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FIXED POINT THEOREM FOR COMPLETE METRIC SPACES USING MEIR-KEELER TYPE CONTRACTIONS

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ABSTRACT

In this paper we obtain a new fixed point theorem for rational expressions using Meir-Keeler type contractions in complete metric spaces. The presented theorem is an extension of the result of Dass and Gupta (1975). We also gave some applications to contractions of integral type.

Mathematical Subject Classification: *54 H10.*

Key Words: *Fixed Point, Meir-Keeler Type Contraction, Complete Metric Space.*

1. INTRODUCTION

Fixed point theory is an important and real topic of nonlinear analysis. If T is self mapping of a metric space (X, d), then a point $x \epsilon X$ is called fixed point of T if $Tx = x$. The application of fixed point theorem has been observed in many fields of engineering and science. The Banach contraction principle is in its core. Many researchers tried to generalize and extend it in different aspects. Also some of them improved in different ways.

Some authors dealt with the contractive condition of Meir-Keeler type [2] [9] [10] [17], some extended the theorem to more generalized metric-type spaces[1][3][5][7][15-16][21] and others applied to common [4], coupled and tripled versions ([6,22] and the references therein).

In **1969** Meir and Keeler [17] established a fixed point theorem in a metric space $(X, d)$for mappings satisfying the following condition, called the Meir-Keeler type contractive condition:
$\forall \epsilon > 0, \exists\, \delta > 0$: $\epsilon \leq d(x, y) < \delta + \epsilon$ implies $d(fx, fy) < \epsilon$
Then T has a unique fixed point $\xi \in X$. Moreover, for all $x \in X$, the sequence$\{T^n x\}$ converges toξ.

The above theorem is a generalization of the Banach contraction principle. Some generalizations of in above theorem exist in literature.

In **1973** Dass and Gupta [12] proved the following fixed point theorem.

Theorem 1.1: Let (X, d)be a complete metric space and let T be a mapping from X into itself satisfying

$$d(Tx, Ty) \leq \alpha d(y, Ty)\frac{1+d(x,Tx)}{1+d(x,y)} + \beta d(x, y),$$

for all $x, y \in X$ where α, β are constants with $\alpha, \beta > 0$ and $\alpha + \beta < 1$. Then T has a unique fixed point $\xi \in X$. Moreover, for all $x \in X$,the sequence $\{T^n x\}$ converges to ξ.

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In **1978** Maiti and Pal [14] generalized a fixed point for maps satisfying the following condition:

$$\forall\varepsilon > 0, \exists\delta > 0: \varepsilon \leq max\{d(x,y), d(x,fx), d(y,fy)\} < \delta + \varepsilon$$

implies $d(fx,fy) < \epsilon$

Later in **1981,** Park and Rhoades in [18] established fixed point theorems for a pair of mappings f, g satisfying a contractive condition that can be reduced to the following generalization of (2) when $f = gf = g$.

$$\forall\varepsilon > 0, \exists\delta > 0;\ \varepsilon \leq max\left\{d(x,y), d(x,fx), d(y,fy), \frac{d(x,fy) + d(y,fx)}{2}\right\} < \delta + \varepsilon$$

implies $d(fx,fy) < \varepsilon$

Branciari [11] obtained a fixed point result for a single mapping satisfying an analogue of Banach's contraction principle for an integral-type inequality. **In 2007** Altun Ishak,, Trkoglu Duran and. Rhoades Billy E. [8] proved following fixed point theorem for weakly compatible maps satisfying a general contractive condition of integral type.

Theorem 1.2: Let A, B, S, and T be self-maps defined on a metric space (X, d) satisfying the following conditions:
(i) $S(X) \subseteq B(X), T(X) \subseteq A(X)$,
(ii) for all $x, y \in X$, there exists a right continuous function $\psi : R^+ \to R^+, \psi(0) = 0$, and $\psi(s) < s$ for $s > 0$ such that

$$\int_0^{d(Sx,Ty)} \varphi(t)dt \leq \psi\left(\int_0^{M(x,y)} \varphi(t)dt\right)$$

where $\phi : R^+ \to R^+$ is a Lebesque integrable mapping which is summable, nonnegative and such that

$$\int_0^{\varepsilon} \varphi(t)dt > 0 \ for\ each\ \varepsilon > 0$$

$$M(x,y) = max\left\{d(Ax,By), d(Sx,Ax), d(Ty,By), \frac{d(Sx,By)+d(Ty,Ax)}{2}\right\}. \quad (1.1)$$

If one of $A(X), B(X), S(X), or\ T(X)$ is a complete subspace of X, then
(1) A and S have a coincidence point, or
(2) B and T have a coincidence point.

Further, if S and A as well as T and B are weakly compatible, then
(3) A, B, S, and T have a unique common fixed point.

In **2011** Thabet Abdeljawad, Erdal Karapınar, and Kenan Tas [3] proved some fixed point theorems for self-mappings satisfying certain contraction principles on a cone Banach space.

Let C be a closed and convex subset of a cone Banach space X with norm $\|x\|_p$and let $d: X \times X \to E$ be such that $(x,y) = \|x - y\|_p$. If there exist a, b, c, s and $T : C \to C$ satisfying the conditions
(1) $0 \leq \frac{s+a-2b+c}{2(a+b)} < 1, a + b \neq 0, a + b + c > 1, s \geq 0$,
(2) $ad(Tx,Ty) + b[d(x,Tx) + d(y,Ty)] + cd(y,Tx) \leq sd(x,y)$ hold for all $x, y \in C$. Then, T has at least one fixed point.

In 2012 Hassen Aydi and Erdal Karapinar [9] established a common fixed point result for four self-maps satisfying a generalized Meir-Keeler type contraction on partial metric spaces. The result is as follows:

Theorem 1.3: Let $A, B, S\ and\ T$ be the self-maps defined on a complete partial metric space (X, p) satisfying the following conditions:

(i) $AX \subseteq TX, BX \subseteq SX$
(ii) for all $\varepsilon > 0, \exists\ \delta > 0$ *such that for all* $x, y \in X$

$$\varepsilon < M(x,y) < \varepsilon + \delta \Longrightarrow p(Ax,By) \leq \varepsilon$$

where $M(x,y) = max\left\{p(Sx,Ty), p(Ax,Sx), p(By,Ty), \frac{1}{2}[p(Sx,By) + p(Ax,Ty)]\right\}$
(iii) for all $x, y \in X\ with M(x,y) > 0 \Longrightarrow p(Ax,By) < M(x,y)$
(iv) $p(Ax,By) \leq max\{a[p(Sx,Ty) + p(Ax,Sx) + p(By,Ty)], b[p(Sx,By) + p(Ax,Ty)]\}$ *for all* $x, y \in X$

$$0 \leq a < \frac{1}{2}, 0 \leq b < \frac{1}{2}$$

If one of the ranges AX, BX, TX and SX is a closed subset of (X, p) then
(I) A and S have a coincidence point,
(II)B and T have a coincidence point,
Moreover, if A and S, as well as, B and T are weakly compatible, then A, B, S and T have a unique common fixed point.

Later **Hassen Aydi and Erdal Karapınar in 2012 [10]** combined partial metric spaces and ordered sets, and discussed the existence and uniqueness of some new Meir-Keeler type tripled fixed-point theorems in the context of partially ordered partial metric spaces. The result is as follows:

Theorem 1.4: Let $(X, p, \leq)$ be a partially ordered complete partial metric space. Suppose that X has the following properties:
(i) if $\{x_n\}$ is a sequence such that $x_{n-1} > x_n$ for each $n = 1,2 \ldots\ldots$ and $x_n \to x$ then $x_n < x$ for each $n = 1,2 \ldots$
(ii) if $\{y_n\}$ is a sequence such that $y_{n-1} < y_n$ for each $n = 1,2 \ldots\ldots\ldots$ and $y_n \to y$ then $y_n > y$ for each $n = 1,2 \ldots..$
Assume that $F : X^3 \to X$ satisfies the following hypotheses:
(i) F has the mixed strict monotone property,
(ii) F is a generalized p-Meir-Keeler type function,
(iii) there exist $x_0, y_0, z_0 \in X$ such that

$$x_0 < F(x_0, y_0, z_0), y_0 \geq \mathrm{F}(y_0, x_0, y_0), z_0 < F(z_{0,}y_0, x_0)$$

Then, F has a tripled fixed point, that is, there exist $x, y, z \in X$ such that

$$(F(x, y, z) = x, F(y, x, y) = y, F(z, y, x) = z$$

Also, $p(x, x) = p(y, y) = p(z, z) = 0$.

Some generalization of above theorem exist in literature.

In this paper, we derive a new fixed point theorem of Meir-Keeler type that generalizes Theorem 2.1.1 of Dass and Gupta in the case $\alpha, \beta \in \left(0, \frac{1}{2}\right)$.

2 MAIN RESULTS

Now we prove our main result.

Theorem 2.1: Let (X, d) be a complete metric space and T be a mapping from X into itself. Let the following hypothesis holds:
Given $\epsilon > 0$, there exists $\delta(\epsilon) > 0$ such that

$$2\epsilon \leq d(Tx, y)\frac{1+d(x,Tx)}{1+d(x,y)} + d(x, y) < 2\epsilon + < \delta(\epsilon) \Longrightarrow d(Tx, Ty) < \epsilon \tag{2.1}$$

Then T has a unique fixed point $\xi \in X$. Moreover, for any $x \in X$, the sequence $\{T^n x\}$ converges to ξ.

Proof: We first observe that (2.1) trivially implies that T satisfies:

$$x \neq y \text{ or } y \neq Ty \text{ implies that } d(Tx, Ty) < \frac{1}{2} d(Tx, y)\frac{1+d(x,Tx)}{1+d(x,y)} + \frac{1}{2} d(x, y) \tag{2.2}$$

Let $x \in X$. Consider the sequence $\{x_n\} = \{T^n x\}$. We will show that $\{x_n\}$ is a Cauchy sequence in X.

If there exists $p \in N$ such that $x_p = x_{p+1,}$ then x_p is a fixed point of T. For this reason, we will assume that $x_p \neq x_{p+1,}$ for all $p \in N$ .Let

$$c_n = d(x_n, x_{n+1}) \forall n \in N$$

From (2.2), we have $c_n = d(Tx_{n-1}, Tx_n) < \frac{1}{2} d(Tx_{n-1}, x_n)\frac{1+d(x_{n-1},x_n)}{1+d(x_{n-1},x_n)} + \frac{1}{2} d(x_{n-1}, x_n) = \frac{1}{2} c_{n-1}$
Then $c_n < c_{n-1}, \forall\, n \in N^*$
and the sequence $\{c_n\}$ is decreasing with n. Suppose now that $c_n \downarrow \varepsilon > 0$ as $n \to \infty$. then $c_n + c_{n-1} \downarrow 2\varepsilon$ as $n \to \infty$. This implies that $\exists\, N \in N^*$ such that

$$2\varepsilon \leq c_N + c_{N-1} < 2\varepsilon + \delta(\varepsilon)$$

We get ,$2\varepsilon \leq d(Tx_{N-1}, x_N)\frac{1+d(x_{N-1},Tx_{N-1})}{1+d(x_{N-1},x_N)} + d(x_{N-1}, x_N) < 2\varepsilon + \delta(\varepsilon)$

From (2.1) we obtain, $d(Tx_{N-1}, Tx_N) = d(x_N, x_{N+1}) = c_N < \varepsilon$ that is a contradiction. Then we deduce that

$$c_n \downarrow 0 \text{ as } n \to \infty \tag{2.3}$$

Let $\varepsilon > 0$. Condition (2.1) will remain true with $\delta(\varepsilon)$ replaced by $\delta'(\varepsilon) = \min(\delta(\varepsilon), \varepsilon, 1)$. From (2.3), $\exists\, k \in N$

$$d(x_m, x_{m+1}) < \frac{\delta'(\varepsilon)}{4}, \forall m \geq k \tag{2.4}$$

Now, we introduce the set $\Lambda \subset X$ defined by $\Lambda = \left\{x_p \middle| p \geq k, d(x_p, x_k) < 2\varepsilon + \frac{\delta'(\varepsilon)}{2}\right\}$

Let us prove that

$$T(\Lambda) \subset \Lambda \tag{2.5}$$

Let $\lambda \in \Lambda$.There exists $p \geq k$ such that $\lambda =$ and $d(x_p, x_k) < 2\varepsilon + \frac{\delta'(\varepsilon)}{2}$.

If $p = k$, we have $T(\lambda) = x_{k+1} \in \Lambda$ by (2.4).Then we will assume that $p > k$. We distinguish two cases.

Case-1:

$$2\varepsilon \leq d(x_p, x_k) < 2\varepsilon + \frac{\delta'(\varepsilon)}{2} \tag{2.6}$$

First, let us prove that

$$\varepsilon \leq \frac{1}{2} d(x_{p+1}, x_k) \frac{1+d(x_p,x_{p+1})}{1+d(x_p,x_k)} + \frac{1}{2} d(x_p, x_k) < \varepsilon + \frac{\delta'(\varepsilon)}{2} \tag{2.7}$$

From (2.6), we have

$$\varepsilon \leq \frac{1}{2} d(x_p, x_k) \leq \frac{1}{2} d(x_{p+1}, x_k) \frac{1+d(x_p,x_{p+1})}{1+d(x_p,x_k)} + \frac{1}{2} d(x_p, x_k) \tag{2.8}$$

On the other hand, we have

$$\begin{aligned}
\frac{1}{2} d(x_{p+1}, x_k) \frac{1+d(x_p,x_{p+1})}{1+d(x_p,x_k)} + \frac{1}{2} d(x_p, x_k) &\leq \frac{1}{2} d(x_{p+1}, x_k) + \frac{1}{2} d(x_{p+1}, x_k) \frac{1+d(x_p,x_{p+1})}{1+d(x_p,x_k)} + \frac{1}{2} d(x_p, x_k) \\
&< \frac{1}{2}[d(x_p, x_k) + d(x_{p+1}, x_k)] + \frac{1}{2} \frac{d(x_{p+1},x_k)}{1+d(x_p,x_k)} d(x_p, x_{p+1}) \\
&= \frac{1}{2} d(x_p, x_{p+1}) + \frac{1}{2} \frac{d(x_{p+1}, x_k)}{1 + d(x_p, x_k)} d(x_p, x_{p+1}) \\
\text{By (2.4)} \quad &< \frac{\delta'(\varepsilon)}{8} + \frac{1}{2} d(x_p, x_{p+1}) \\
\text{By (2.4)} \quad &< \frac{\delta'(\varepsilon)}{8} + \frac{\delta'(\varepsilon)}{8} \\
&< \frac{\delta'(\varepsilon)}{4} < \frac{\delta'(\varepsilon)}{2} < \varepsilon + \frac{\delta'(\varepsilon)}{2})
\end{aligned}$$

Then

$$\frac{1}{2} d(x_{p+1}, x_k) \frac{1+d(x_p,x_{p+1})}{1+d(x_p,x_k)} + \frac{1}{2} d(x_p, x_k) < \varepsilon + \frac{\delta'(\varepsilon)}{2} \tag{2.9}$$

It follows from (2.8)-(2.9) that (2.7) holds. Then

$$2\varepsilon \leq d(Tx_p, x_k) \frac{1+d(x_p,Tx_p)}{1+d(x_p,x_k)} + d(x_p, x_k) < 2\varepsilon + \delta'(\varepsilon)$$

which implies by (2.1) that

$$d(Tx_p, Tx_k) < \varepsilon \tag{2.10}$$

Now, we have

$$\begin{aligned}
d(Tx_p, x_k) &\leq d(Tx_p, Tx_k) + d(Tx_k, x_k) \\
\text{By (3.10) and (3.4)} \quad &< \varepsilon + \frac{\delta'(\varepsilon)}{4} \\
&< 2\varepsilon + \frac{\delta'(\varepsilon)}{2}
\end{aligned}$$

This implies that $T\lambda = Tx_p = x_{p+1} \in \Lambda$

Case-II:

$$d(x_p, x_k) < 2\varepsilon \tag{2.11}$$

From (2.2), we have:

$$\begin{aligned}
d(Tx_p, x_k) &\leq d(Tx_p, Tx_k) + d(Tx_k, x_k) \\
&< \frac{1}{2} d(x_{p+1}, x_k) \frac{1+d(x_p,x_{p+1})}{1+d(x_p,x_k)} + \frac{1}{2} d(x_p, x_k) + d(x_{k+1}, x_k) \\
&< \frac{1}{2} d(x_{p+1}, x_k) + \frac{1}{2} \frac{d(x_{p+1},x_k) d(x_p,x_{p+1})}{1+d(x_p,x_k)} + \frac{1}{2} d(x_p, x_k) + d(x_{k+1}, x_k) \\
&< \frac{1}{2} d(x_{p+1}, x_p) + \frac{1}{2} \frac{d(x_{p+1},x_k) d(x_p,x_{p+1})}{1+d(x_p,x_k)} + d(x_p, x_k) + d(x_{k+1}, x_k) \\
&< \frac{1}{2} d(x_{p+1}, x_p) + d(x_p, x_k) + d(x_{k+1}, x_k) + \frac{1}{2} d(x_{p+1}, x_p)
\end{aligned}$$

By (2.4) and (2. 11) $\quad < 2\varepsilon + \frac{\delta'(\varepsilon)}{2}$

This implies that $= Tx_p = x_{p+1} \in \Lambda$. Hence (2.5) holds and

$$d(x_m, x_k) < 2\varepsilon + \frac{\delta'(\varepsilon)}{2}, \quad \forall m > k \tag{2.12}$$

Now, for all $(m,n) \in N^2$ such that $m > n > k$, by (2.12), we get:

$$d(x_m, x_n) \le d(x_m, x_k) + d(x_n, x_k) < 4\,\varepsilon + \delta'(\varepsilon) < 5\varepsilon$$

This implies that $\{x_n\}$ is a Cauchy sequence in X.

Since (X,d) is complete, there exists $\xi \in X$ such that $\{x_n\}$ converges to ξ. From (2.2), we have

$$d(T\xi,\xi) \le d(T\xi, Tx_n) + d(x_{n+1},\xi)$$
$$< \frac{1}{2} d(T\xi, x_{n+1}) \frac{1+d(\xi,T\xi)}{1+d(\xi,x_n}) + \frac{1}{2} d(\xi,x_n) + d(x_{n+1},\xi).$$

Now let $n \to \infty$, we get $d(T\xi,\xi) \le 0$,

which implies that $\xi = T\xi$, i.e. ξ is a fixed point of T.

Suppose now that η is another fixed point of T. From (2.2). we get

$$d(\xi,\eta) = d(T\xi, T\eta) < \frac{1}{2} d(\xi,\eta) \frac{1+d(\xi,\xi)}{1+d(\xi,\eta)} + \frac{1}{2} d(\xi,\eta) = \frac{1}{2} d(\xi,\eta)$$

Which is a contradiction. Then the uniqueness of the fixed point is proved. This makes end to the proof.

Now we will show that the result of Dass and Gupta [12] (when $\alpha,\beta \in (0,1/2)$ is a particular case of theorem 2.2.1.

Corollary 2.2: (Dass and Gupta [12])

Let (X,d) be a complete metric space and T be a mapping from X into itself. We assume that the mapping T satisfies:
For all $x,y \in X$,

$$d(Tx,Ty) \le k\left(d(Tx,y)\frac{1+d(x,Tx)}{1+d(x,y)} + d(x,y)\right) \tag{2.13}$$

where $k \in (0,1/2)$ is a constant. Then T has a unique fixed point $\xi \in X$. Moreover, for any $x \in X$, the sequence $\{T^n x\}$ converges to ξ.

Proof: Fix $\varepsilon > 0$, we take: $\delta(\varepsilon) = \varepsilon\left(\frac{1}{k} - 2\right)$
Assume that

$$2\,\varepsilon \le d(Tx,y)\frac{1+d(x,Tx)}{1+d(x,y)} + d(x,y) < 2\varepsilon + \delta(\varepsilon)$$

From (2.13), we have

$$d(Tx,Ty) \le k\left(d(Tx,y)\frac{1+d(x,Tx)}{1+d(x,y)} + d(x,y)\right) < k(2\varepsilon + \delta(\varepsilon) = 2\varepsilon k + \varepsilon k\left(\frac{1}{k} - 2\right) = \varepsilon$$

Then condition (2.1) of Theorem 2.1 satisfied. This makes end to the proof.

3. APPLICATION TO MEIR-KEELER CONTRACTIONS OF INTEGRAL TYPE

In recent years, Branciari [11] initiated a study of contractive condition of integral type, giving an integral version of the Banach contraction principle that could be extended to more general contractive conditions. More precisely, he established the following result.

Theorem 3.1: (Branciari [11])
Let (X,d) be a complete metric space, $k \in (0,1)$, and let T be a mapping from X into itself such that for each $x,y \in X$,

$$\int_0^{d(Tx,Ty)} \varphi(t)dt \le k \int_0^{d(x,y)} \varphi(t)dt \tag{3.1}$$

where φ is a locally integrable function from $[0,\infty]$ into itself and such that for all $\varepsilon > 0$,

$$\int_0^{\varepsilon} \varphi(t)dt > 0.$$

Then T admits a unique fixed point $\xi \in X$ such that for each $x \in X$, the sequence $\{T^n x\}$ converges to ξ.

Putting $\varphi(t) = 1$ in the previous theorem, we retrieve the Banach fixed point theorem.

Later on, the authors in [8, 13, 19, 20, 24] established fixed point theorems involving more general contractive conditions.

Suzuki [23] showed that Meir-Keeler contractions of integral type are still Meir-Keeler contractions and so proved that Theorem 3.1 of Branciari is a particular case of the Meir-Keeler fixed point theorem [17]. In this section, following the idea of Suzuki [23], we will show that Theorem 2.1 allows us to obtain an integral version of Corollary 2.2.

We start by proving the following result.

Theorem 3.2: Let $(X;\ d)$ be a metric space and let T be a mapping from X into itself. Assume that there exists a function θ from [0, ∞) into itself satisfying the following:
(i) $\theta(0) = 0$ and $\theta(t) > 0$ for every $t > 0$
(ii)θ is non -decreasing and right continuous.
(iii) for every$\varepsilon > 0$, there exists $\delta(\varepsilon) > 0$ such that

$$2\varepsilon \le \theta\left(d(Tx,y)\frac{1+d(x,Tx)}{1+d(x,y)} + d(x,y)\right) < 2\epsilon + < \ \delta(\epsilon) \Longrightarrow \theta\big(2d(Tx,Ty)\big) < 2\epsilon$$

for all $x, y\ \in X$.
Then (2.1) is satisfied.

Proof:-Fix $\varepsilon > 0$. Since $\theta(2\varepsilon) > 0$, by (iii), there exists $\alpha > 0$ such that

$$\theta(2\varepsilon) \le \theta\left(d(Tx,y)\frac{1+d(x,Tx)}{1+d(x,y)} + d(x,y)\right) < \theta(2\varepsilon) + \alpha \Longrightarrow \theta\big(2d(Tx,Ty)\big) < \theta(2\varepsilon) \tag{3.2}$$

From the right continuity of θ , there exists $\delta > 0$ such that $\theta(2\varepsilon + \delta) < \theta(2\varepsilon) + \alpha$. Fix $x, y \in X$ such that

$$2\varepsilon \le d(Tx,y)\frac{1+d(x,Tx)}{1+d(x,y)} + d(x,y) < 2\varepsilon + \delta$$

Since θ is nondecreasing, we get:

$$\theta(2\varepsilon) \le\ \theta\left(d(Tx,y)\frac{1+d(x,Tx)}{1+d(x,y)} + d(x,y)\right) \le\ \theta(2\varepsilon+\delta) < \theta(2\varepsilon) + \alpha$$

Then by (3.2), we have:

$$\theta(2d(Tx,Ty)\ ) < \theta(2\varepsilon)$$

which implies that $d(Tx,Ty) < \varepsilon$. Then (2.1)is satisfied. This completes the proof.

Since a function $t \mapsto \int_0^t \varphi(s)ds$ is absolutely continuous, we obtain the following,

Corollary 3.3: Let (X,d) be a metric space and let T be a mapping from X into itself. Let φ be a locally integrable function from$[0,+\infty)$ into itself such that $\int_0^t \varphi(s)ds > 0$ for all $t > 0$. Assume that for each $\varepsilon > 0$, there exists $\delta(\varepsilon) > 0$ such that

$$2\varepsilon \le \int_0^{d(Tx,y)\frac{1+d(x,Tx)}{1+d(x,y)}+d(x,y)} \varphi(t)dt < 2\varepsilon + \delta(\varepsilon) \Longrightarrow \int_0^{2d(Tx,Ty)} \varphi(t)dt < 2\varepsilon \tag{3.3}$$

Then (2.1) is satisfied.

Now we are able to obtain the integral version of corollary 3.3. We have the following result.

Corollary 3.4: Let (X,d) be a metric space and let T be a mapping from X into itself. Let φ be a locally integrable function from$[0,\infty)$ into itself

$$\int_{\mathbf{0}}^{\mathbf{2}\boldsymbol{d}(\boldsymbol{Tx},\boldsymbol{Ty})} \varphi(t)dt \le c\int_0^{d(Tx,y)\frac{1+d(x,Tx)}{1+d(x,y)}+d(x,y)} \varphi(t)dt, \tag{3.4}$$

where $c \in (0,1)$ is a constant. Then T has a unique fixed point $\xi \in X$. Moreover, for any $x \in X$, the sequence $\{T^n x\}$ converges to ξ.

Proof: Fix$\varepsilon > 0$. It is easily to check that (2.1) is satisfied with $\delta(\varepsilon) = 2\varepsilon\left(\frac{1}{c} - 1\right)$.Then (2.1) is satisfied and we can apply Theorems.

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