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ANTI Q-FUZZY B – IDEALS IN B – ALGEBRA

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ABSTRACT

In this paper, we introduce the notion of anti Q-fuzzy B–Ideals of B-algebras, lower level B – Ideal and prove some results on these. We show that a Q-fuzzy subset of a B-algebra is a Q-fuzzy B-ideal if and only if the complement of this Q-fuzzy subset is an anti Q-fuzzy B-ideal.

Keywords: *B–algebra, B–Ideal, Fuzzy B–Ideal, Anti Fuzzy B–Ideal, Q–Fuzzy B–Ideal, Anti Q–Fuzzy B–Ideal.*

1. INTRODUCTION

After the introduction of fuzzy subsets by L.A. Zadeh[11], several researchers explored on the generalization of the notion of fuzzy subset. K. Atanassov[2] ,introduced the Intuitionistic fuzzy sets. F. Adam and N. Hassan[1], introduced the Q-fuzzy soft set. Muthuraj, P.M. Sitharselvam, M.S. Muthuraman[6], introduced the notion Anti Q-fuzzy group and its lower level subgroups, J.R.Cho and H.S.Kim[4] discussed relations between B-algebras and other topics, especially quasi-groups. Jiayinpeng[5], introduced the Intuitionistic Fuzzy B-algebras. H.K.Park and H.S.Kin[8],introduced the notion of Quadratic B-algebras. Sun ShinAhn and KeumseongBang[9] have discussed the fuzzy subalgebra in B-algebra. C.Yamini and S.Kailasavalli[10] ,introduced the notion of Fuzzy B-ideals. R.Biswas[3] ,introduced the concept of anti-fuzzy subgroups of groups. P.Muthuraj, M.Sridharan, M.S.Muthuraman and P.M.SitharSelvam[7] ,introduce the notion of Anti Q-Fuzzy BG-Ideals in BG-Algebra. Modifying their idea, in this paper, we apply the idea to B-Algebra. We introduce the notion of Anti Q-Fuzzy B-ideals of B-Algebras, lower level cuts of a Q-Fuzzy set, and prove some results on these.

2. PRELIMINARIES

In this section we give some basic definitions and preliminaries of B-algebras and introduce Q-fuzzy B-ideal.

Definition 2.1: (Jung R. Cho and H.S.Kim [4]) A B-algebra is a non-empty set X with a constant 0 and a binary operation '*' satisfying the following axioms:

(i) $x * x = 0$
(ii) $x * 0 = x$
(iii) $(x * y) * z = x * (z * (0 * y)), for\ all\ x, y, z \in X$

For brevity we also call X a B-algebra. In X we can define a binary relation "≤" by $x \le y$ if and only if $x * y = 0$.

Definition 2.2: (Jung R. Cho and H.S.Kim [4]) A non-empty subset M of a B-algebra X is called a sub-algebra of X if $x * y \in M$ for any $x, y \in M$.

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Definition 2.3: (JiayinPeng [5]) Let α be a fuzzy set in a B-algebra. Then α is called a fuzzy subalgebra of X if $\alpha(x * y) \geq \alpha(x) \wedge \alpha(y)\ for\ all\ x, y \in X$.

Definition 2.4: (Jung R. Cho and H.S.Kim [4]) A non-empty subset N of a B-algebra X is called a B-ideal of X if it satisfies for $x, y, z \in X$

(i) $0 \in N$

(ii) $(x * y) \in N and\ (z * x) \in N$ implies $(y * z) \in N$

Definition 2.5: (L. A. Zadeh [1]) Let X be a non-empty set. A fuzzy subset α of the set X is a mapping $\alpha: X \to [0,1]$

Definition 2.6: (F. Adam and N. Hassan [1]) Let Q and G be any two sets. A mapping $A: G \times Q \to [0,1]$ is called a Q-fuzzy set in G.

Definition 2.7: (R.Muthuraj *et.al* [7]) Let α be a Q-fuzzy set in B-algebra. Then α is called a Q-fuzzy sub-algebra of X if $\alpha(x * y, q) \geq \min\{\alpha(x, q), \alpha(y, q)\}, for\ all\ x, y \in X\ \&\ q \in Q$

Definition 2.8: (R.Muthuraj *et.al* [7]) Let α be a Q-fuzzy set in set X. Then the complement $\bar{\alpha}$ is the Q-fuzzy subset of X given by $\bar{\alpha}(x, q) = 1 - \alpha(x, q) for\ all\ x \in X\ \&\ q \in Q$.

Definition 2.9: (R. Muthuraj *et.al* [6]) Let α be a Q-fuzzy set in a set X. For $S \in [0,1]$, the set $\alpha_s = \{x \in \alpha(x, q) \geq S\ for\ all\ q \in Q\}$ is called a level subset of α.

Definition 2.10: (R.Muthuraj *et.al* [7]) A Q-fuzzy set α in X is called Q-fuzzy B-ideal of X if it satisfies the following axioms:

(i) $\alpha(0, q) \geq \alpha(x, q)$

(ii) $\alpha(y * z, q) \geq \min\{\alpha(x * y, q), \alpha(z * x, q)\}, for\ all\ x, y \in X\ and\ q \in Q$.

3. ANTI Q-FUZZY B-IDEALS

De**finition 3.1:** A Q-fuzzy set α of a B-algebra X is called and Anti Q-Fuzzy subalgebra of X if
$\alpha(x * y, q) \leq \max\{\alpha(x, q), \alpha(y, q)\} for\ all\ x, y \in X\ and\ q \in Q$.

Definition 3.2: A Q-fuzzy set α in X is called an anti Q-fuzzy B-ideal of X if it satisfies the following axioms:

(i) $\alpha(0, q) \leq \alpha(x, q)$

(ii) $\alpha(y * z, q) \leq \max\{\alpha(x * y, q), \alpha(z * x, q)\}, for\ all\ x, y \in X\ and\ q \in Q$

Theorem 3.3: Every anti Q-fuzzy B-ideal of a B-algebra X is order preserving.

Proof: Let α be an anti Q-fuzzy B-ideal of a B-algebra X.

Let $x, y \in X$ and $q \in Q$ be such that $y \leq x$ if and only if $y * x = 0$

Now,

$$\begin{aligned} \alpha(y, q) &= \alpha(0 * y, q) \\ &\leq \max\{\alpha(x * 0, q), \alpha(y * x, q)\} \\ &\leq \max\{\alpha(x, q), \alpha(0, q)\} \\ &\leq \alpha(x, q) \end{aligned}$$

$$\Rightarrow \alpha(y, q) \leq \alpha(y, q)$$

Theorem 3.4: A Q-fuzzy subset α of a B-algebra X is a Q-fuzzy B-ideal of X if and only if its complement $\bar{\alpha}$ is an anti Q-fuzzy B-ideal of X.

Proof: Let α be a Q-fuzzy B-ideal of X and let $x, y, z \in X$ and $q \in Q$.

To prove:

$\bar{\alpha}$ is an anti Q-fuzzy B-ideal of X.

(i) $$\begin{aligned} \bar{\alpha}(0, q) &= 1 - \alpha(0, q) \\ &\leq 1 - \alpha(x, q) \\ &= \bar{\alpha}(x, q) \end{aligned}$$

(ii) $$\begin{aligned} \bar{\alpha}(y * z, q) &= 1 - \alpha(y * z, q) \\ &\leq 1 - \min\{\alpha(x * y, q), \alpha(z * x, q)\} \\ &\leq 1 + \max\{-\alpha(x * y, q), -\alpha(z * x, q)\} \\ &\leq \max\{1 - \alpha(x * y, q), 1 - \alpha(z * x, q)\} \\ &= \max\{\bar{\alpha}(x * y, q), \bar{\alpha}(z * x, q)\} \end{aligned}$$

$$\Rightarrow \bar{\alpha}(y * z, q) \leq \max\{\bar{\alpha}(x * y, q), \bar{\alpha}(z * x, q)\}$$

Thus, $\bar{\alpha}$ is an anti Q-fuzzy B-ideal of X.
The converse part can also be prepared similarly.
Hence, the proof.

Definition 3.5: Let α be a Q-fuzzy subset of a B-algebra X. For $S \in [0,1]$, the set $\alpha^S = \{x \in X/\mu(x,q) \leq S$ is called a lower level cut of α.

Clearly, $\alpha' = X$ and $\alpha_S^S \cup \alpha^S = X$for $S \in [0,1]$. If $S_1 \leq S_2$then $\alpha^{S1} \subseteq \alpha^{S2}$.

Theorem 3.6: Let α be a Q-fuzzy subset of a B-algebra X. If α is an anti Q-fuzzy B-ideal of X, then the lower level cut α^S is a B-ideal of X for all $S \in [0,1]$, $S \geq \alpha(0,q)$.

Proof: Let α be an anti Q-fuzzy B-ideal of X. Then for all $x, y \in X$and $q \in Q$,

(i) $\alpha(0,q) \leq \alpha(x,q)$
(ii) $\alpha(y*z,q) \leq \max\{\alpha(x*y,q), \alpha(z*x,q)\}$

To prove:
$\alpha^S$is a B-ideal of X.

Let $x, y, z \in \alpha^S$

$\Rightarrow \alpha(x,q) \leq S$

(i) Since $\alpha(0,q) \leq \alpha(x,q)$
$\leq S$
$\Rightarrow \alpha(0,q) \leq S$
$\Rightarrow 0 \in \alpha^S$

(ii) $x*y \in \alpha^S \& z*x \in \alpha^S$
$\Rightarrow \alpha(x*y,q) \leq S \& \alpha(z*x,q) \leq S$
$\alpha(y*z,q) \leq \max\{\alpha(x*y,q), \alpha(z*x,q)\}$
$\leq \max\{S,S\}$
$= S$
$\Rightarrow \alpha(y*z,q) \leq S$
$\Rightarrow y*z \in \alpha^S$

Thus, α^S is a B-ideal of X.
Hence, the proof

Theorem 3.7: Let α be a Q-fuzzy subset of a B-algebra X. If for each $S \in [0,1], S \geq \alpha(0,q)$ the lower level cut α^S is a B-ideal of X, then S is an anti Q-fuzzy B-ideal of X.

Proof:

$\alpha^S$is a B-ideal of X.
$0 \in \alpha^S$
$x*y \in \alpha^S \& z*x \in \alpha^S \Rightarrow y*z \in \alpha^S$

To prove
$\alpha^S$is an anti Q-fuzzy B-ideal of X.

For all $x, y \in X$ and $q \in Q$

(i) $x*y \in \alpha^S \& z*x \in \alpha^S$
$\Rightarrow \alpha(x*y,,q) \leq S \& \alpha(z*x,q) \leq S$

Let $\alpha(x*y,q) = S \& \alpha(z*x,q) = S$
$y*z \in \alpha^S$
$\Rightarrow \alpha(y*z,q) \leq S$
$= \max\{S,S\}$
$\leq \max\{\alpha(x*y,q), \alpha(z*x,q)\}$
$\Rightarrow \alpha(y*z,q) \leq \max\{\alpha(x*y,q), \alpha(z*x,q)\}$

(ii) $0 \in \mu^t$
Since $x*x = 0$
$\alpha(0,q) = \alpha(x*x,q)$
$\leq \max\{\alpha(x,q), \alpha(x,q)\}$
$= \alpha(x,q)$
$\Rightarrow \alpha(0,q) \leq \alpha(x,q)$
$\Rightarrow \alpha^S$is an anti Q-fuzzy B-ideal of S.

Hence, the proof.

4. CONCLUSION

This paper tried to define the Anti Q-Fuzzy B-Ideals on B-Algebra and proved some theorems on them. This Concept can further be generalized to normalization of Q-fuzzy B-Ideals in B-Algebra using n-fold translation and multiplication.

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