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ENTIRE RATE SEQUENCE SPACE OF INTERVAL NUMBERS

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ABSTRACT

In this paper we introduced the new concept of interval valued sequence space $\Gamma_{\pi}(IR)$ where (π_k) is a sequence of positive numbers. We present the different properties like completeness, solidness, AB property etc.

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1. INTRODUCTION

The power of interval arithmetic lies in its implementation on computers. In particular, outwardly rounded interval arithmetic allows rigorous enclosures for the ranges of operations and functions. This makes a qualitative difference in scientific computations, since the results are now intervals in which the exact result must lie. It also enables the use of computations for proving automated theorem.

Interval arithmetic is a tool in numerical computing where the rules for the arithmetic of intervals are explicitly stated. It was first suggested by P.S.Dwyer [10] in 1951. Development of interval arithmetic as a formal system and evidence of its value as a computational device was provided by R.E.Moore [34], [35] in 1959 and 1962.

A set consisting of a closed interval of real numbers x such that $a \le x \le b$ is called an interval number. A real interval can also be considered as a set. We denote the set of all real valued closed intervals by $I\Re$. Any element of $I\Re$ may be called closed interval and denoted by $\hat{x}$. That is, $\hat{x} = [x_l, x_r] = \{x \in \Re : x_l \le x \le x_r\}$. An interval number $\hat{x}$ is a closed subset of real numbers. Let x_l and x_r be respectively referred to as the infimum (lower bound) and supremum (upper bound) of the interval number $\hat{x}$. If $\hat{x} = [0,0]$, then $\hat{x}$ is said to be a zero interval. It is denoted by $\hat{0}$.

For $\bar{x}_1, \bar{x}_2 \in I\Re$, we define $\bar{x}_1 = \bar{x}_2$ if and only if $x_{1l} = x_{2l}$ *and* $x_{1r} = x_{2r}$

$$\bar{x}_1 + \bar{x}_2 = \{x \in \Re : x_{1l} + x_{2l} \le x \le x_{1r} + x_{2r})\}$$

$$\bar{x}_1 \times \bar{x}_2 = \{x \in \Re : \min(x_{1l}x_{2l}, x_{1l}x_{2r}, x_{1r}x_{2l}, x_{1r}x_{2r}) \le x \le \max(x_{1l}x_{2l}, x_{1l}x_{2r}, x_{1r}x_{2l}, x_{1r}x_{2r})\}$$

The set of all interval numbers $I\Re$ is a complete metric space defined by

$$d(\bar{x}_1, \bar{x}_2) = \max\{|\bar{x}_{1l} - \bar{x}_{2l}|, |\bar{x}_{1r} - \bar{x}_{2r}|\}$$

In the special case $\bar{x}_1 = [a,a]$ and $\bar{x}_2 = [b,b]$, we obtain usual metric of $\Re$.

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Let us define transformation f: N $\rightarrow \Re$, k $\rightarrow$ f(k)= $\bar{x}_k$, then $\bar{x} = (\bar{x}_k)$ is called sequence of interval numbers. $\bar{x}_k$ is called kth term of sequence $\bar{x} = (\bar{x}_k)$, ω^i denotes the set of all interval numbers with real terms and the algebraic properties of ω^i are in[7].

A sequence $\bar{x} = (\bar{x}_k)$ of interval numbers is said to be convergent to the interval number $\bar{x}_0$ if for each ε >0 there exists a positive integer k$_0$ such that $d(\bar{x}_k, \bar{x}_0) < \varepsilon$ for all k$\geq$k$_0$ and we denote it by $\lim_k \bar{x}_k = \bar{x}_0$. Equivalently $\lim_k \bar{x}_k = \bar{x}_0$ iff $\lim_k x_{kl} = x_{0l}$ *and* $\lim_k x_{kr} = x_{0r}$.

2. MAIN RESULTS

The entire sequence space of symmetric interval numbers is denoted by $\Gamma_\pi(IR)$ where (π_k) is a sequence of positive numbers and the set of functional of interval numbers is denoted by $\Lambda_\pi(IR)$.

$$\Gamma_\pi(IR) = \left\{ \tilde{x} = \left(\frac{\tilde{x}_k}{\pi_k}\right) \in w(IR) : \lim_k \left(D\left(\frac{\tilde{x}_k}{\pi_k}, \tilde{0}\right)\right) = \tilde{0} \right\}$$

$$\Lambda_\pi(IR) = \left\{ \tilde{x} = \left(\frac{\tilde{x}_k}{\pi_k}\right) \in w(IR) : \sup_k \left(D\left(\frac{\tilde{x}_k}{\pi_k}, \tilde{0}\right)\right) < \infty \right\}$$

where $D\left(\frac{\tilde{x}_k}{\pi_k}, \frac{\tilde{y}_k}{\pi_k}\right) = \max\left\{ \left|\frac{\underline{x}_k - \underline{y}_k}{\pi_k}\right|^{1/k} \left|\frac{\overline{x}_k - \overline{y}_k}{\pi_k}\right|^{1/k} \right\}$

The metric $\tilde{d}$ is defined by

$$\tilde{d}\left(\frac{\tilde{x}_k}{\pi_k}, \frac{\tilde{y}_k}{\pi_k}\right) = \sup_k \max\left\{ \left|\frac{\underline{x}_k - \underline{y}_k}{\pi_k}\right|^{1/k} \left|\frac{\overline{x}_k - \overline{y}_k}{\pi_k}\right|^{\frac{1}{k}} \right\} = \sup_k D\left(\frac{\tilde{x}_k}{\pi_k}, \frac{\tilde{y}_k}{\pi_k}\right) \tag{2.1}$$

which satisfies the metric space axioms .

Theorem 2.1: The sequence space of interval numbers $\Gamma_\pi(IR)$ is a complete metric space with respect to the metric defined by (2.1).

Proof: Let $\left(\frac{\tilde{x}^{(n)}}{\pi^{(n)}}\right)$ be ainterval number fundamental sequence in $\Gamma_\pi(IR)$.Then for a given $\varepsilon > 0$ there exists a positive integer n_0 such that

$$\tilde{d}\left(\frac{\tilde{x}_k^{(n)}}{\pi_k^{(n)}}, \frac{\tilde{x}_k^{(m)}}{\pi_k^{(m)}}\right) = \sup_k D\left(\frac{\tilde{x}_k^{(n)}}{\pi_k^{(n)}}, \frac{\tilde{x}_k^{(m)}}{\pi_k^{(m)}}\right) < \varepsilon \text{ for all } n, m \geq n_0$$

For each k, we have $D\left(\frac{\tilde{x}_k^{(n)}}{\pi_k^{(n)}}, \frac{\tilde{x}_k^{(m)}}{\pi_k^{(m)}}\right) < \varepsilon$ for all $n, m \geq n_0$

$$\max\left\{ \left|\frac{\underline{x}_k^{(n)} - \underline{x}^{(m)}}{\pi_k}\right|^{1/k}, \left|\frac{\overline{x}_k^{(n)} - \overline{x}_k^{(m)}}{\pi_k}\right|^{1/k} \right\} < \varepsilon \text{ for all } n, m \geq n_0$$

$$\left|\frac{\underline{x}_k^{(n)} - \underline{x}_k^{(m)}}{\pi_k}\right|^{1/k} < \varepsilon^k \ and \ \left|\frac{\overline{x}_k^{(n)} - \overline{x}_k^{(m)}}{\pi_k}\right|^{1/k} < \varepsilon^k \text{ for all } n, m \geq n_0$$

This leads to the fact $\frac{\tilde{x}_k^{(n)}}{\pi_k^{(n)}}$ is a interval number fundamental sequence in IR .Since IR is a complete metric space, $\frac{\tilde{x}_k^{(n)}}{\pi_k^{(n)}}$ is convergent $\lim_n \frac{\tilde{x}_k^{(n)}}{\pi_k^{(n)}} = \frac{\tilde{x}_k}{\pi_k}$ for each $k \in \mathrm{N}$.

Hence $\sup_k D\left(\frac{\tilde{x}_k^{(n)}}{\pi_k^{(n)}}, \frac{\tilde{x}_k}{\pi_k}\right) < \varepsilon$

So $\frac{\tilde{x}_k^{(n)}}{\pi_k^{(n)}} \to \frac{\tilde{x}_k}{\pi_k} as\ n \to \infty\ \ in\ \ \Gamma_\pi(IR)$

We have to show that $\tilde{x}_k = \left(\frac{\tilde{x}_k}{\pi^k}\right) \in \Gamma_\pi(IR)$

Since $\tilde{x}_k \in \Gamma_\pi(IR)$, we have $\tilde{d}\left(\frac{\tilde{x}_k^{(n)}}{\pi_k^{(n)}}, \tilde{0}\right) < \varepsilon$

Consider

$$\begin{aligned}
\tilde{d}\left(\frac{\tilde{x}_k}{\pi_k}, \tilde{0}\right) &= \sup_k D\left(\frac{\tilde{x}_k}{\pi_k}, \tilde{0}\right) \\
&\le \sup_k D\left(\frac{\tilde{x}_k^{(n)}}{\pi_k^{(n)}}, \frac{\tilde{x}_k}{\pi_k}\right) + \sup_k D\left(\frac{\tilde{x}_k^{(n)}}{\pi_k^{(n)}}, \tilde{0}\right) \\
&< \varepsilon + \varepsilon \\
&= 2\varepsilon
\end{aligned}$$

Hence $(\tilde{x}_k) \in \Gamma_\pi(IR)$.This completes the proof.

Theorem 2.2: A necessary and sufficient condition that $D\left(\sum \frac{\tilde{x}_k}{\pi_k}\frac{\tilde{y}_k}{\pi_k}, \tilde{0}\right)$ should be convergent for every $\left(\frac{\tilde{x}_k}{\pi_k}\right)$ in which $\lim_k \left(D\left(\frac{\tilde{x}_k}{\pi_k}, \tilde{0}\right)\right) = 0$ is that $D\left(\frac{\tilde{y}_k}{\pi_k}, \tilde{0}\right)$ should be bounded.

Proof: Suppose $D\left(\frac{\tilde{y}_k}{\pi_k}, \tilde{0}\right)$ is bounded. Then there is an M so that $D\left(\frac{\tilde{y}_k}{\pi_k}, \tilde{0}\right) \le M\ \ for\ k \ge 1$.

Since $D\left(\frac{\tilde{x}_k}{\pi_k}, \tilde{0}\right) \to \tilde{0}$ as $k \to \infty$, there exists a positive integer k_0 so that

$$D\left(\frac{\tilde{x}_k}{\pi_k}, \tilde{0}\right) \le \frac{1}{2M}, k \ge k_0$$

$$[D\left(\frac{\tilde{x}_k}{\pi_k}\frac{\tilde{y}_k}{\pi_k}, \tilde{0}\right)]^k \le [D\left(\frac{\tilde{x}_k}{\pi_k}, \tilde{0}\right)]^k [D\left(\frac{\tilde{y}_k}{\pi_k}, \tilde{0}\right)]^k$$

$$< \left(\frac{1}{2M}\right)^k M^k = \frac{1}{2^k}$$

Then $\sum \left[D\left(\frac{\tilde{x}_k}{\pi_k}\frac{\tilde{y}_k}{\pi_k}, \tilde{0}\right)\right]^k$ converges

Conversely,

Suppose $D\left(\frac{\tilde{y}_k}{\pi_k}, \tilde{0}\right)$ is not bounded .Then there is an increasing sequence $\{k_p\}$ of integers such that

$$D\left(\frac{\tilde{y}_{k_p}}{\pi_{k_p}}, \tilde{0}\right) \ge p,\ p = 1,2,..$$

That is, $\left[D\left(\frac{\tilde{y}_{k_p}}{\pi_{k_p}},\tilde{0}\right)\right]^{k_p} \geq p^{k_p}$, $p=1,2,..$

Take $\frac{\tilde{x}_k}{\pi_k}=\begin{cases}\left[0,\frac{1}{p^k}\right] & if\ k=k_p \\ [0,0] & if\ k\neq k_p\end{cases}$

Then $\lim_k\left(D\left(\frac{\tilde{x}_k}{\pi_k},\tilde{0}\right)\right)=0$

But $\left[D\left(\frac{\tilde{x}_k}{\pi_k},\tilde{0}\right)\right]^{k_p}\left[D\left(\frac{\tilde{y}_k}{\pi_k},\tilde{0}\right)\right]^{k_p}=\left[\left(\frac{1}{p^{k_p}}\right)^{1/k_p}\right]^{k_p}\left[\left(p^{k_p}\right)^{1/k_p}\right]^{k_p}=\frac{1}{p^{k_p}}.p^{k_p}=1 for\ k=k_p$

So that $\sum\left[D\left(\frac{\tilde{x}_k}{\pi_k}\frac{\tilde{y}_k}{\pi_k},\tilde{0}\right)\right]^k$ does not converge.

Hence $D\left(\frac{\tilde{y}_k}{\pi_k},\tilde{0}\right)$ is bounded.

Theorem 2.3: The sequence spaces of interval numbers $\Gamma_\pi(IR)$ and $\Lambda_\pi(IR)$ are solid.

Proof: Consider, $\Gamma_\pi(IR)$. Now let $\tilde{d}\left(\frac{\tilde{y}_k}{\pi_k},\tilde{0}\right)\leq\tilde{d}\left(\frac{\tilde{x}_k}{\pi_k},\tilde{0}\right)$ for all $k\in N$ and for some $\tilde{x}\in\Gamma_\pi(IR)$.

Then, $\max\left\{\left|\underline{y}_k\right|^{1/k}\left|\overline{y}_k\right|^{1/k}\right\}\leq\max\left\{\left|\underline{x}_k\right|^{1/k}\left|\overline{x}_k\right|^{1/k}\right\}, \underline{y}_k\leq\underline{x}_k\ and\ \overline{y}_k\leq\overline{x}_k$

$$D\left(\frac{\tilde{y}_k}{\pi_k},\tilde{0}\right)\leq D\left(\frac{\tilde{x}_k}{\pi_k},\tilde{0}\right)$$

$$\lim_{k\to\infty}D\left(\frac{\tilde{y}_k}{\pi_k},\tilde{0}\right)\leq\lim_{k\to\infty}D\left(\frac{\tilde{x}_k}{\pi_k},\tilde{0}\right)=0$$

It is clear that $\tilde{y}_k\in\Gamma_\pi(IR)$

Therefore $\Gamma_\pi(IR)$ is solid.

Similarly, it can be proved that $\Lambda_\pi(IR)$ is solid.

Theorem 2.4: The sequence of interval numbers $(\tilde{e}_1,\tilde{e}_2....\tilde{e}_k,..)$ is schauder base for $\Gamma_\pi(IR)$, where $\tilde{e}_k=\{\tilde{0},\tilde{0},...[1,1],\tilde{0},...\}$.

Proof: Let $\tilde{x}_k=\left(\frac{\tilde{x}_k}{\pi_k}\right)\in\Gamma_\pi(IR)$. Therefore for every $\varepsilon>0$ there exists a positive integer $n\in N$ such that $k\geq n$,

$$\tilde{d}\left(\frac{\tilde{x}_k}{\pi_k},\tilde{0}\right)=\sup_k D\left(\frac{\tilde{x}_k}{\pi_k},\tilde{0}\right)<\varepsilon$$

It is enough to show that $\lim_{k\to\infty}\tilde{d}\left[\frac{\tilde{x}_k}{\pi_k}-\sum\left(\tilde{e}_k\frac{\tilde{x}_k}{\pi_k}\right),\tilde{0}\right]=0$

Now,

$$\tilde{d}\left[\frac{\tilde{x}_k}{\pi_k}-\sum_{k=1}^{n}\left(\tilde{e}_k\,\frac{\tilde{x}_k}{\pi_k}\,,\,\tilde{0}\right)\right]=\tilde{d}\,[([\underline{x}_1,\overline{x}_1],[\underline{x}_2,\overline{x}_2],......[\underline{x}_k,\overline{x}_k],...)-([\underline{x}_1,\overline{x}_1],[\underline{x}_2,\overline{x}_2],......[\underline{x}_n,\overline{x}_n]),\tilde{0}]$$

$$=\tilde{d}[([\tilde{0},\tilde{0},......[\underline{x}_{n+1},\overline{x}_{n+1}],[\underline{x}_{n+2},\overline{x}_{n+2}]),\tilde{0}]$$

$$=\sup_{k\geq n+1}\max\left\{|\underline{x}_k|^{1/k}|\overline{x}_k|^{1/k}\right\}\to\tilde{0}\ as\ n\to\infty$$

We have , $\frac{\tilde{x}_k}{\pi_k}=\sum_{k=1}^{\infty}\tilde{e}_k\,\frac{\tilde{x}_k}{\pi_k}$ (2.2)

Let us show the uniqueness of the representation given by (5.2) for $\tilde{x}=\left(\frac{\tilde{x}_k}{\pi_k}\right)\in\Gamma_\pi\,(IR)$

Suppose that there exists a representation $\tilde{x}\,\frac{\tilde{x}_k}{\pi_k}=\sum_{k=1}^{\infty}\tilde{e}_k\,\frac{\tilde{y}_k}{\pi_k}$, then,

$$\tilde{d}\left(\sum_{k=1}^{n}\left(\frac{\tilde{x}_k}{\pi_k}-\frac{\tilde{y}_k}{\pi_k}\right)\tilde{e}_k\,,\,\tilde{0}\right)=\sum_{k=1}^{n}\tilde{d}\left(\left(\frac{\tilde{x}_k}{\pi_k}-\frac{\tilde{y}_k}{\pi_k}\right)\tilde{e}_k\,,\,\tilde{0}\right)$$

$$=\sup_{k\geq n+1}\max\left\{\left|\frac{\underline{y}_k-\underline{x}_k}{\pi_k}\right|^{\frac{1}{k}}\left|\frac{\overline{y}_k-\overline{x}_k}{\pi_k}\right|^{\frac{1}{k}}\right\}\to 0\ as\ n\to\infty$$

$$\left|\frac{\underline{y}_k-\underline{x}_k}{\pi_k}\right|^{\frac{1}{k}}\to 0\ and\ \left|\frac{\overline{y}_k-\overline{x}_k}{\pi_k}\right|^{\frac{1}{k}}\to 0\ as\ n\to\infty$$

Therefore $\underline{y}_k=\underline{x}_k\ and\ \overline{y}_k=\overline{x}_k$ .That is $\frac{\tilde{y}}{\pi}=\frac{\tilde{x}}{\pi}$.

Theorem 2.5: The β dual of the sequence space of interval numbers $\Gamma_\pi\,(IR)$ is $\Lambda_\pi\,(IR)$.

Proof: Let us suppose that $\tilde{y}=\left(\frac{\tilde{y}_k}{\pi_k}\right)\in\Lambda_\pi\,(IR)$ for every $\tilde{x}=\left(\frac{\tilde{x}_k}{\pi_k}\right)\in\Gamma_\pi\,(IR)$, then

$$\lim_n D\left(\sum_{k=1}^{n}\frac{\tilde{x}_k}{\pi_k}\,\frac{\tilde{y}_k}{\pi_k}\,,\,\tilde{0}\right)=\lim_n D(\sum_{k=1}^{n}[\underline{y}_k,\overline{y}_k],[\underline{x}_k,\overline{x}_k],\tilde{0})$$

$$=\lim_n D\left(\sum_{k=1}^{n}\left[\min(\underline{y}_k\underline{x}_k,\underline{y}_k\overline{x}_k,\overline{y}_k\underline{x}_k,\overline{y}_k\overline{x}_k)\,,\,\max(\underline{y}_k\underline{x}_k,\underline{y}_k\overline{x}_k,\overline{y}_k\underline{x}_k,\overline{y}_k\overline{x}_k)\right],\tilde{0}\right)$$

$$=\lim_n\max\left\{\left|\sum_{k=1}^{n}\underline{y}_k\underline{x}_k\right|^{1/k},\left|\sum_{k=1}^{n}\underline{y}_k\overline{x}_k\right|^{1/k},\left|\sum_{k=1}^{n}\underline{y}_k\overline{x}_k\right|,\left|\sum_{k=1}^{n}\overline{y}_k\overline{x}_k\right|^{1/k}\right\}$$

$$\leq\lim_n\max\left\{\sum_{k=1}^{n}\left|\underline{y}_k\underline{x}_k\right|^{1/k},\sum_{k=1}^{n}\left|\underline{y}_k\overline{x}_k\right|^{1/k},\sum_{k=1}^{n}\left|\underline{y}_k\overline{x}_k\right|^{1/k},\sum_{k=1}^{n}\left|\overline{y}_k\overline{x}_k\right|^{1/k}\right\}$$

$$=\lim_n M\,\max\left\{\sum_{k=1}^{n}|\underline{x}_k|^{1/k},\sum_{k=1}^{n}|\overline{x}_k|^{1/k}\right\}$$

Where $M=\max\{M_1,M_2\}$ $M_1=\sup_k\left|\underline{y}_k\right|^{1/k},M_2=\sup_k\left|\overline{y}_k\right|^{1/k}$

$$\lim_n D\left(\sum_{k=1}^{n}\frac{\tilde{x}_k}{\pi_k}\frac{\tilde{y}_k}{\pi_k},\tilde{0}\right) \le M\lim_n D\left(\sum_{k=1}^{n}\frac{\tilde{x}_k}{\pi_k},\tilde{0}\right)$$

$$= MD\left(\sum_{k=1}^{\infty}\frac{\tilde{x}_k}{\pi_k},\tilde{0}\right) = M\sum_{k=1}^{\infty}D\left(\frac{\tilde{x}_k}{\pi_k},\tilde{0}\right) < \infty$$

Therefore, we get $\tilde{x}_k\ \tilde{y}_k \in cs(IR)$.

Hence $(\tilde{x}_k) \in [\Gamma_\pi(IR)]^\beta$

$$\Lambda_\pi(IR) \subset [\Gamma_\pi(IR)]^\beta \tag{2.3}$$

Let $\tilde{y} = \left(\frac{\tilde{y}_k}{\pi_k}\right) \in [\Gamma_\pi(IR)]^\beta$, then $\sum D\left(\frac{\tilde{x}_k}{\pi_k}\frac{\tilde{y}_k}{\pi_k},\tilde{0}\right)$ converges for every $\tilde{x} = \left(\frac{\tilde{x}_k}{\pi_k}\right) \in \Gamma_\pi(IR)$

By theorem 2.2, $D\left(\frac{\tilde{y}_k}{\pi_k},\tilde{0}\right)$ is bounded.

$\sup_k D\left(\frac{\tilde{y}_k}{\pi_k},\tilde{0}\right)$ exists, then $\tilde{y} = \left(\frac{\tilde{y}_k}{\pi_k}\right) \in \Lambda_\pi(IR)$

$$[\Gamma_\pi(IR)]^\beta \subset \Lambda_\pi(IR) \tag{2.4}$$

From equations (2.3) and (2.4), $[\Gamma_\pi(IR)]^\beta = \Lambda_\pi(IR)$

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