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A COMMON FIXED POINT THEOREM IN INTUITIONISTIC MENGER (PQM) SPACE WITH USING PROPERTY (E.A.)

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ABSTRACT

In this paper deals a common fixed point theorem in intuitionistic menger space.

Key words: *fixed point, common fixed point theorem , menger space , intuitionistic menger space.*

INTRODUCTION

- There have been a number of generalizations of metric space. One such generalization is Menger space introduced in 1942 by Menger [98] who used distribution functions instead of nonnegative real numbers as values of the metric.
- In fact, he replaced the distance function $d: X \times X \to \mathfrak{R}^+$ with a distribution function $F_{p,q}: \mathfrak{R} \to [0,1]$ wherein for any number x, the value $F_{p,q}(x)$ describes the probability that the distance between p and q is less than.
- Schweizer and Sklar [118] studied this concept and then the important development of Menger space theory was due to Sehgal and Bharucha-Reid [122]. Sessa [123] introduced weakly commuting maps in metric spaces.
- Jungck [81] enlarged this concept to compatible maps. The notion of compatible maps in Menger spaces has been introduced by Mishra [99].
- Aamri and Moutawakil [1] and Liu *et al*. [92] respectively defined the property (E.A) and common property (E.A) and proved some common fixed point theorems in metric spaces.
- Imdad *et al.* [77] extended the results of Aamri and Moutawakil [1] to semi-metric spaces. Most recently, Kubiaczyk and Sharma [90] defined the property (E.A) in PM spaces and
- used the same to prove some results on common fixed points wherein authors claim their results for strict contractions which are in fact proved for contractions.
- Kutukcu *et al.* [91] defined the notion of intuitionistic Menger spaces with the help of t-norms and t−conorms as a generalization of Menger spaces due to Menger [96].On the other hand Rezaiyan et. al. [106] prove fixed point theorem for Menger (PQM) space which is modified by Mihet [98].
- The aim of this paper is to prove a hard and fast purpose theorem in Intuitionistic Menger (PQM) area mistreatment property E.A. for this 1st we have a tendency to offer some definitions and notable results that square measure utilized in this paper

Definition 7.1.1: A binary operation $T : [0,1] \times [0,1] \to [0,1]$, is a t-norm if T satisfies the following conditions:
71.1 (i) T is commutative and associative.
7.1.1 (ii) $T(a,1) = a$ for all $a \in [0,1]$.
7.1.1 (iii) $T(a,b) \leq T(c,d)$ whenever $a \leq c$ and $b \leq d$
For $a, b, c, d \in [0,1]$.

Definition 7.1.2: A binary operation $S : [0,1] \times [0,1] \to [0,1]$ is a t−conorm if S satisfies the following conditions:
7.1.2 (i) S is commutative and associative.
7.1.2 (ii) $S(a,0) = a$ for all $a \in [0,1]$
7.1.2 (iii) $S(a,b) \leq S(c,d)$ whenever $a \leq c$ and $b \leq d$
For $a, b, c, d \in [0,1]$.

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Remark 7.1.3: The concepts of t−norm T and t−conorm S are known as the axiomatic skeletons that we use $T : [0,1] \times [0,1] \to [0,1]$, for characterizing fuzzy intersections and unions respectively. Throughout this chapter, we will denote $R = (-\infty, \infty)$ and $R^+ = [0, \infty)$.

Definition 7.1.4: A distance distribution function is a function $F: R \to R^+$, which is left continuous on R, non-decreasing $T : [0,1] \times [0,1] \to [0,1]$, and $\inf_{t\in R} F(t) = 0$, $\sup_{t\in R} F(t) = 1$. We will denote by D, the family of all distance distribution functions and by H a special element of D defined by

$$H(t) = \begin{cases} 0, & if\ t \le 0 \\ 1, & if\ t > 0 \end{cases}$$

If X is a nonempty set, $F: X \times X \to D$ is called a probabilistic distance on X.

Definition 7.1.5: A non-distance distrib $T : [0,1] \times [0,1] \to [0,1]$, ution function is a function $L: R \to R^+$, which is right continuous on R, non-increasing and $\inf_{t\in R} L(t) = 1$, $\sup_{t\in R} L(t) = 0$.

We will denote by E, the family of all non-distance distribution functions and by G a special element of E defined by

$$G(t) = \begin{cases} 1, & if\ t \le 0 \\ 0, & if\ t > 0 \end{cases}$$

If X is a nonempty set, $L: X \times X \to D$ is called a probabilistic non-distance on X.

Definition 7.1.6 A triple $(X, F, L,)$ is said to be **an intuitionistic Menger(PQM)** space if X is a nonempty set, F is a probabilistic distance and L is probabilistic non-distance on X satisfying the following conditions:
For all $x, y, z \in X$ and $s, t > 0$

7.1.6 (i) $F_{x,y}(t) + L_{x,y}(t) \le 1$
7.1.6 (ii) $F_{x,y}(t) = 0$
7.1.6 (iii) $F_{x,y}(t) = H(t)$for all $t > 0$ $if\ and\ only\ if\ x = y$
7.1.6 (iv) $F_{x,y}(t) = F_{y,x}(t)$
7.1.6 (v) $L_{x,y}(0) = 1$
71.6 (vi) $L_{x,y}(t) = G(t)$for all $t > 0$ $if\ and\ only\ if\ x = y$
7.1.6 (vii) $L_{x,y}(t) = L_{y,x}(t)$
If in addition, we have the triangle inequalities:
7.1.6 (viii) $F_{x,y}(t + s) \ge T\left(F_{x,z}(t), F_{z,y}(t)\right)$.
7.1.6 (ix) $L_{x,y}(t + s) \le S(L_{x,z}(t), L_{z,y}(t))$.

Here T is a t−norm and S is a t−conorm. Then (X, F, L, T, S) is said to be an

INTUITIONISTIC MENGER (PQM) SPACE

The Functions $F_{x,y}(t)$ and $L_{x,y}(t)$ Denote The degree of nearness and degree of non-nearness between x and y with respect to t respectively.

Remark 7.1.7: Every Menger (PQM) space (X, F, T) is an intuitionistic Menger (PQM) space of the form $(X, F, 1 - F, T, S)$ such that t−norm T and t−conorm S are associated
i.e. $S(x, y) = 1 - T(1 - x, 1 - y)$ for any $x, y \in X$.

Example 7.1.8: (Induced intuitionistic Menger (PQM)space) Let (X, d)be a metric space. Then the metric d induces a distance distribution function F defined by

$$F_{x,y}(t) = H(t - d(x, y))$$

and a non-distance distribution function L defined by

$$L_{x,y}(t) = G(t - d(x, y)) \text{ for all } x, y, \in X \text{ and } t \ge 0.$$

Then (X, F, L) IS AN INTUITIONISTIC MENGER (PQM) SPACE.

We call this Intuitionistic Menger (PQM) space induced by a metric d the induced intuitionistic Menger (PQM) space. If t−norm T is $T(a, b) = \min\{a, b\}$ and t−conorm S is $S(a, b) = \min\{1, a + b\}$ for all $a, b \in [0, 1]$, then (X, F, L, T_M, S_M) is an intuitionistic Menger (PQM)space.

Remark 7.1.9: Note that the above examples hold even with the t−norm $T(a, b) = \min\{a, b\}$ and t−conorm $S(a, b) = \max\{a, b\}$, and hence (X, F, L, T, S) is an INTUITIONISTIC MENGER SPACE WITH RESPECT TO ANY T−NORM AND T−CONORM.Also note that, $T : [0,1] \times [0,1] \to [0,1]$, in the above example the t−norm T and t−conorm S are not associated.

Definition 7.1.10: Let (X, F, L, T, S) be a Intuitionistic Menger (PQM) space.
(i) A sequence $\{x_n\}$ in X is $T : [0,1] \times [0,1] \to [0,1]$, said to be convergent to x in X, if for every $\varepsilon > 0$, $\lambda > 0$, there exists positive integer N such that

$$F_{x_n,x}(\varepsilon) > 1 - \lambda \text{ and } L_{x_n,x}(\varepsilon) < \lambda \text{ whenever } n \geq N.$$

we write $x_n \to x$ as $n \to \infty$ or $\lim_{n\to\infty} x_n = x$.
(ii) A sequence $\{x_n\}$ in X is called cauchy sequence, if for every $\varepsilon > 0$, $\lambda > 0$,, there exists positive integer N such that

$$F_{x_n,x_m}(\varepsilon) > 1 - \lambda \text{ and } L_{x_n,x_m}(\varepsilon) < \lambda \text{ whenever } n \geq N \geq m \geq N.$$

(iii) A Menger (PQM) space $(X, F, T)$is said to be complete if and only if every cauchy sequence in X is convergent to a point in X.

Lemma 7.1.11: Let (X, F, L, T, S) be an intuitionistic Menger space. If there is a constant $k \in (0,1)$ such that for $x, y \in X$, $t > 0$, $F_{x,y}(kt) \geq F_{x,y}(t)$ and $L_{x,y}(kt) \leq L_{x,y}(t)$, then $x = y$.

Lemma 7.1.12: let (X, F, L, T, S) be an intuitionistic menger space. then $f_{x,y}(t)$ and $l_{x,y}(t)$ are continuous functions on $x \times x \to (0, \infty)$.

Definition 7.1.13: Let (X, F, L, T, S) be a intuitionistic menger (pqm) space such that the t-norm t and t−conorm s is continuous and P, Q be mappings from X into itself. Then, P and Q are said to be compatible if $\lim_{n\to\infty} F_{PQx_n,QPx_n}(x) = 1$ and $\lim_{n\to\infty} L_{PQx_n,QPx_n}(x) = 0$ for all $x > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = z$ for some $z \in X$.

Definition 7.1.14: Two self mappings P and Q are said to be weakly compatible if they commute at their $T : [0,1] \times [0,1] \to [0,1]$, coincidence points that is $Px = Qx$. For some $x \in X$ implies $PQx = QPx$.

Definition 7.1.15: let P and Q be two self mappings of a menger space (X, F, L, T, S) we say that p and q satisfy the property (E.A) if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = z$$

for some $z \in X$.

Example 7.1.16: Let $X = [0, +\infty)$. Define $P, Q : X \to X$ by

$$Px = \frac{7x}{5} \text{ and } x = \frac{3x}{8}, \forall x \in X.$$

Consider the sequence $x_n = \frac{1}{n}$.Clearly

$$\lim_{n\to\infty} x_n = Px_n = \lim_{n\to\infty} x_n = Qx_n = 0$$

Then P and Q satisfy (E, A).

Example 7.1.17: Let $X = [2, +\infty)$. Define $P, Q : X \to X$ by

$$Px = x + 1 \text{ and } x = 2x + 1 \ \forall x \in X.$$

Suppose that the property (E.A.) holds.

Then there is $T : [0,1] \times [0,1] \to [0,1]$, a sequence $\{x_n\}$ in X satisfying

$$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = z \text{ for some } z \in X$$

Therefore $\lim_{n\to\infty} x_n = z - 1$ and $\lim_{n\to\infty} x_n = \frac{z-1}{2}$.

Thus, $z = 1$, which is a contradiction since $1 \notin X$.

Hence P and Q do not satisfy (E.A.). $T : [0,1] \times [0,1] \to [0,1]$,

7.2. COMMON FIXED POINT THEOREM IN INTUITIONISTIC MENGER SPACES

Theorem 7.2.1: Let (X, F, L, T, S) be a Intuitionistic Menger (PQM) space with $T(x, y) = \min\{x, y\}$ and $S(x, y) = \max\{x, y\}$ for all $x, y \in [0, 1]$. Let A, B, P and Q be mappings from X into itself such that:

7.2.1 (I) $A(X) \subset P(X)$ and $B(X) \subset Q(X)$.
7.2.1 (II) (A, Q) or (B, P) satisfies the property (E.A).
7.2.1 (III) There exists a number $k \in (0, 1)$ such that

$$\min\left\{\begin{matrix}(F_{Au,Bv}\,kx)(F_{Qu,Pv}(x), F_{Qu,Bv}(x), F_{Pv,Bv}(x), \\ F_{Au,Qu}(x), F_{Au,Pv}(x), F_{Qu,Qu}(x)\end{matrix}\right\}^2 \geq 0$$

$$\min\left\{\begin{matrix}L_{Au,Bv}(kx)L_{Qu,Pv}(x), L_{Qu,Bv}(x), L_{Pv,Bv}(x), \\ L_{Au,Qu}(x), L_{Au,Pv}(x)L_{Qu,Qu}(x)\end{matrix}\right\}^2 \leq 0$$

for all $u, v \in X$.

7.2.1 (IV) (A , Q) and (B , P) are weakly compatible,

7.2.1 (V) One of A(X), B(X), Q(X) or P(X) is a closed subset of X.

Then A, B , P and Q have a unique common fixed point in X.

Proof: Suppose that (B , P) satisfies the $T : [0,1] \times [0,1] \to [0,1]$, property (E.A). Then there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = z$$

for some $z \in X$.

Since $B(X) \subset Q(X)$, there exists in X a sequence $\{y_n\}$ such that

$$Bx_n = Qy_n.$$

Hence $\lim_{n\to\infty} Qy_n = z$.

Let us show that $\lim_{n\to\infty} Ay_n = z$.

$$\min\left\{\begin{matrix}F_{Ay_n,Bx_n}(kx)F_{Qy_n,Px_n}(x), F_{Qy_n,Bx_n}(x), F_{Px_n,Bx_n}(x), \\ F_{Ay_n,Qy_n}(x), F_{Ay_n,Px_n}(x)F_{Qy_n,Qx_n}(x)\end{matrix}\right\}^2 \geq 0$$

$$\geq \min\left\{\begin{matrix}F_{Ay_n,Bx_n}(kx)F_{Bx_n,Px_n}(x), F_{Px_n,Bx_n}(x), \\ F_{Ay_n,Bx_n}(x), F_{Ay_n,Px_n}(x)\end{matrix}\right\}^2$$

$$\geq \{F_{Ay_n,Bx_n}(x)\}^2 \min\left\{\begin{matrix}L_{Ay_n,Bx_n}(kx)L_{Qy_n,Px_n}(x), L_{Qy_n,Bx_n}(x), L_{Px_n,Bx_n}(x), \\ L_{Ay_n,Qy_n}(x), L_{Ay_n,Px_n}(x)L_{Qy_n,Qx_n}(x)\end{matrix}\right\}^2 \leq 0$$

$$\leq \min\left\{\begin{matrix}L_{Ay_n,Bx_n}(kx)L_{Bx_n,Px_n}(x), L_{Px_n,Bx_n}(x), \\ L_{Ay_n,Bx_n}(x), L_{Ay_n,Px_n}(x)\end{matrix} \leq 0\right\}^2$$

$$\leq \min\left\{\begin{matrix}L_{Ay_n,Bx_n}(kx)L_{Bx_n,Px_n}(x), L_{Px_n,Bx_n}(x), \\ L_{Ay_n,Bx_n}(x), L_{Ay_n,Px_n}(x)\end{matrix}\right\}^2$$

$$\leq \{L_{Ay_n,Bx_n}(x)\}^2 \{\text{k belong to (zero , one)}$$

Therefore with the Lemma (7.1.11) $Ay_n = Bx_n$.

Letting $n \to \infty$, we obtain

$$\min\left\{\begin{matrix}L_{Ay_n,Bx_n}(kx)L_{Qy_n,Px_n}(x), L_{Qy_n,Bx_n}(x), L_{Px_n,Bx_n}(x), \\ L_{Ay_n,Qy_n}(x), L_{Ay_n,Px_n}(x)L_{Qy_n,Qx_n}(x)\end{matrix}\right\}^2 \leq 0$$

$$\leq \min\left\{\begin{matrix}L_{Ay_n,Bx_n}(kx)L_{Bx_n,Px_n}(x), L_{Px_n,Bx_n}(x), \\ L_{Ay_n,Bx_n}(x), L_{Ay_n,Px_n}(x)\end{matrix}\right\}^2 \leq 0$$

$$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Ay_n = z. \{\text{k belong to (zero, one)}$$

Suppose Q(X)is a closed subset of X. Then $z = Qu$ for some $u \in X$.

Subsequently, we have

$$\min\left\{\begin{matrix}L_{Ay_n,Bx_n}(kx)L_{Qy_n,Px_n}(x), L_{Qy_n,Bx_n}(x), L_{Px_n,Bx_n}(x), \\ L_{Ay_n,Qy_n}(x), L_{Ay_n,Px_n}(x)L_{Qy_n,Bx_n}(x)\end{matrix}\right\}^2 \leq 0$$

$$\min\left\{\begin{matrix}L_{Ay_n,Bx_n}(kx)L_{Bx_n,Px_n}(x), L_{Px_n,Bx_n}(x), \\ L_{Ay_n,Bx_n}(x), L_{Ay_n,Px_n}(x)\end{matrix}\right\}^2 \leq 0 \{\text{k belong to (zero, one)}$$

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$

We have

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$

$$\min\begin{Bmatrix} F_{Au,Bx_n}(kx)F_{Qu,Px_n}(x), F_{Qu,Bx_n}(x), F_{Px_n,Bx_n}(x), \\ F_{Au,Qu}(x), F_{Au,Px_n}(x)F_{Qy_n,Qx_n}(x) \end{Bmatrix}^2 \geq 0$$

$$\min\begin{Bmatrix} L_{Au,Bx_n}(kx)L_{Qu,Px_n}(x), L_{Qu,Bx_n}(x), L_{Px_n,Bx_n}(x), \\ L_{Au,Qu}(x), L_{Au,Px_n}(x)L_{Qy_n,Qx_n}(x) \end{Bmatrix}^2 \leq 0 \text{ \{k belong to (zero, one)}$$

Letting $n \to \infty$, we obtain

$$\{F_{Au,Su}(kx)F_{Au,Su}(x)\}^2 \geq 0$$
$$\{L_{Au,Su}(kx)L_{Au,Su}(x)\}^2 \leq 0$$

Therefore with the Lemma (7.1.11) we have

$$Au = Qu.$$

The weak compatibility of A and Q implies that

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$
$$AQu = QAu$$

and then $AAu = AQu = QAu = QQu$.

On the other hand, since $A(X) \subset P(X)$, there exists a point $v \in X$, such that $Au = Pv$.

We claim that $Pv = Bv$.

We have

$$\min\begin{Bmatrix} F_{Au,Bv}(kx)F_{Qu,Pv}(x), F_{Qu,Bv}(x), F_{Pv,Bv}(x), \\ F_{Au,Qu}(x), F_{Au,Pv}(x)F_{Qy_n,Qx_n}(x) \end{Bmatrix}^2 \geq 0$$
$$\geq \{F(Au, Bv)(x)\}^2 \text{ \{k belong to (zero , one)}$$
$$\min\begin{Bmatrix} L_{Au,Bv}(kx)\, L_{Qu,Pv}(x), L_{Qu,Bv}(x), L_{Pv,Bv}(x), \\ L_{Au,Qu}(x), L_{Au,Pv}(x)F_{Qy_n,Qx_n}(x) \end{Bmatrix}^2 \leq 0$$
$$\leq \{ L_{Au,Bv}(x)\}.^2 \text{\{k belong to (zero, one)}$$

Therefore, with the Lemma (7.1.11) we have $Au = Bv$.

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$

Thus $Au = Qu = Pv = Bv$.

The weak compatibility of B and P implies that

$$BPv = PBv \text{ and } PPv = PBv = BPv = BBv.$$

Let us show that Au is a common fixed point of A, B, P and Q.

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$

We have

$$\text{Min}\begin{Bmatrix} F_{AAu,Bv}(kx)\, F_{QAu,Pv}(x), F_{QAu,Bv}(x), F_{Pv,Bv}(x), \\ F_{AAu,QAu}(x), F_{AAu,Pv}(x)F_{Qy_n,Qx_n}(x) \end{Bmatrix}^2 \geq 0 \quad \text{\{k belong to (zero , one)}$$
$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$
$$\{F_{Au,AAu}(kx)\, F_{AAu,Av}(x)\}^2 \geq 0$$
$$\min\begin{Bmatrix} L_{AAu,Bv}(kx)\, L_{QAu,Pv}(x), L_{QAu,Bv}(x), L_{Pv,Bv}(x), \\ L_{AAu,QAu}(x), L_{AAu,Pv}(x)L_{Qy_n,Qx_n}(x) \end{Bmatrix}^2 \leq 0 \text{ \{k belong to (zero, one)\}}$$
$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$
$$\{L_{Au,AAu}(kx)\}^2 \quad \leq \{L_{AAu,Av}(x)\}^2$$

Therefore, we have

$$Au = AAu = QAu$$

That is Au is a common fixed point of A and Q.

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$

Similarly, we can prove that Bv is a common fixed point of B and P.

Since $\mathrm{Au} = \mathrm{Bv}$, we conclude that Au is a common fixed point of $\mathrm{A, B, P}$ and Q.

The proof is similar when P(X) is assumed to be a closed subset of X.

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$

The cases in which A(x) or B(x) is closed subset of X are similar to the cases in which P(X) or Q(X), respectively, is closed

Since $\quad A(X) \subset P(X)$ and $B(X) \subset Q(X)$.

If $Au = Bu = Su = Lu = u$ and $Av = Bv = Sv = Lv = v$.

We have

$$\min\begin{Bmatrix} F_{Au,Bv}(kx)\, F_{Qu,Pv}(x), F_{Qu,Bv}(x), F_{Pv,Bv}(x), \\ F_{Au,Qu}(x), F_{Au,Pv}(x) F_{Qy_n,Qx_n}(x) \end{Bmatrix}^2 \geq 0$$

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu \geq \{F_{u,v}(x)\}^2.$$

$$\min\begin{Bmatrix} L_{Au,Bv}(kx)\, L_{Qu,Pv}(x), L_{Qu,Bv}(x), L_{Pv,Bv}(x), \\ L_{Au,Qu}(x), L_{Au,Pv}(x) L_{Qy_n,Qx_n}(x) \end{Bmatrix}^2 \leq 0$$

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu \leq \{L_{u,v}(x)\}^2.$$

Thus we have $u = v$ and the common fixed point is unique.

This completes the proof of the theorem.

For three mapping, we have the following result:

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$

Corollary 7.2.2: Let (X, F, L, T, S) be an Intuitionistic Menger (PQM) space with $T(x, y) = \min\{x, y\}$ and $S(x, y) = \max\{x, y\}$ for all $x, y \in [0,1]$.

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$

Let A, B and P be mappings from X into itself such that:

7.2.2 (I) $A(X) \subset P(X)$ and $B(X) \subset P(X)$

7.2.2 (II) (A, P) or (B, P) satisfies the property (E.A.),

7.2.2 (III) There exists a number $k \in (0,1)$such that

$$\min\begin{Bmatrix} F_{Au,Bv}(kx) F_{Pu,Pv}(x), F_{Pu,Bv}(x), \\ F_{Pv,Bv}(x), F_{Au,Pu}(x), F_{Au,Pv}(x) F_{Qy_n,Qx_n}(x) \end{Bmatrix} \geq 0$$

$$\min\begin{Bmatrix} L_{Au,Bv}(kx)\, L_{Pu,Pv}(x), L_{Pu,Bv}(x), \\ L_{Pv,Bv}(x), L_{Au,Pu}(x), L_{Au,Pv}(x) F_{Qy_n,Qx_n}(x) \end{Bmatrix} \leq 0$$

for all $u, v \in X$.

7.2.2 (IV) (A, P) and (B, P) are weakly compatible,

7.2.2 (V) One of $A(X), B(X)$ or$P(X)$ is a closed subset of X.

Then A, B and P have a unique common fixed point in X.

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$

Corollary 7.2.3: Let (X, F, L, T, S) be a Intuitionistic Menger (PQM) space with $T(x, y) = \min\{x, y\}$ and $S(x, y) = \max\{x, y\}$ for all $x, y \in [0,1]$.

Let Aand P be mappings from X into itself such that:

7.2.3 (I) $A(X) \subset P(X)$.

7.2.3 (II) (A, P) satisfies the property (E.A),

7.2.3 (III) There exists a number $k \in (0,1)$ such that

$$\min\begin{Bmatrix} F_{Au,Av}(kx) F_{Pu,Pv}(x), F_{Pu,Av}(x), \\ F_{Pv,Av}(x), F_{Au,Pu}(x), F_{Au,Pv}(x) F_{Qy_n,Qx_n}(x) \end{Bmatrix}^2 \geq 0$$

$$\min\left\{\begin{matrix} L_{Au,Av}(kx)L_{Pu,Pv}(x), L_{Pu,Av}(x), \\ L_{Pv,Av}(x), L_{Au,Pu}(x), L_{Au,Pv}(x)L_{Qy_n,Qx_n}(x) \end{matrix}\right\}^2 \leq 0$$

for all $u, v \in X$.

7.2.3 (IV) (A, P) be weakly compatible,

7.2.3 (V) One of $A(X)$ or$P(X)$ is a closed subset of X.

Then A and P have a unique common fixed point in X.

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$

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