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α^*- REGULAR & α^*- NORMAL SPACE

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ABSTRACT

The purpose of this paper is to introduce new space namely α-regular, α*-normal, using α* - open sets and investigate their properties. We also study the relationships among themselves.*

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I. INTRODUCTION

Separation axioms are useful in classifying topological spaces. Maheswari and Prasad introduces the notation of s-regular and s- normal spaces using semi-open sets, Dorsett introduces the concept of semi – regular and semi – normal spaced and investigate their propertied.

In this paper, we define α* - regular, α*- normal, using α*- open sets and investigate their basic properties. We further study the relationshiops among themselves

II. PRILIMINARIES

Throughout this paper (X, τ) will always denote a topological space on which no separation axioms are assumed, unless explicitly stated. If A is a subset of the space (X, τ), Cl(A) and Int (A) respectively denote the closure and the interior of A in X.

Definition 2.1[7]: A subset A of a topological space (X, τ) is called
- (i) generalized closed (briefly g - closed) if Cl(A)⊆U whenever A⊆U and U is open in X.
- (ii) generalized open (briefly g - open) if X \ A is g - closed in X.

Definition 2.2: Let A be a subset of X. Then
- (i) generalized closure [5] of A is defined as the intersection of all g - closed sets containing A and is denoted by Cl*(A).
- (ii) generalized interior of A is defined as the union of all g - open subsets of A and is denoted by Int*(A).

Definition 2.3: A subset A of a topological space (X, τ) is called
- (i) α *-open [8] A⊆Int*(Cl(Int*(A))).
- (ii) α *-closed [8]) if Cl*(Int(Cl*(A)) ⊆ A.

The class of all α *-open (resp. α *-closed) sets is denoted by α *O (X, τ) (resp. α*C (X, τ)).

The α *-interior of A is defined as the union of all α *-open sets of X contained in A. It is denoted by α *Int(A). The α *-closure of A is defined as the intersection of all α *-closed sets in X containing A. It is denoted by α *Cl(A).

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Definition 2.4: A topological space X is said to be regular if for every pair consisting of a point x and a closed set B not containing x, there are disjoint open sets U and V in X containing x and B respectively. [5]

Definition 2.5: A topological space X is said to be normal if for every pair of disjoint closed sets A and B in X, there are disjoint open sets U and V in X containing A and B respectively. [5]

Definition 2.6: A function f: X⟶Y is said to be

(i) closed [5] if f (V) is closed in Y for every closed set V in X.
(ii) α *-continuous [6] if f $^{-1}$(V) is α *-open in X for every open set V in Y.
(iii) α *-open [7] if f (V) is α *-open in Y for every open set V in X.
(iv) pre- α *-open [7] if the image of every α *-open set of X is α *-open in Y.
(v) contra-pre- α *-open [7] if f(V) is α * - closed in Y for every α *-open set V in X.
(vi) pre- α *-closed [7] if f(V) is α *-closed in Y for every α *-closed set V in X.

III. REGULAR SPACES ASSOCIATE WITH α* - OPEN SETS

In this section we introduce the concepts of α*-regular and α* - regular spaces. Also we investigate their basic properties and study their relationship with already existing concepts.

Definition 3.1: A Space X is said to be α^*- regular if for every pair consisting of a point x and a α^*- closed set B not containing x, there are disjoint α^*-open sets U & V in X containg x & B respectively.

Theorem 3.2: In a topological space X, the following are equivalent.

1) X is α^*- regular.
2) For $x \epsilon X$ and every α^*-open set U containing x, there exist a α^*-open set V containing x such that $\alpha^* Cl(V) \subseteq U$.
3) For every set A & α^*-open set B such that $A \cap B \neq \varphi$, there existes a α^*-open set U such that $A \cap U \neq \varphi$ and $\alpha^* Cl(U) \subseteq B$.
4) For every non - empty set A and α^*- closed sets B such that $A \cap B = \varphi$, there exist disjoint α^*- open set U and V such that $A \cap U \neq \varphi$ and B ⊆ V.

Proof:
(i)⟹(ii): Let U be a α^*-open set containing x. Then $B = X \setminus U$ is a α^*- closed not containing x. Since X is α^*- regular, there exist disjoint α^*-open sets V and W containing x and B respectively. If $y \epsilon$ B, W is a α^*- openset containing y that does not intersect V. Therefore $\alpha^* Cl \subseteq U$.

(ii) ⟹(iii): Let $A \cap B \neq \varphi$ and B is α^*- open. Let x $A \cap B$. Then by assumption, there exists a α^*- open set U containing x such that $\alpha^* Cl \subseteq B$.Since x ⊆ A, $A \cap U \neq \varphi$. This proves (iii).

(iii) ⟹ (iv): Suppose $A \cap B = \varphi$, where A is non – emety and B is α^*- closed. Then $X \setminus B$ is α^*- open set and $\cap (X \setminus B) \neq \varphi$. By (iii) there exist a α^* - open set U such that $A \cap U \neq \varphi$, and $\subseteq \alpha^* Cl(U) \subseteq X \setminus B$. Put $V = X \setminus \alpha^* Cl(U)$. Hence V is a α^*-open set containing B such that $U \cap V = U \cap (X \setminus \alpha^* Cl(U)) \subseteq X \setminus U = \varphi$.This proves (iv).

(iv) ⟹(i): Let B be α^*- closed and $x \notin B$. Take A=$\{x\}$. Then $A \cap B = \varphi$.By (iv), there exist disjoint α^*- open sets U and V such that $U \cap A \neq \varphi$ and B ⊆ V. Since $U \cap V \neq \varphi$, $x \in U$. This proves that X is α^*- regular.

Theorem 3.3: Let X be a α^*- regular space.

(i) Every α^*- open set in X is a union of α^*- closed sets.
(ii) Every α^*- closed set in X is an intersection of α^*- open sets.

Proof:

(i) Suppose X is α^*- regular. Let G be a α^*- open set and x ∈ G. Then F = X \ G is α^*- closed and x ∉ F. Since X is α^*- regular, there exist disjoint α^*-open sets U_x and V in X such that x ∈ U_x and F ⊆ V. Since $U_x \cap F \subseteq U_x \cap V = \phi$, we have $U_x \subseteq X \setminus F = G$. Take $V_x = \alpha^* Cl(U_x)$. Then V_x is α^* - closed and $V_x \cap V = \phi$. Now F ⊆ V implies that $V_x \cap F \subseteq V_x \cap V = \phi$. It follows that $x \in V_x \subseteq X \setminus F = G$. This proves that $G = \cup \{V_x : x \in G\}$. Thus G is a union of α^* - closed sets.

(ii) Follows from (i) and set theoretic properties.

Theorem 3.4: If f is a α^* - irresolute and pre - α^* - closed injection of a topological space X into a α^* - regular space Y, then X is α^* - regular.

Proof: Let x ∈ X and A be a α^* - closed set in X not containing x. Since f is pre- α^* -closed, f (A) is a α^* - closed set in Y not containing f (x). Since Y is α^* - regular, there exist disjoint α^*-open sets V_1 and V_2 in Y such that f(x) ∈V_1 and f(A)⊆V_2. Since f is α^* - irresolute, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint α^* - open sets in X containing x and A respectively. Hence X is α^* - regular.

Theorem 3.5: If f is a α^* - continuous and closed injection of a topological space X into a regular space Y and if every α^* - closed set in X is closed, then X is α^* - regular.

Proof: Let x ∈ X and A be a α*- closed set in X not containing x. Then by assumption, A is closed in X. Since f is closed, f (A) is a closed set in Y not containing f(x). Since Y is regular, there exist disjoint open sets V_1 and V_2 in Y such that f (x) ∈ V_1 and f (A) ⊆ V_2. Since f is α*-continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint α*-open sets in X containing x and A respectively. Hence X is α*- regular.

Theorem 3.6: If f: X⟶Y is a α*-irresolute bijection which is pre-α*-open and X is α*-regular. Then Y is also α*-regular.

Proof: Let f : X⟶Y be a α*- irresolute bijection which is α* - open and X be α* - regular. Let y ∈Y and B be a α*-closed set in Y not containing y. Since f is α*- irresolute, f^{-1}(B) is a α*-closed set in X not containing f^{-1}(y). Since X is α*-regular, there exist disjoint α*-open sets U_1 and U_2 containing f^{-1}(y) and f^{-1}(B) ⊆ U_2 respectively. Since f is pre-α*-open, $f(U_1)$ and $f(U_2)$ are disjoint α*- open sets in Y containing y and B respectively. Hence Y is α*-regular.

Theorem 3.7: If f is a continuous α*-open bijection of a regular space X into a space Y and if every α*-closed set in Y is closed, then Y is α*-regular.

Proof: Let y∈Y and B be a α*-closed set in Y not containing y. Then by assumption, B is closed in Y. Since f is a continuous bijection, f^{-1}(B) is a closed set in X not containing the point f^{-1}(y). Since X is regular, there exist disjoint open sets U_1 and U_2 in X such that f^{-1}(y)∈U_1 and f^{-1}(B) ⊆ U_2. Since f is α*-open, $f(U_1)$ and $f(U_2)$ are disjoint α*-open sets in Y containing x and B respectively. Hence Y is α*-regular.

IV. NORMAL SPACES ASSOCIATED WITH α*- OPEN SETS

In this section we introduce a normal spaces namely α* - normal spaces and investigate their basic properties.

Definition 4.1: A space X is said to be ***α*-normal*** if for every pair of disjoint α*-closed sets A and B in X, there are disjoint α*-open sets U and V in X containing A and B respectively.

Theorem 4.2: In a topological space X, the following are equivalent:
- (i) X is α*-normal.
- (ii) For every α*-closed set A in X and every α*-open set U containing A, there exists a α*-open set V containing A such that *α*Cl*(V) ⊆ U.
- (iii) For each pair of disjoint α*-closed sets A and B in X, there exists a α*-open set U containing A such that *α*Cl*(U)∩B=ϕ.
- (iv) For each pair of disjoint α*-closed sets A and B in X, there exist α*-open sets U and V containing A and B respectively such that *α*Cl*(U)∩*α*Cl*(V)=ϕ.

Proof:
(i) ⟹ (ii): Let U be a α*-open set containing the α*-closed set A. Then B=X\U is a α*-closed set disjoint from A. Since X is α*-normal, there exist disjoint α*-open sets V and W containing A and B respectively. Then *α*Cl*(V) is disjoint from B, since if y∈B, the set W is a α*-open set containing y disjoint from V. Hence *α*Cl*(V) ⊆ U.

(ii) ⟹ (iii): Let A and B be disjoint α*-closed sets in X. Then X\B is a α*-open set containing A. By (ii), there exists a α*-open set U containing A such that *α*Cl*(U) ⊆ X\B. Hence *α*Cl*(U)∩B=ϕ. This proves (iii).

(iii) ⟹ (iv): Let A and B be disjoint α*-closed sets in X. Then, by (iii), there exists a α*-open set U containing A such that *α*Cl*(U)∩B=ϕ. Since *α*Cl*(U) is α*-closed**,** B and *α*Cl*(U) are disjoint α*-closed sets in X. Again by (iii), there exists a α*-open set V containing B such that *α*Cl*(U) ∩*α*Cl*(V) = ϕ. This proves (iv).

(iv) ⟹ (i): Let A and B be the disjoint α*-closed sets in X. By (iv), there exist α*-open sets U and V containing A and B respectively such that *α*Cl*(U) ∩ *α*Cl*(V)=ϕ. Since U∩V ⊆ *α*Cl*(U) ∩ *α*Cl*(V), U and V are disjoint α* - open sets containing A and B respectively. Thus X is α*-normal.

Theorem 4.3: For a space X, then the following are equivalent:

(i) X is α*-normal.
(ii) For any two α*-open sets U and V whose union is X, there exist α*-closed subsets A of U and B of V whose union is also X.

Proof:

(i) ⟹ (ii): Let U and V be two α*-open sets in a α*- normal space X such that X = U ∪ V. Then X \ U, X \ V are disjoint α*- closed sets. Since X is α*- normal, there exist disjoint α*-open sets G_1 and G_2 such that $X\setminus U\subseteq G_1$ and $X\setminus V\subseteq G_2$. Let A =X \ G_1 and B = X \ G_2. Then A and B are α*- closed subsets of U and V respectively such that A∪B=X. This proves (ii).

(ii) ⟹ (i): Let A and B be disjoint α*-closed sets in X. Then X \ A and X \ B are α*- open sets whose union is X. By (ii), there exists α*- closed sets F_1 and F_2 such that $F_1 \subseteq X \setminus A$, $F_2 \subseteq X \setminus B$ and $F_1 \cup F_2 = X$. Then X \ F_1 and X \ F_2 are disjoint α*- open sets containing A and B respectively. Therefore X is α*-normal.

Theorem 4.4: If *f* is an injective and α*-irresolute and pre-α*-closed mapping of a topological space X into a α*-normal space Y, then X is α*-normal.

Proof: Let *f* be an injective and α*-irresolute and pre-α*-closed mapping of a topological space X into a α*-normal space Y. Let A and B be disjoint α*-closed sets in X. Since *f* is a pre-α*-closed function, *f* (A) and *f* (B) are disjoint α*-closed sets in Y. Since Y is α*-normal, there exist disjoint α*-open sets V_1 and V_2 in Y containing *f* (A) and *f* (B) respectively. Since *f* is α*-irresolute, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint α*-open sets in X containing A and B respectively. Hence X is α*-normal.

Theorem 4.5: If *f* is an injective and α*-continuous and closed mapping of a topological space X into a normal space Y and if every α*-closed set in X is closed, then X is α*- normal.

Proof: Let A and B be disjoint α*-closed sets in X. By assumption, A and B are closed in X. Then *f* (A) and *f* (B) are disjoint closed sets in Y. Since Y is normal, there exist disjoint open sets V_1 and V_2 in Y such that $f(A) \subseteq V_1$ and $f(B) \subseteq V_2$.Then $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint α*-open sets in X containing A and B respectively. Hence X is α*- normal.

Theorem 4.6: If *f* : X⟶Y is a α*- irresolute injection which is pre-α*-open and X is α*- normal, then Y is also α*-normal.

Proof: Let *f* : X⟶Y be a α*-irresolute surjection which is α*- open and X be α*- normal. Let A and B be disjoint α*- closed sets in Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint α*- closed sets in X. Since X is α*-normal, there exist disjoint α*-open sets U_1 and U_2 containing $f^{-1}(A)$ and $f^{-1}(B)$ respectively. Since *f* is pre - α*- open, $f(U_1)$ and $f(U_2)$ are disjoint α*- open sets in Y containing A and B respectively. Hence Y is α* - normal.

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