{-}1/10\text{-}4/0\text{-}1/10\text{-}3/6$

$\qquad = 3/6/\bar{1}/7/6 = 3/6\text{-}1/10\text{-}1/7/6$

$\qquad = 35976$

$3\bar{2}\bar{3}4\bar{3} = 3\text{-}1/10\text{-}2/\bar{3}/4\text{-}1/10\text{-}3$

$\qquad = 2/8\text{-}1/10\text{-}3/3/7 = 27737$

and $7\bar{4}\bar{4}\bar{4}\bar{2} = 7\text{-}1/10\text{-}4/\bar{4}/\bar{4}/\bar{2}$

$\qquad = 6/6\text{-}1/10\text{-}4/\bar{4}/\bar{2}$

$\qquad = 6/5/6\text{-}1/10\text{-}4/\bar{2}$

$\qquad = 6/5/5/6\text{-}1/10\text{-}2 = 65558$

Then,

$$
\begin{array}{ccccc}
1 & 7 & 7 & 8 & 6 \\
3 & 5 & 9 & 7 & 6 \\
2 & 7 & 7 & 3 & 7 \\
+\,6 & 5 & 5 & 5 & 8 \\
\hline
14 & 7 & 0 & 5 & 7
\end{array}
$$

(B) Subtraction:

Type – I

Calculation with Vinculum	**Calculation by removing Vinculum**

(i) Evaluate:

$$
\begin{array}{cc}
5 & \bar{4} \\
-\,2 & 8 \\
\hline
2 & \bar{2}
\end{array}
$$

$\because \ \bar{4}\text{-}8 = \text{-}4\text{-}8 = \text{-}12 = \overline{12} = \bar{1}\bar{2}$

Here, carry number $= \bar{1}$ and $5 - 2 + \bar{1} = 5 - 2 - 1 = 2$

By removing the vinculum in $2\bar{2}$.

$\therefore \ 2\bar{2} = 2\text{-}1/10\text{-}2$

$\qquad = 18$

$$
\begin{array}{cc}
5 & \bar{4} \\
-\,2 & 8 \\
\hline
\end{array}
$$

By removing the vinculum in $5\bar{4}$.

$\therefore \ 5\bar{4} = 5\text{-}1/10\text{-}4 = 46$

Then,

$$
\begin{array}{cc}
4 & 6 \\
-\,2 & 8 \\
\hline
1 & 8
\end{array}
$$

(ii) Evaluate:

$$
\begin{array}{r}
4 \quad \overline{3} \\
- 2 \quad 9 \\
\hline
1 \quad \overline{2}
\end{array}
$$

By removing the vinculum in $1\overline{2}$.

$\therefore \ 1\overline{2} = 1\text{-}1/10\text{-}2$

$\qquad = 08$

(iii) Evaluate:

$$
\begin{array}{r}
5 \quad 7 \quad \overline{2} \\
- 1 \quad 3 \quad 9 \\
\hline
4 \quad 3 \quad \overline{1}
\end{array}
$$

By removing the vinculum in $43\overline{1}$.

$\therefore \ 43\overline{1} = 4/3\text{-}1/10\text{-}1$

$\qquad = 429$

(iv) Evaluate:

$$
\begin{array}{r}
5 \quad \overline{4} \quad 1 \\
- 3 \quad 9 \quad 4 \\
\hline
1 \quad \overline{3} \quad \overline{3}
\end{array}
$$

By removing the vinculum in $1\overline{3}\,\overline{3}$.

$\therefore \ 1\overline{3}\,\overline{3} = 1\text{-}1/10\text{-}3/\,\overline{3} = 0/7\text{-}1/10\text{-}3$

$\qquad = 067$

(v) Evaluate:

$$
\begin{array}{r}
9 \quad \overline{3} \quad \overline{2} \\
- 4 \quad 8 \quad 0 \\
\hline
4 \quad \overline{1} \quad \overline{2}
\end{array}
$$

$\because \ \overline{2}\text{-}0 = \text{-}2\text{-}0 = \text{-}2 = \overline{2}$ and $\overline{3}\text{-}8 = \text{-}3\text{-}8 = \text{-}11 = \overline{11} = \overline{1}\,\overline{1}$

By removing the vinculum in $4\overline{1}\,\overline{2}$.

$\therefore \ 4\overline{1}\,\overline{2} = 4\text{-}1/10\text{-}1/\,\overline{2} = 3/9\text{-}1/10\text{-}2$

$\qquad = 388$

$$
\begin{array}{r}
4 \quad \overline{3} \\
- 2 \quad 9 \\
\hline
\end{array}
$$

By removing the vinculum in $4\overline{3}$.

$\therefore \ 4\overline{3} = 4\text{-}1/10\text{-}3 = 37$

Then,

$$
\begin{array}{r}
3 \quad 7 \\
- 2 \quad 9 \\
\hline
0 \quad 8
\end{array}
$$

$$
\begin{array}{r}
5 \quad 7 \quad \overline{2} \\
- 1 \quad 3 \quad 9 \\
\hline
\end{array}
$$

By removing the vinculum in $57\overline{2}$.

$\therefore \ 57\overline{2} = 5/7\text{-}1/10\text{-}2 = 568$

Then,

$$
\begin{array}{r}
5 \quad 6 \quad 8 \\
- 1 \quad 3 \quad 9 \\
\hline
4 \quad 2 \quad 9
\end{array}
$$

$$
\begin{array}{r}
5 \quad \overline{4} \quad 1 \\
- 3 \quad 9 \quad 4 \\
\hline
\end{array}
$$

By removing the vinculum in $5\overline{4}1$.

$\therefore \ 5\overline{4}1 = 5\text{-}1/10\text{-}4/1 = 461$

Then,

$$
\begin{array}{r}
4 \quad 6 \quad 1 \\
- 3 \quad 9 \quad 4 \\
\hline
0 \quad 6 \quad 7
\end{array}
$$

$$
\begin{array}{r}
9 \quad \overline{3} \quad \overline{2} \\
- 4 \quad 8 \quad 0 \\
\hline
\end{array}
$$

By removing the vinculum in $9\overline{3}\,\overline{2}$.

$\therefore \ 9\overline{3}\,\overline{2} = 9\text{-}1/10\text{-}3/\,\overline{2} = 8/7/\,\overline{2}$

$\qquad = 8/7\text{-}1/10\text{-}2 = 868$

Then,

$$
\begin{array}{r}
8 \quad 6 \quad 8 \\
- 4 \quad 8 \quad 0 \\
\hline
3 \quad 8 \quad 8
\end{array}
$$

Type – II

Calculation with Vinculum	**Calculation by removing Vinculum**

(i) Evaluate:

$$\begin{array}{cc} 7 & 8 \\ 3 & \overline{4} \\ \hline 5 & 2 \end{array}$$

$\because$ $\overline{4}$ = -4   and $8 - \overline{4} = 8 - (-4) = 8 + 4 = 12$

$$\begin{array}{cc} 7 & 8 \\ 3 & \overline{4} \\ \hline \end{array}$$

By removing the vinculum in $3\overline{4}$.

$\therefore$ $3\overline{4}$ = 3-1/10-4 = 26
Then,

$$\begin{array}{cc} 7 & 8 \\ 2 & 6 \\ \hline 5 & 2 \end{array}$$

(ii) Evaluate:

$$\begin{array}{ccc} 7 & 8 & 9 \\ 3 & 5 & \overline{4} \\ \hline 4 & 4 & 3 \end{array}$$

$$\begin{array}{ccc} 7 & 8 & 9 \\ 3 & 5 & \overline{4} \\ \hline \end{array}$$

By removing the vinculum in $35\overline{4}$.

$\therefore$ $35\overline{4}$ = 3/5-1/10-4 = 346
Then,

$$\begin{array}{ccc} 7 & 8 & 9 \\ 3 & 4 & 6 \\ \hline 4 & 4 & 3 \end{array}$$

(iii) Evaluate:

$$\begin{array}{ccc} 5 & 8 & 2 \\ 1 & \overline{4} & 6 \\ \hline 5 & 2 & \overline{4} \end{array}$$

By removing the vinculum in $52\overline{4}$.

$\therefore$ $52\overline{4}$ = 5/2-1/10-4
     = 516

$$\begin{array}{ccc} 5 & 8 & 2 \\ 1 & \overline{4} & 6 \\ \hline \end{array}$$

By removing the vinculum in $1\overline{4}6$.

$\therefore$ $1\overline{4}6$ = 1-1/10-4/6 = 066
Then,

$$\begin{array}{ccc} 5 & 8 & 2 \\ 0 & 6 & 6 \\ \hline 5 & 1 & 6 \end{array}$$

(iv) Evaluate:

$$\begin{array}{ccc} 8 & 9 & 2 \\ 5 & \overline{3} & \overline{4} \\ \hline 4 & 2 & 6 \end{array}$$

$$\begin{array}{ccc} 8 & 9 & 2 \\ 5 & \overline{3} & \overline{4} \\ \hline \end{array}$$

By removing the vinculum in $5\overline{3}\overline{4}$.

$\therefore$ $5\overline{3}\overline{4}$ = 5-1/10-3/$\overline{4}$ = 4/7/$\overline{4}$
   = 4/7-1/10-4 = 466
Then,

$$\begin{array}{ccc} 8 & 9 & 2 \\ 4 & 6 & 6 \\ \hline 4 & 2 & 6 \end{array}$$

Type – III

| **Calculation with Vinculum** | **Calculation by removing Vinculum** |

(i) Evaluate:

$$\begin{array}{cc} 9 & \overline{2} \\ -5 & \overline{4} \\ \hline 4 & 2 \end{array}$$

∵ $\overline{2}$ - $\overline{4}$ = -2 - (-4) = -2 + 4 = 2.

$$\begin{array}{cc} 9 & \overline{2} \\ -5 & \overline{4} \\ \hline \end{array}$$

By removing the vinculum in $9\overline{2}$ and $5\overline{4}$

∴ $9\overline{2}$ = 9-1/10-2 = 88

and $5\overline{4}$ = 5-1/10-4 = 46

Then,

$$\begin{array}{cc} 8 & 8 \\ -4 & 6 \\ \hline 4 & 2 \end{array}$$

(ii) Evaluate:

$$\begin{array}{ccc} 9 & \overline{3} & \overline{2} \\ -4 & \overline{4} & 6 \\ \hline 5 & 1 & \overline{8} \end{array}$$

∵ $\overline{2}$ - 6 = -2 - 6 = -8 = $\overline{8}$ and $\overline{3}$ - $\overline{4}$ = -3 - (-4) = -3 + 4 = 1

By removing the vinculum in $51\overline{8}$

∴ $51\overline{8}$ = 5/1-1/10-8

    = 502

$$\begin{array}{ccc} 9 & \overline{3} & \overline{2} \\ -4 & \overline{4} & 6 \\ \hline \end{array}$$

By removing the vinculum in $9\overline{3}\overline{2}$ and $4\overline{4}6$

∴ $9\overline{3}\overline{2}$ = 9-1/10-3/$\overline{2}$ = 8/7/$\overline{2}$

        = 8/7-1/10-2 = 868

and $4\overline{4}6$ = 4-1/10-4/6 = 366

Then,

$$\begin{array}{ccc} 8 & 6 & 8 \\ -3 & 6 & 6 \\ \hline 5 & 0 & 2 \end{array}$$

(iii) Evaluate:

$$\begin{array}{ccc} 6 & 2 & \overline{3} \\ -3 & \overline{4} & \overline{1} \\ \hline 3 & 6 & \overline{2} \end{array}$$

By removing the vinculum in $36\overline{2}$.

∴ $36\overline{2}$ = 3/6-1/10-2

    = 358

$$\begin{array}{ccc} 6 & 2 & \overline{3} \\ -3 & \overline{4} & \overline{1} \\ \hline \end{array}$$

By removing the vinculum in $62\overline{3}$ and $3\overline{4}\overline{1}$.

∴ $62\overline{3}$ = 6/2-1/10-3 = 617

and $3\overline{4}\overline{1}$ = 3-1/10-4/$\overline{1}$ = 2/6/$\overline{1}$

        = 2/6-1/10-1 = 259

Then,

$$\begin{array}{ccc} 6 & 1 & 7 \\ -2 & 5 & 9 \\ \hline 3 & 5 & 8 \end{array}$$

(iv) Evaluate:

$$\begin{array}{ccc} 9 & \overline{3} & 0 \\ -7 & 4 & \overline{4} \\ \hline 2 & \overline{7} & 4 \end{array}$$

By removing the vinculum in $2\overline{7}4$.

∴ $2\overline{7}4$ = 2-1/10-7/4

    = 134

$$\begin{array}{ccc} 9 & \overline{3} & 0 \\ -7 & 4 & \overline{4} \\ \hline \end{array}$$

By removing the vinculum in $9\overline{3}0$ and $74\overline{4}$.

∴ $9\overline{3}0$ = 9-1/10-3/0 = 870

and $7\overline{4}\overline{4}$ = 7/4-1/10-4 = 736

Then,

```
    8   7   0
-   7   3   6
─────────────
    1   3   4
```

(v) Evaluate:

```
    7   4̄   1̄
    4   3̄   2̄
-  ─────────────
    3   1̄   1
```

By removing the vinculum in $3\overline{1}1$.

∴ $3\overline{1}1$ = 3-1/10-1/1

     = 291

```
    7   4̄   1̄
    4   3̄   2̄
-  ─────────────
```

By removing the vinculum in $7\overline{4}\overline{1}$

and $4\overline{3}\overline{2}$.

∴ $7\overline{4}\overline{1}$ = 7-1/10-4/$\overline{1}$ = 6/6-1/10-1 = 659

and $4\overline{3}\overline{2}$ = 4-1/10-3/$\overline{2}$ = 3/7-1/10-2

     = 368

Then,

```
    6   5   9
-   3   6   8
─────────────
    2   9   1
```

(C) Multiplication:

Calculation with Vinculum

(i) Evaluate:

```
    3   4̄
×       3
─────────────
    8   2̄
```

∵ 3 X $\overline{4}$ = 3 X -4 = -12 = $\overline{12}$ = $\overline{1}\overline{2}$

and 3 X 3 + $\overline{1}$ = 9 + (-1) = 9 - 1 = 8

By removing the vinculum in $8\overline{2}$.

∴ $8\overline{2}$ = 8-1/10-2 = 78

Calculation by removing Vinculum

```
    3   4̄
×       3
─────────────
```

By removing the vinculum in $3\overline{4}$

∴ $3\overline{4}$ = 3-1/10-4 = 26

Then,

```
    2   6
×       3
─────────────
    7   8
```

(ii) Evaluate:

```
            8   4̄
×       6̄   3̄
─────────────────
        2̄   3̄   2
    4   6   4̄   0
─────────────────
    4   4   7̄   2
```

∵ $\overline{3}$ X $\overline{4}$ = -3 X (-4) = 12

By removing the vinculum in

∴ $44\overline{7}2$ = 4/4-1/10-7/2

       = 4332

```
            8   4̄
×       6̄   3̄
─────────────────
```

By removing the vinculum in $8\overline{4}$ and $6\overline{3}$

∴ $8\overline{4}$ = 8-1/10-4 = 76

and $6\overline{3}$ = 6-1/10-3 = 57

Then,

```
            7   6
×       5   7
─────────────────
        5   3   2
    3   8   0   0
─────────────────
    4   3   3   2
```

(iii) Evaluate:

$$
\begin{array}{rrr}
3 & 2 & \bar{4} \\
\times\ 3 & \bar{2} \\
\hline
\bar{6} & \bar{4} & 8 \\
9 & 5 & \bar{2} & 0 \\
\hline
9 & \bar{1} & \bar{6} & 8 \\
\end{array}
$$

By removing the vinculum in $9\bar{1}\bar{6}8$.

∴ $9\bar{1}\bar{6}8$ = 9-1/10-1/$\bar{6}$/8

   = 8/9/$\bar{6}$/8

   = 8/9-1/10-6/8

   = 8848

$$
\begin{array}{rrr}
3 & 2 & \bar{4} \\
\times\ 3 & \bar{2} \\
\hline
\end{array}
$$

By removing the vinculum in $32\bar{4}$ and $3\bar{2}$.

∴ $32\bar{4}$ = 3/2-1/10-4 = 316

and $3\bar{2}$ = 3-1/10-2 = 28

Then,

$$
\begin{array}{rrr}
3 & 1 & 6 \\
\times\ 2 & 8 \\
\hline
2 & 5 & 2 & 8 \\
6 & 3 & 2 & 0 \\
\hline
8 & 8 & 4 & 8 \\
\end{array}
$$

(iv) Evaluate:

$$
\begin{array}{rrr}
5 & \bar{4} & 3 \\
\times\ 5 & \bar{4} \\
\hline
\bar{1} & \bar{9} & 5 & \bar{2} \\
2 & 4 & \bar{9} & 5 & 0 \\
\hline
2 & 2 & \bar{7} & 0 & \bar{2} \\
\end{array}
$$

By removing the vinculum in $22\bar{7}0\bar{2}$.

∴ $22\bar{7}0\bar{2}$ = 2/2-1/10-7/0-1/10-2

   = 2/1/3/$\bar{1}$/8

   = 2/1/3-1/10-1/8

   = 21298

$$
\begin{array}{rrr}
5 & \bar{4} & 3 \\
\times\ 5 & \bar{4} \\
\hline
\end{array}
$$

By removing the vinculum in $5\bar{4}3$ and $5\bar{4}$.

∴ $5\bar{4}3$ = 5-1/10-4/3 = 463

and $5\bar{4}$ = 5-1/10-4 = 46

Then,

$$
\begin{array}{rrr}
4 & 6 & 3 \\
\times\ 4 & 6 \\
\hline
2 & 7 & 7 & 8 \\
1 & 8 & 5 & 2 & 0 \\
\hline
2 & 1 & 2 & 9 & 8 \\
\end{array}
$$

(v) Evaluate:

$$
\begin{array}{rrrr}
3 & 4 & \bar{2} \\
\times\ 4 & \bar{1} & \bar{4} \\
\hline
\bar{1} & \bar{3} & \bar{6} & 8 \\
\bar{3} & \bar{4} & 2 & 0 \\
1 & 3 & 6 & \bar{8} & 0 & 0 \\
\hline
1 & 3 & 1 & \bar{5} & \bar{4} & 8 \\
\end{array}
$$

By removing the vinculum in $131\bar{5}\bar{4}8$.

∴ $131\bar{5}\bar{4}8$ = 1/3/1-1/10-5/$\bar{4}$/8

   = 1/3/0/5/$\bar{4}$/8

   = 1/3/0/5-1/10-4/8

   = 130468

$$
\begin{array}{rrr}
3 & 4 & \bar{2} \\
\times\ 4 & \bar{1} & \bar{4} \\
\hline
\end{array}
$$

By removing the vinculum in $34\bar{2}$ and $4\bar{1}\bar{4}$.

∴ $34\bar{2}$ = 3/4-1/10-2 = 338

and $4\bar{1}\bar{4}$ = 4-1/10-1/$\bar{4}$ = 3/9-1/10-4

   = 386

Then,

```
              3  3  8
         ×    3  8  6
      ─────────────────
              2  0  2  8
           2  7  0  4  0
        1  0  1  4  0  0
      ─────────────────
        1  3  0  4  6  8
```

3. VERTICALLY AND CROSSWISE VEDIC METHOD FOR MULTIPLICATION IN VINCULUM NUMBERS

For vertically and crosswise product of $m\overline{n}$ and $p\overline{q}\overline{r}$, we have to make number of digits of two number to be same. For this we include 0 (Zero) at hundreds place of $m\overline{n}$ and $m\overline{n}$ is written as $0m\overline{n}$. Multiplication is written as –

$$\begin{array}{ccc} p & \overline{q} & \overline{r} \\ \times \quad 0 & m & \overline{n} \end{array}$$

Multiplication process is as follow

(i) Step – 1. $\quad \begin{array}{cc} p & \overline{q} \\ 0 & m \end{array} \left(\begin{array}{c} \overline{r} \\ \overline{n} \end{array}\right) = \overline{r} \times \overline{n} = ab$

$\qquad\qquad a, b \in N \cup \{0\}, \ ab = 1 \times b + 10 \times a$

Here, a be the carry number and b is included in the result.

Fig.–1

(ii) Step – 2. $\quad \begin{array}{c} p \\ 0 \end{array} \left(\begin{array}{cc} \overline{q} & \overline{r} \\ m & \overline{n} \end{array}\right) = \overline{n} \times \overline{q} + \overline{r} \times m = cd$

$\qquad\qquad$ and $\ cd + a = ef$

Here, e be the carry number and f is included in the result left to b of step – 1.

Fig.–2

(iii) Step – 3. $\quad \left(\begin{array}{ccc} p & \overline{q} & \overline{r} \\ 0 & m & \overline{n} \end{array}\right) = \overline{n} \times p + \overline{r} \times 0 + \overline{q} \times \overline{n} = gh$

$\qquad\qquad$ and $gh + e = ij$

Here, i be the carry number and j is included in the result left to f of step – 2.

Fig.–3

(iv) Step – 4. $\quad \left(\begin{array}{cc} p & \overline{q} \\ 0 & m \end{array}\right) \begin{array}{c} \overline{r} \\ \overline{n} \end{array} = m \times p + \overline{q} \times 0 = kl$

$\qquad\qquad$ and $kl + i = st$

Fig.–4

Here, s be the carry number and t is included in the result left to j of step – 3.

(v) Step – 5. $\quad \left(\begin{array}{c} p \\ 0 \end{array}\right) \begin{array}{cc} \overline{q} & \overline{r} \\ m & \overline{n} \end{array} = p \times 0 = 0$

$\qquad\qquad$ and $o + s = s$

Here, s is included in the result left to t of step – 4.
Hence,

$$\therefore \ m\overline{n} \times p\overline{q}\overline{r} = \text{stifb}$$

Fig.–5

Example – $\quad$ Evaluate: $\quad 6\overline{3} \times 32\overline{4}$

We write

$$\begin{array}{ccc} 3 & \overline{2} & \overline{4} \\ \times \quad 0 & 6 & \overline{3} \end{array}$$
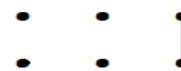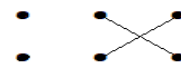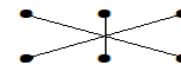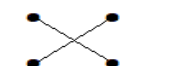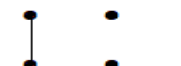
(i) Step – 1. $\quad \begin{matrix} 3 & \bar{2} & \left(\bar{4}\right) \\ 0 & 6 & \left(\bar{3}\right) \end{matrix} = \bar{4} \times \bar{3} = 12$

Here, 1 be the carry number and 2 is included in the result.

Fig.–6

(ii) Step – 2. $\quad \begin{matrix} 3 & \left(\bar{2} & \bar{4}\right) \\ 0 & \left(6 & \bar{3}\right) \end{matrix} = \bar{3} \times \bar{2} + \bar{4} \times 6 = \overline{18}$
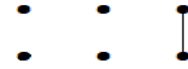
$$\text{and } \overline{18} + 1 = \overline{17}$$

Fig.–7

Here, $\bar{1}$ be the carry number and $\bar{7}$ is included in the result left to 2 of step – 1.

(iii) Step – 3. $\quad \begin{pmatrix} 3 & \bar{2} & \bar{4} \\ 0 & 6 & \bar{3} \end{pmatrix} = \bar{3} \times 3 + \bar{4} \times 0 + 6 \times \bar{2} = \overline{21}$

$$\text{and } \overline{21} + \bar{1} = \overline{22}$$

Fig.–8

Here, $\bar{2}$ be the carry number and $\bar{2}$ is included in the result left to $\bar{7}$ of step – 2.

(iv) Step – 4. $\quad \begin{pmatrix} 3 & \bar{2} \\ 0 & 6 \end{pmatrix} \begin{matrix} \bar{4} \\ \bar{3} \end{matrix} = 6 \times 3 + \bar{2} \times 0 = 18$
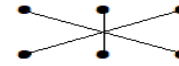
$$\text{and } 18 + \bar{2} = 16$$

Fig.–9

Here, 1 be the carry number and 6 is included in the result left to $\bar{2}$ of step – 3.

(v) Step – 5. $\quad \begin{pmatrix} 3 \\ 0 \end{pmatrix} \begin{matrix} \bar{2} & \bar{4} \\ 6 & \bar{3} \end{matrix} = 3 \times 0 = 0$
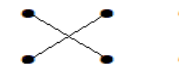
$$\text{and } 0 + 1 = 1$$

Fig.–10

Here, 1 is included in the result left to 6 of step – 4.
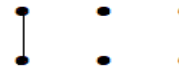
Hence,

$$\therefore\ 6\bar{3} \times 32\bar{4} = 16\bar{2}\bar{7}2$$

By removing the vinculum in $16\bar{2}\bar{7}2$

$$\therefore\ 16\bar{2}\bar{7}2 = 1/6\text{-}1/10\text{-}2/\bar{7}/2 = 1/5/8/\bar{7}/2 = 1/5/8\text{-}1/10\text{-}7/2 = 15732.$$

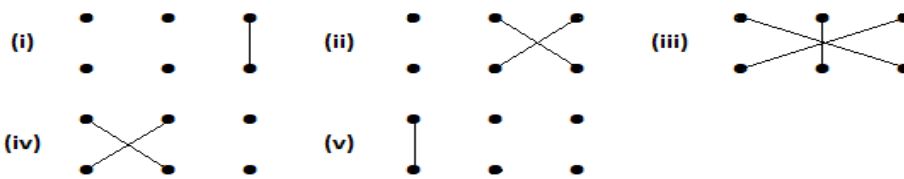In short we may calculate this multiplication with the help of above process as follow step –

Fig.–11

$$\begin{matrix} & 3 & \bar{2} & \bar{4} \\ \times & 0 & 6 & \bar{3} \\ \hline 1 & 6 & \bar{2} & \bar{7} & 2 \end{matrix}$$

$$\therefore\ \text{Answer} = 16\bar{2}\bar{7}2$$

By removing the vinculum in $16\bar{2}\bar{7}2$

$$\therefore\ 16\bar{2}\bar{7}2 = 1/6\text{-}1/10\text{-}2/\bar{7}/2 = 1/5/8/\bar{7}/2 = 1/5/8\text{-}1/10\text{-}7/2 = 15732.$$

Multiplication with vinculum Method:

$$
\begin{array}{ccc}
 & 3 & \overline{2} & \overline{4} \\
\times & & 6 & \overline{3} \\
\hline
 & \overline{9} & 7 & 2 \\
1 & 7 & \overline{4} & \overline{4} & 0 \\
\hline
1 & 6 & \overline{3} & 3 & 2 \\
\end{array}
$$

By removing the vinculum in $16\overline{3}32$.

$\therefore 16\overline{3}32 = 1/6\text{-}1/10\text{-}3/3/2 = 15732$

Example – Evaluate: $\quad 8\overline{4} \times 6\overline{3}$

Multiplication process is done by following steps -

Fig.–12

$$
\begin{array}{ccc}
 & 8 & \overline{4} \\
\times & 6 & \overline{3} \\
\hline
44 & \overline{7} & 2 \\
\end{array}
$$

$\therefore$ Answer $= 44\overline{7}2$.

4. CONCLUSION

We have performed addition, subtraction and multiplication of numbers with involvement of vinculum in which carry number appears. During the process by vertically and crosswise Vedic method it becomes simpler.

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