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Applications of b#-Open set

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ABSTRACT

Using the concept of b#-open sets we introduce and study topological properties of b#-limit points, b#-derived sets, b#-closure, b#-border, b#-Frontier and Db#- exterior and discuss their relations with one another.

Keywords: *b#-limit points, b#-derived sets, b#-closure, b#-border, b#-Frontier and Db#- exterior.*

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1. INTRODUCTION

In the year 1996, Andrijivic introduced [1] and studied b-open sets. Following this Usha Paraeswari *et.al* [2] introduced the concept of b#- open sets. In this paper we introduce the notions of b#-limit points, b#-derived sets, b#-closure, b#-border, b#-Frontier and b#- exterior by using the concept of b#-open set.

2. PRELIMINARIES

Throughout this paper X denotes a topological space on which no separation axiom is assumed. For any subset A of X, *cl*(A) denotes the closure of A and *int*(A) denotes the interior of A in the topological space X. Further X \ A denotes the complement of A in X. The following definitions and results are very useful in the subsequent sections.

Definition 2.1 [2]: A subset A of a space X is called b#- open if A= cl(int (A)) $\cup$ int(cl(A)) and their complement is called b#- closed. That is A is b#-closed if A = cl(int (A))$\cap$int(cl(A)).

Definition 2.2[3]: The b#-interior of A, denoted by b#-*int*(A), is defined to be the union of all b#-open sets contained in A. That is b#-*int*(A)= $\bigcup$ {B: B $\subseteq$ A and B is b#-open}.

The next Lemma gives the properties of b#-interior.

Lemma 2.3[3]:
(i) b#-*int*(ϕ)= ϕ.
(ii) b#-*int*(X)= X.
(iii) b#-*int*(A) $\subseteq$ A.
(iv) b#-interior of a set A is not always b#-open.
(v) If A is b#-open then b#-*int*(A)=A.

Lemma 2.4[3]: Let X be a space. Then for any two sub sets A and B of X we have
(i) If A $\subseteq$ B then b#-*int*(A) $\subseteq$ b#-*int*(B).
(ii) b#-*int*(b#-*int*(A))= b#-*int*(A).
(iii) b#-*int*(A$\cap$B) $\subseteq$ b#-*int*(A) $\bigcap$ b#-*int*(B).
(iv) b#-*int*(A $\bigcup$ B) $\supseteq$ b#-*int*(A) $\bigcup$ b#-*int*(B).

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Definition 2.5[3]: The $b^{\#}$-closure of A, denoted by $b^{\#}$-*cl*(A), is defined to be the intersection of all $b^{\#}$-closed sets containing A. That is $b^{\#}$-*cl*(A)=$\bigcap$ {B: A $\subseteq$ B and B is $b^{\#}$-closed}.

Lemma 2.6[3]: Let X be a space. Then for any sub set A of X we have

(i) X\ $b^{\#}$-*int*(A)= $b^{\#}$-*cl*(X\ A).
(ii) X \ $b^{\#}$-*cl*(A)= $b^{\#}$-*int*(X\ A).

Remarks 2.7[3]:

(i) $b^{\#}$-*cl*(ϕ)= ϕ,
(ii) $b^{\#}$-*cl*(X)= X.
(iii) A$\subseteq b^{\#}$-*cl*(A).
(iv) $b^{\#}$-closure of a set A is not always $b^{\#}$-closed.
(v) If A is $b^{\#}$-closed then $b^{\#}$-*cl*(A)=A.

Lemma 2.8[3]: Let X be a space. Then for any two sub sets A and B of X we have

(i) If A$\subseteq$B then $b^{\#}$-*cl*(A)$\subseteq$ $b^{\#}$-*cl*(B).
(ii) $b^{\#}$-*cl*($b^{\#}$-*cl*(A))= $b^{\#}$-*cl*(A).
(iii) $b^{\#}$-*cl*(A$\cup$B)$\supseteq$ $b^{\#}$-*cl*(A)$\cup b^{\#}$-*cl*(B).
(iv) $b^{\#}$-*cl*(A$\cap$B)$\subseteq$ $b^{\#}$-*cl*(A)$\cap$ $b^{\#}$-*cl*(B).

3. $b^{\#}$- limit points

Definition 3.1: Let A be a subset of a topological space (X, τ) and x be a point of X. A point x$\in$X is said to be a $b^{\#}$-limit point of A if every $b^{\#}$-neighborhood of x intersects A in some point other than x itself. That is U$\bigcap$(A/{x})$\neq \phi$ for all U$\in b^{\#}$-O(X, τ).

The set of all $b^{\#}$-limit points of A is called the $b^{\#}$-derived set of A and is denoted by $Db^{\#}$-(A).

Remark 3.2: A subset A of X, a point x $\in$ X is not a $b^{\#}$-limit point of A if and only if there exists a $b^{\#}$-open set G in X such that x $\in$G and G$\bigcap$(A/{x}) = ϕ that is x$\in$G and G$\bigcap$A=ϕ or G$\bigcap$A={x} that is x$\in$G and G$\bigcap$A$\subseteq$ {x}.

Theorem 3.3: Let τ_1 and τ_2 be topologies on X such that $\tau_1^{b\#} \subseteq \tau_2^{b\#}$. For any subset A of X, every $b^{\#}$-limit point of A with respect to τ_2 is a $b^{\#}$-limit point of A with respect to τ_1.

Proof: Let x be a $b^{\#}$-limit point of A with respect to τ_2. Then U$\bigcap$(A/{x})$\neq \phi$ for every U$\in \tau_2^{b\#}$ such that x$\in$U. But $\tau_1^{b\#} \subseteq \tau_2^{b\#}$, we have U$\bigcap$(A/{x})$\neq \phi$ for every U$\in \tau_1^{b\#}$ such that x$\in$U. Hence x is a $b^{\#}$-limit point of A with respect to τ_1.

Theorem 3.4: For any sub sets A and B of (X, τ) the following holds.

(i) If A$\subseteq$B then $Db^{\#}$-(A) $\subseteq$ $Db^{\#}$- (B).
(ii) $Db^{\#}$- (A)$\bigcup$ $Db^{\#}$- (B) $\subseteq$ $Db^{\#}$- (A$\bigcup$B).
(iii) $Db^{\#}$- (A$\bigcap$B) $\subseteq$ $Db^{\#}$- (A) $\bigcap$ $Db^{\#}$- (B).
(iv) $Db^{\#}$- ($Db^{\#}$- (A))/A$\subseteq$ $Db^{\#}$- (A).
(v) $Db^{\#}$- (A$\bigcup$ $Db^{\#}$- (A)) $\subseteq$A$\bigcup$ $Db^{\#}$- (A).

Proof: Let x$\in$ $Db^{\#}$- (A) and let U$\in \tau^{b\#}$ with x$\in$U. Then U$\bigcap$(A/{x})$\neq \phi$. Since A$\subseteq$B, we have U$\bigcap$(B/{x})$\neq \phi$. This implies that x$\in$ $Db^{\#}$-(B). This proves (i).

Now to prove (ii). Since A$\subseteq$A$\bigcup$B and B$\subseteq$A$\bigcup$B. Using (i), $Db^{\#}$- (A) $\subseteq Db^{\#}$- (A$\bigcup$B) and $Db^{\#}$- (B)$\subseteq Db^{\#}$- (A$\bigcup$B) that is $Db^{\#}$- (A) $\bigcup Db^{\#}$- (B) $\subseteq Db^{\#}$- (A$\bigcup$B). This proves (ii).

Next we have to prove (iii). Since A$\bigcap$B$\subseteq$A and A$\bigcap$B$\subseteq$B. Using (i), $Db^{\#}$- (A$\bigcap$B) $\subseteq Db^{\#}$- (A) and $Db^{\#}$- (A$\bigcap$B) $\subseteq Db^{\#}$- (B). Thus we get $Db^{\#}$- (A$\bigcap$B) $\subseteq Db^{\#}$- (A) $\bigcap Db^{\#}$- (B). Hence (iii). Next to prove (iv). Let x$\in Db^{\#}$-($Db^{\#}$-(A))/A and let U$\in \tau^{b\#}$ with x$\in$U. Then U$\bigcap$($Db^{\#}$- (A)/{x})$\neq \phi$. Let y$\in$ U$\bigcap$(D $b^{\#}$- (A)/{x}). Then y $\in$U and y $\in$ D $b^{\#}$- (A) and U $\bigcap$(A/{y})$\neq \phi$. If we take z$\in$ U$\bigcap$(A/{y}), then x $\neq$ z because x$\notin$A.
Hence U$\bigcap$(A/{x}) $\neq \phi$. Therefore x$\in$D $b^{\#}$- (A). Hence (iv).

Next to prove (v). Let $x \in D\ b^{\#}$- $(A \bigcup D\ b^{\#}$- $(A))$. If $x \in A$, the result is obvious. Assume that $x \notin A$. Then $U \bigcap (A \bigcup Db^{\#}$- $(A)/\{x\}) \neq \phi$ for all $U \in \tau^{b\#}$ with $x \in U$. Hence $U \bigcap (A/\{x\}) \neq \phi$ or $U \bigcap (Db^{\#}$-$(A)/\{x\}) \neq \phi$. The first case implies $x \in Db^{\#}$- (A). Then the second case implies $x \in Db^{\#}$- $(D\ b^{\#}$-(A)). Since $x \notin A$, by (iv)
$x \in Db^{\#}$-$(Db^{\#}$- $(A))/A \subseteq D\ b^{\#}$- (A). This proves (v).

The reverse inclusion of (i) and the converse of (ii), (iii) and (iv) are not true as shown by the following examples.

Example 3.5: Let X= {a, b, c, d}. Consider the topology τ={Φ, X, {a, b, c}, {a}, {b, c}}. The $b^{\#}$-open sets are Φ, X, {d, b, c}, {a, d} and $b^{\#}$-closed sets are Φ, X, {a}, {b, c}. Let A= {a, d} and B= {b, c}. Then $Db^{\#}$-(A) = {a, b, c} and $Db^{\#}$-(B) = {b, c}. So $Db^{\#}$-(B)⊆ $Db^{\#}$-(A) but B⊄A.

Also $Db^{\#}$-(A∪B)= {a, b, c, d}⊄ $Db^{\#}$-(A)∪ $Db^{\#}$-(B). Again let A_1= {a, b} and B_1= {a, c}. $Db^{\#}$-(A_1)= {c, d} and $Db^{\#}$-(B_1) = {b, d}. Therefore $Db^{\#}$-(A)∩ $Db^{\#}$- (B)⊄ $Db^{\#}$-(A∩B).

Let A_2= {a, c}. $Db^{\#}$-(A_2) = {b, d} and $Db^{\#}$($Db^{\#}$-(A)) = {a, b, c}. Thus $Db^{\#}$-(A)⊄ $Db^{\#}$($Db^{\#}$-(A))\A.

Theorem 3.6: Let A be a sub set of (X, τ) and $x \in X$. Then the following are equivalent.

(i) If for all $U \in \tau^{b\#}$, $x \in U$ then $A \bigcap U \neq \phi$.

(ii) $x \in b^{\#}$-cl(A).

Proof: Suppose (i) holds. If $x \notin b^{\#}$-cl(A), then there exists a $b^{\#}$-closed set F such that $A \subseteq F$ and $x \notin F$. Hence X/F is a $b^{\#}$-open set containing x and $A \bigcap (X/F) \subseteq A \bigcap (X/A) = \phi$. This is a contradiction to our assumption. This proves (i) $\Rightarrow$ (ii). The proof of (ii) $\Rightarrow$ (i) is from the Definition 3.1.

Corollary 3.7: For any sub set A of X we have $Db^{\#}$- (A) $\subseteq b^{\#}$-cl(A).

Proof: Let $x \in Db^{\#}$- (A). By Definition 3.1, there exists $x \in U$ such that $U \bigcap (A/\{x\}) \neq \phi$. That is $U \bigcap A \neq \phi$. So by Theorem 3.6, $x \in b^{\#}$-cl(A).

Theorem 3.8: For any sub set A of X, $b^{\#}$-cl(A)= $A \bigcup Db^{\#}$- (A).

Proof: Let $x \in b^{\#}$-cl(A). Assume that $x \notin A$ and let $U \in \tau^{b\#}$ with $x \in U$. Then $U \bigcap (A/\{x\}) \neq \phi$ and so $x \in Db^{\#}$-(A). Hence $b^{\#}$-cl(A) $\subseteq A \bigcup Db^{\#}$- (A). Conversely since $A \subseteq b^{\#}$-cl(A) and $Db^{\#}$- (A) $\subseteq b^{\#}$-cl(A). This proves the theorem.

Definition 3.9[3]: A space X is said to be $b^{\#}$-closed preserving if every $b^{\#}$-closure of a subset is $b^{\#}$-closed.

Theorem 3.10: Let A and B be a sub sets of (X, τ). If A is $b^{\#}$-closed preserving then $b^{\#}$-cl$(A \bigcap B) \subseteq A \bigcap b^{\#}$-cl(A).

Proof: If A is $b^{\#}$-closed preserving then $b^{\#}$-cl(A)=A and so $b^{\#}$-cl$(A \bigcap B) \subseteq b^{\#}$-cl(A) $\bigcap b^{\#}$-cl(B)=$A \bigcap b^{\#}$-cl(B).

Theorem 3.11: For every sub set A of X we have A is $b^{\#}$-closed then $Db^{\#}$- (A) $\subseteq$ A.

Proof: Assume that A is $b^{\#}$-closed. Let $x \in X/A$. Then X/A is $b^{\#}$-open, $(X/A) \bigcap (A/\{x\}) = \phi$. Therefore x is not a $b^{\#}$-limit point of A. That is $x \notin Db^{\#}$- (A). Hence $Db^{\#}$- (A) $\subseteq$ A.

Corollary 3.12: The converse of the above theorem is true if A is $b^{\#}$-closed preserving.

Theorem 3.13: Let A be a sub set of (X, τ). If a point $x \in X$ is a $b^{\#}$-limit point of A\x then x is also a $b^{\#}$-limit point of A.

Proof: If x is a $b^{\#}$-limit point of A/{x} then by Definition 3.1, there exists a $b^{\#}$-open set U such that $x \in U$ and $U \bigcap [(A/\{x\})/\{x\}] \neq \phi$. That is x is a $b^{\#}$-limit point of A/{x}.

4. $b^{\#}$-interior, $b^{\#}$-border and $b^{\#}$-Frontier

Definition 4.1: Let A be a sub set of a topological space (X, τ). A point $x \in X$ is called a $b^{\#}$-interior point of A if there exists a $b^{\#}$-open set U such that $x \in U \subseteq A$. The set of all $b^{\#}$-interior points of A is called $b^{\#}$- interior of A and is denoted by $b^{\#}$-int(A).

Definition 4.2: For any sub set A of X, the set $b^{\#}$-b(A)= A/ $b^{\#}$-int(A) is called the $b^{\#}$-border of A and the set $b^{\#}$-Fr(A)= $b^{\#}$-cl(A)/ $b^{\#}$-int(A) is called the $b^{\#}$-Frontier of A.

Remark 4.3: If A is a $b^{\#}$-closed preserving sub set of X then $b^{\#}$-b(A) = $b^{\#}$-Fr(A).

Proposition 4.4: For a sub set A of X the following statements holds.

(i) $b^{\#}$-int(A) $\bigcap$ $b^{\#}$-b(A)= ϕ.
(ii) $b^{\#}$-int($b^{\#}$-b(A))= ϕ.
(iii) $b^{\#}$-b($b^{\#}$-b(A))= $b^{\#}$-b(A).
(iv) $b^{\#}$-b(A)=A $\bigcap$ $b^{\#}$-cl(X/A).

Proof: By Definition of 4.2, (i) holds. Now to prove (ii).
If x $\in$ $b^{\#}$-int($b^{\#}$-b(A)) then x $\in$ $b^{\#}$-b(A) $\subseteq$ A and x $\in$ $b^{\#}$-int(A). Thus x $\in$ $b^{\#}$-int(A) $\bigcap$ $b^{\#}$-b(A) but by (i), $b^{\#}$-int(A) $\bigcap$ $b^{\#}$-b(A)= ϕ which is a contradiction. Hence $b^{\#}$-int($b^{\#}$-b(A))= ϕ. This proves (ii).

Now to prove (iii). By Definition 4.2, $b^{\#}$-b($b^{\#}$-b(A))= $b^{\#}$-b(A)/ $b^{\#}$-int($b^{\#}$-b(A)).

Using (ii), $b^{\#}$-b($b^{\#}$-b(A)) = $b^{\#}$-b(A). This proves (iii). Now to prove (iv). Using Definition 4.2,
$b^{\#}$-b(A)=A/ $b^{\#}$-int(A)=A/[X/ $b^{\#}$-cl(X/A)]= A $\bigcap$ $b^{\#}$-cl(X/A). This proves (iv).

Theorem 4.5: For a sub set A of (X, τ), the following conditions holds.

(i) $b^{\#}$-int(A) $\bigcap$ $b^{\#}$-Fr(A)= ϕ.
(ii) $b^{\#}$-b(A) $\subseteq$ $b^{\#}$-Fr(A).
(iii) $b^{\#}$-Fr(A)= $b^{\#}$-b(A) $\bigcup$ (Db#- (A)/ $b^{\#}$-int(A)).
(iv) $b^{\#}$-Fr(A)= $b^{\#}$-cl(A) $\bigcap$ $b^{\#}$-cl(X/A).
(v) $b^{\#}$-Fr(A)= $b^{\#}$-Fr(X/A).
(vi) $b^{\#}$-Fr($b^{\#}$-int(A)) $\subseteq$ $b^{\#}$-Fr(A).
(vii) $b^{\#}$-int(A)=A/ $b^{\#}$-Fr(A).

Proof: Using Definition 4.2, $b^{\#}$-int(A) $\bigcap$ $b^{\#}$-Fr(A)= $b^{\#}$-int(A) $\bigcap$[$b^{\#}$-cl(A)/ $b^{\#}$-int(A)]= ϕ. This proves (i). Now to prove (ii).

Since A $\subseteq$ $b^{\#}$-cl(A) we have $b^{\#}$-b(A)= A/ $b^{\#}$-int(A) $\subseteq$ $b^{\#}$-cl(A)/ $b^{\#}$-int(A)= $b^{\#}$-Fr(A). This proves (ii). Now to prove (iii).

By Definition 4.2, $b^{\#}$-Fr(A)= $b^{\#}$-cl(A)/ $b^{\#}$-int(A) = (A $\bigcup$ D$b^{\#}$- (A))/ $b^{\#}$-int(A) = (A/$b^{\#}$-int(A)) $\bigcup$ (D$b^{\#}$- (A)/ $b^{\#}$-int(A)) = $b^{\#}$-b(A) $\bigcup$ (D$b^{\#}$- (A)/ $b^{\#}$-int(A)). Hence (iii) is proved. Now to prove (iv).

Using Lemma 2.6, we have
$b^{\#}$-cl(A) $\bigcap$ $b^{\#}$-cl(X/A)=$b^{\#}$-cl(A) $\bigcap$ (X/$b^{\#}$-int(A))=$b^{\#}$-cl(A)/$b^{\#}$-int(A)= $b^{\#}$-Fr(A). This proves (iv). Using (iv),
$b^{\#}$-Fr(X/A) = $b^{\#}$-cl(X/A) $\bigcap$ $b^{\#}$-cl(A)= $b^{\#}$-Fr(A). Hence (v) is proved.

Using Lemma 2.4, $b^{\#}$-Fr($b^{\#}$-int(A))= $b^{\#}$-cl($b^{\#}$-int(A))/ $b^{\#}$-int($b^{\#}$-int(A)) $\subseteq$ $b^{\#}$-cl(A)/ $b^{\#}$-int(A)= $b^{\#}$-Fr(A). This proves (vi). Now A/$b^{\#}$-Fr(A)=A/($b^{\#}$-cl(A)/$b^{\#}$-int(A))=A $\bigcap$ ((X/$b^{\#}$-cl(A) $\bigcup$ $b^{\#}$-int(A))= ϕ $\bigcup$ (A $\bigcup$ $b^{\#}$-int(A))=$b^{\#}$-int(A). This completes the proof.

The converse of (ii) and (vi) of Theorem 4.5 is not true in general as seen in the following Example.

Example 4.6: Consider the same topological space in Example 3.5. Let A= {c}. Then $b^{\#}$-Fr(A)= {b, c}, $b^{\#}$-b(A)= {c}, $b^{\#}$-int(A)= Φ and $b^{\#}$-Fr($b^{\#}$-int(A))= Φ. Thus $b^{\#}$-Fr(A) $\not\subset$ $b^{\#}$-b(A) and $b^{\#}$-Fr(A) $\not\subset$ $b^{\#}$-Fr($b^{\#}$-int(A)).

5. $b^{\#}$- exterior

Definition 5.1: For a sub set A of (X, τ), the $b^{\#}$-interior of X/A is called $b^{\#}$- exterior of A and is denoted by $b^{\#}$-ext(A), that is $b^{\#}$-ext(A)= $b^{\#}$-int(X/A).

Theorem 5.2: For sub sets A and B of X the following assertions are valid.

(i) $b^{\#}$-ext(A)= X/ $b^{\#}$-cl(A).

(ii) $b^{\#}$-ext($b^{\#}$-ext(A))= $b^{\#}$-int($b^{\#}$-cl(A)) $\supseteq$ $b^{\#}$-int(A).

(iii) A $\subseteq$ B implies $b^{\#}$-ext(A) $\subseteq$ $b^{\#}$-ext(B).

(iv) $b^{\#}$-ext(A $\bigcup$ B) $\subseteq$ $b^{\#}$-ext(A) $\bigcap$ $b^{\#}$-ext(B).

(v) $b^{\#}$-ext(A $\bigcap$ B) $\supseteq$ $b^{\#}$-ext(A) $\bigcup$ $b^{\#}$-ext(B).

(vi) $b^{\#}$-ext(X)= ϕ, $b^{\#}$-ext(ϕ)= X.

(vii) X= $b^{\#}$-int(A) $\bigcup$ $b^{\#}$-ext(A) $\bigcup$ $b^{\#}$-Fr(A).

Proof: By Definition 5.1 and Lemma 2.6, $b^{\#}$-ext(A)= $b^{\#}$-int(X/A)= X/$b^{\#}$-cl(A). This proves (i). Now to prove (ii). Using Lemma 2.6, we get $b^{\#}$-ext($b^{\#}$-ext(A))= $b^{\#}$-ext($b^{\#}$-int(X/A)) = $b^{\#}$-int(X/$b^{\#}$-int(X/A))= $b^{\#}$-int($b^{\#}$-cl(A)) $\supseteq$ $b^{\#}$-int(A). This proves (ii).

Now to prove (iii). Assume that A $\subseteq$ B. Then $b^{\#}$-ext(B)= $b^{\#}$-int(X/B) $\subseteq$ $b^{\#}$-int(X/A)= $b^{\#}$-ext(A). Hence (iii) is proved.

Now to prove (iv). $b^{\#}$-ext(A $\bigcup$ B)= $b^{\#}$-int(X/(A $\bigcup$ B)) = $b^{\#}$-int((X/A) $\bigcap$ (X/B)) $\subseteq$ $b^{\#}$-int(X/A) $\bigcap$ $b^{\#}$- int(X/B)= $b^{\#}$-ext(A) $\bigcap$ $b^{\#}$-ext(B). Hence (iv) is proved.

Now to prove (v). $b^{\#}$-ext(A $\bigcap$ B)= $b^{\#}$-int(X/(A $\bigcap$ B))= $b^{\#}$-int((X/A) $\bigcup$ (X/B)) $\supseteq$ $b^{\#}$-int(X/A) $\bigcup$ $b^{\#}$-int(X/B) = $b^{\#}$-ext(A) $\bigcup$ $b^{\#}$-ext(B). Thus (v) is proved. Using Definition 5.1, (vi) is proved.

Now to prove (vii). $b^{\#}$-int(A) $\bigcup$ $b^{\#}$-ext(A) $\bigcup$ $b^{\#}$-Fr(A)= $b^{\#}$-int(A) $\bigcup$ ($b^{\#}$-cl(A)/ $b^{\#}$-int(A)) $\bigcup$ $b^{\#}$-ext(A) = ($b^{\#}$-int(A) $\bigcup$ $b^{\#}$-cl(A)) $\cup$ $b^{\#}$-ext(A)= $b^{\#}$-cl(A) $\cup$ $b^{\#}$-cl(X\A)= X since by Definition 4.2 and Lemma 2.6.

Examples can be easily constructed for the reverse inclusion of Theorem 5.2(iii) and (iv).

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