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SOME CHARACTERIZATION OF FUZZY MEAN RESIDUAL LIFE ORDER AND FUZZY PROPORTIONAL MEAN RESIDUAL LIFE ORDER

D. RAJAN[1] AND D. VIJAYABALAN*[2]

[1]Associate Professor, [2]Research Scholar,
P.G. & Research Department of Mathematics, T.B.M.L. College, Porayar - 609307, India.

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ABSTRACT

In this paper, we have recalled some of the known stochastic orders and the shifted version of them, so discussed their relations. Also, we obtained some applications of proportional hazard rate ordering

Keywords: *Fuzzy random variables, Fuzzy Hazard rate order, Shifted fuzzy Hazard rate order, Fuzzy Mean Residual Life Order, Shifted and Proportional Fuzzy Mean Residual Life Order.*

1. INTRODUCTION

In life testing situations, the mean additional life time given that a component has survived until time t is a function of t, called the mean residual life. We consider the special case of mean residual life in terms of fuzzy random variables. More specifically, if the fuzzy random variables X represent the life of a component, then the mean residual life is given by m_α(t) = E[$X_\alpha - t/X_\alpha > t$]

The mean residual life has been employed in life length by various authors, e.g. Bryson and Siddigui (1969), Hollander and Proschan (1975) and Muth (1977). Limiting properties of the mean residual life have been studied by Meillijson (1972), Bradley and Gutpa [2], (2002). A smooth estimator of the mean residual life is given by Chaubey and Sen (1999).

It is well known that the failure rate function can be expressed quite well in terms of mean residual life and its derivative. However, the inverse problem – namely that of expressing the mean residual life in terms of the failure rate typically involves an integral of the complicated expressions.

2. PRELIMINARIES

Definition 2.1: If X is a random variable with a survival function $\bar{F}$ and a finite mean μ, the mean residual life of X at t is defined as

$$m(t) = \begin{cases} E[X_\alpha - t/X_\alpha > t], & \text{for } t<t^* \\ 0 & otherwise \end{cases}$$

Where t^* = sup{t:$\bar{F}$(t) > 0}. Note that if X ia an almost surely positive random variable, then $m(0)$ = μ. By the finiteness of μ we have that m(t) <∞ for all t < ∞. However, it is possible that $m(\infty) \equiv \lim_{t\to\infty} m(t) = \infty$. A useful observation is that

$$m(t) = \int_t^\infty \bar{F}(x)dx/\bar{F}(t). \text{ When } t^* = \infty$$

Although in definition, there is no restriction on the support of X, the mean residual life function is usually of interest when X is a nonnegative random variable. In that case X can be thought of as a lifetime of a device and then expreses the conditional expected residual life of the device and at time t given that the device is still alive at time t. Clearly, m (t) ≥ 0, but not every nonnegative function is a *mean residual life* (mrl) function corresponding to some random variable. In fact, a function n is an mrl function of some non- negative random variable with an absolutely continuous distribution function if, and only if, m satisfies the following properties:

Corresponding Author: D. Vijayabalan*[2], [2]Research Scholar,
P.G. & Research Department of Mathematics, T.B.M.L. College, Porayar - 609307, India.

(i) $0 \leq m(t) < \infty$ for all $t \geq 0$,
(ii) $m(0) > 0$,
(iii) m is continuous,
(iv) $m(t) + t$ is increasing on [0,∞], and
(v) When there exist a t_0 such that $m(t_0) = 0$, $m(t) = 0$ for all $t \geq t_0$ otherwise when there does not exist such that a t_0 with $m(t_0) = 0$, then $\int_0^\infty \frac{1}{m(t)} dt = \infty$.

2.2. FUZZY NUMBERS

Let X be a universal set and $S_X = \{x \epsilon X: f(x;\theta) > 0\}$ be the support of X. A fuzzy subset (briefly, a fuzzy set) $\tilde{x}$ of S_X is defined by its membership function $\mu_{\tilde{x}} : S_X \to [0,1]$. We denote the α –cuts of $\tilde{x}$ by $\tilde{x}_\alpha = \{x: \mu_{\tilde{x}}(x) \geq \alpha\}$, and $\tilde{x}_0$ is the closure of the set $\{x: \mu_{\tilde{x}}(x) > 0\}$. then $f(x;\theta)$ is a fuzzy set.

The fuzzy set $\tilde{x}$ is called a normal fuzzy set if there exists $x \in S_X$ such that $\mu_{\tilde{x}}(x) = 1$, and called convex fuzzy sets $\mu_{\tilde{x}}(\lambda x + (1-\lambda)y) \geq \min\{\mu_{\tilde{x}}(x), \mu_{\tilde{x}}(y)\}$ for every $x, y \in S_X$ and λ ∈ [0,1]. The fuzzy set $\tilde{x}$ is called fuzzy number if it is normal and convex fuzzy set and its α –cuts are bounded for all α ∈ [0,1] . In addition, if $\tilde{x}$ is a fuzzy number and the support of its membership function $\mu_{\tilde{x}}$ is compact, then we called $\tilde{x}$ as a bounded fuzzy numbers.

If $\tilde{x}$ is a closed and bounded fuzzy number with $\tilde{x}_\alpha^L = min\{x: x \in \tilde{x}_\alpha\}$ and $\tilde{x}_\alpha^U = max\{x: x \in \tilde{x}_\alpha\}$ and its membership function be strictly increasing on the interval $[x_\alpha^L, x_1^L]$ and strictly decreasing on the interval $[x_\alpha^U, x_1^U]$, then $\tilde{x}$ is called a canonical fuzzy number.

2.3. FUZZY RANDOM VARIABLE

The fuzzy number $\tilde{x}$ with membership function $\mu_{\tilde{x}}(r)$ can be induced by any real number $x \in S_X$ such that $\mu_{\tilde{x}}(x) = 1$ and $\mu_{\tilde{x}}(r) < 1$ for $r \neq x$. We denote the set of all fuzzy real numbers induced by real number $x \in S_X$ by F(S_X).

The relation ∼ on F(S_X) define as $\tilde{x}_1 \sim \tilde{x}_2$ if and only if $\tilde{x}_1$ and $\tilde{x}_2$ are induced by the same real number x. Then ∼ is an equivalence relation, which induce the equivalence classes $[\tilde{x}] = \{\tilde{a}: \tilde{a} \sim \tilde{x}\}$. The set (F($S_X$)/∼) called a fuzzy real number system. In practice, we take only one element $\tilde{x}$ from each equivalence class $[\tilde{x}]$ to form the fuzzy real number system (F(S_X)/∼). If the fuzzy real number system (F(S_X)/∼) consists all of the canonical fuzzy real numbers, then we call (F(S_X)/∼) as the canonical fuzzy real number system.

Let be a random variables with support S_X and (S_X) is the set of all canonical fuzzy numbers induce the real numbers in S_X. A fuzzy random variable is a function $X: \to (S_X)$ where for all α∈ [0,1].

$$\{(\omega, x): \omega \in \psi, x \in \tilde{X}_\alpha(\omega)\} \in Ƒ \times \mathcal{B}$$

Noting that F(S_X) is the support of the fuzzy random variable $\tilde{X}$ and hence, each α –cut set of $\tilde{X}$ depends on the random variable X .

3. SOME PROPERTIES OF HAZARD RATE ORDER

3.1. FUZZY HAZARD RATE ORDER:

Let X and Y are two non negative fuzzy random variables with continuous distribution functions and with Hazard rate functions $\tilde{r}(\tilde{x})$ and $\widetilde{q}(\tilde{x})$ respectively, then X is smaller than Y in Hazard rate order. Denoted as $X \leq_{FHR} Y$. If

$$\frac{min\left\{\min_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^U\right)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^U\right)\right\}} \leq \frac{min\left\{\min_{\alpha\leq\beta\leq1} g\left(\tilde{y}_\beta^L\right), \min_{\alpha\leq\beta\leq1} g\left(\tilde{y}_\beta^U\right)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^U\right)\right\}}$$

and

$$\frac{max\left\{\max_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^L\right), \max_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^U\right)\right\}}{max\left\{\max_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^L\right), \max_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^U\right)\right\}} \leq \frac{max\left\{\max_{\alpha\leq\beta\leq1} g\left(\tilde{y}_\beta^L\right), \max_{\alpha\leq\beta\leq1} g\left(\tilde{y}_\beta^U\right)\right\}}{max\left\{\max_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^L\right), \max_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^U\right)\right\}}$$

For each α, β ∈ (0,1]∩Q, where $\bar{F}, f$ are the survival and density functions of X respectively and $\bar{G}$, g are the survival and density functions of Y respectively.

3.2. SHIFTED FUZZY HAZARD RATE ORDER:

Let X and Y are two non negative fuzzy random variables with continuous distribution functions and with Shifted fuzzy Hazard rate functions $\tilde{r}(\tilde{x})$ and $\widetilde{q}(\tilde{x})$ respectively, then X is smaller than Y in Hazard rate order. Denoted as X $\leq_{SFHR\uparrow}$ Y. If

$$\frac{min\left\{\min_{\alpha\leq\beta\leq1} f(a+\tilde{x}_\alpha^L)g(\tilde{x}_\alpha^U), \min_{\alpha\leq\beta\leq1} f\left(a+\tilde{x}_\beta^U\right)g(\tilde{x}_\alpha^L)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}(a+\tilde{x}_\alpha^L)\bar{G}(\tilde{x}_\alpha^U), \min_{\alpha\leq\beta\leq1} \bar{F}\left(a+\tilde{x}_\beta^U\right)\bar{G}(\tilde{x}_\alpha^L)\right\}} \geq \frac{min\left\{\min_{\alpha\leq\beta\leq1} g(a+\tilde{x}_\alpha^U)f(\tilde{x}_\alpha^L), \min_{\alpha\leq\beta\leq1} g(a+\tilde{x}_\alpha^L)f(\tilde{x}_\alpha^U)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{G}(a+\tilde{x}_\alpha^U)\bar{F}(\tilde{x}_\alpha^L), \min_{\alpha\leq\beta\leq1} \bar{G}\left(a+\tilde{x}_\beta^L\right)\bar{F}(\tilde{x}_\alpha^U)\right\}}$$

and

$$\frac{max\left\{\max_{\alpha\leq\beta\leq1} f(a+\tilde{x}_\alpha^L)g(\tilde{x}_\alpha^U), \max_{\alpha\leq\beta\leq1} f\left(a+\tilde{x}_\beta^U\right)g(\tilde{x}_\alpha^L)\right\}}{max\left\{\max_{\alpha\leq\beta\leq1} \bar{F}(a+\tilde{x}_\alpha^L)\bar{G}(\tilde{x}_\alpha^U), \max_{\alpha\leq\beta\leq1} \bar{F}\left(a+\tilde{x}_\beta^U\right)\bar{G}(\tilde{x}_\alpha^L)\right\}} \geq \frac{max\left\{\max_{\alpha\leq\beta\leq1} g(a+\tilde{x}_\alpha^U)f(\tilde{x}_\alpha^L), \max_{\alpha\leq\beta\leq1} g(a+\tilde{x}_\alpha^L)f(\tilde{x}_\alpha^U)\right\}}{max\left\{\max_{\alpha\leq\beta\leq1} \bar{G}(a+\tilde{x}_\alpha^U)\bar{F}(\tilde{x}_\alpha^L), \max_{\alpha\leq\beta\leq1} \bar{G}\left(a+\tilde{x}_\beta^L\right)\bar{F}(\tilde{x}_\alpha^U)\right\}}$$

For each α, β Є (0, 1] ∩Q, where $\bar{F}, f$ are the survival and density functions of X respectively and $\bar{G}, g$ are the survival and density functions of Y respectively.

3.3. SHIFTED PROPORTIONAL FUZZY HAZARD RATE ORDER:

Let X and Y are two non negative fuzzy random variables with continuous distribution functions and with Shifted fuzzy Hazard rate functions $\tilde{r}(\tilde{x})$ and $\widetilde{q}(\tilde{x})$ respectively, then X is smaller than Y in Hazard rate order. Denoted as X $\leq_{PFHR\updownarrow}$Y. If,

$$\frac{min\left\{\min_{\alpha\leq\beta\leq1} f(a+\tilde{x}_\alpha^L)g(\lambda\tilde{x}_\alpha^U), \min_{\alpha\leq\beta\leq1} f\left(a+\tilde{x}_\beta^U\right)g(\widetilde{\lambda x}_\alpha^L)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}(a+\tilde{x}_\alpha^L)\bar{G}(\lambda\tilde{x}_\alpha^U), \min_{\alpha\leq\beta\leq1} \bar{F}\left(a+\tilde{x}_\beta^U\right)\bar{G}(\widetilde{\lambda x}_\alpha^L)\right\}} \geq \frac{min\left\{\min_{\alpha\leq\beta\leq1} g(a+\tilde{x}_\alpha^U)f(\lambda\tilde{x}_\alpha^L), \min_{\alpha\leq\beta\leq1} g(a+\tilde{x}_\alpha^L)f(\lambda\tilde{x}_\alpha^U)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{G}(a+\tilde{x}_\alpha^U)\bar{F}(\lambda\tilde{x}_\alpha^L), \min_{\alpha\leq\beta\leq1} \bar{G}\left(a+\tilde{x}_\beta^L\right)\bar{F}(\lambda\tilde{x}_\alpha^U)\right\}}$$

and

$$\frac{max\left\{\max_{\alpha\leq\beta\leq1} f(a+\tilde{x}_\alpha^L)g(\widetilde{\lambda x}_\alpha^U), \max_{\alpha\leq\beta\leq1} f\left(a+\tilde{x}_\beta^U\right)g(\lambda\tilde{x}_\alpha^L)\right\}}{max\left\{\max_{\alpha\leq\beta\leq1} \bar{F}(a+\tilde{x}_\alpha^L)\bar{G}(\lambda\tilde{x}_\alpha^U), \max_{\alpha\leq\beta\leq1} \bar{F}\left(a+\tilde{x}_\beta^U\right)\bar{G}(\lambda\tilde{x}_\alpha^L)\right\}} \geq \frac{max\left\{\max_{\alpha\leq\beta\leq1} g(a+\tilde{x}_\alpha^U)f(\lambda\tilde{x}_\alpha^L), \max_{\alpha\leq\beta\leq1} g(a+\tilde{x}_\alpha^L)f(\lambda\tilde{x}_\alpha^U)\right\}}{max\left\{\max_{\alpha\leq\beta\leq1} \bar{G}(a+\tilde{x}_\alpha^U)\bar{F}(\lambda\tilde{x}_\alpha^L), \max_{\alpha\leq\beta\leq1} \bar{G}\left(a+\tilde{x}_\beta^L\right)\bar{F}(\lambda\tilde{x}_\alpha^U)\right\}}$$

for each α, β Є (0,1]∩Q, where $\bar{F}, f$ are the survival and density functions of X respectively and $\bar{G}, g$ are the survival and density functions of Y respectively.

4. SOME PROPERTIES FUZZY MEAN RESIDUAL LIFE ORDER ORDER

4.1. FUZZY MEAN RESIDUAL LIFE ORDER ORDER

Let X and Y are two non negative fuzzy random variables with continuous distribution functions and with fuzzy mean residual life order functions $\bar{F}(\tilde{x})$ and$\bar{G}$ $(\tilde{x})$ respectively, then X is smaller than Y in Mean residual life order. Denoted as $X \leq_{FMLR} Y$. If

$$\frac{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}dx} \leq \frac{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{y}_\beta^L\right), \min_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{y}_\beta^U\right)\right\}dy}{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{y}_\beta^L\right), \min_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{y}_\beta^U\right)\right\}dy}$$

and

$$\frac{\int_x^\infty max\left\{\max_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{x}_\beta^L\right), \max_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty max\left\{\max_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{x}_\beta^L\right), \max_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}dx} \leq \frac{\int_x^\infty max\left\{\max_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{y}_\beta^L\right), \max_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{y}_\beta^U\right)\right\}dy}{\int_x^\infty max\left\{\max_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{y}_\beta^L\right), m_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{y}_\beta^U\right)\right\}dy},$$

increases in x over $\{x: \int_x^\infty \bar{F}(x)dx, \int_x^\infty \bar{F}(x)dx > o\}$. For each α, β Є (0,1]∩Q, where $\bar{F}, \bar{G}$ are the survival functions of X and Y respectively.

Definition 4.2: X is said to be Mean residual life aging faster than Y if $\frac{\bar{F}_X(\tilde{x})}{\bar{G}_Y(\tilde{x})}$ is increasing in $x \geq 0$ or ultimately Mean residual life aging faster than Y if above holds for sufficiently large x.

4.3. UP PROPORTIONAL FUZZY MEAN RESIDUAL LIFE ORDER ORDER:

Let X and Y are two non negative fuzzy random variables with continuous distribution functions and with fuzzy mean residual life order functions $\bar{F}(\tilde{x})$ and$\bar{G}$ $(\tilde{x})$ respectively, then X is smaller than Y in Mean residual life order. Denoted as $X \leq_{UPFMLR} Y$. If

$$\frac{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{x}_\beta^L-t/\tilde{x}_\beta^L>0\right), \min_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{x}_\beta^L-t/\tilde{x}_\beta^L>0\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{x}_\beta^L-t/\tilde{x}_\beta^L>0\right), \min_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{x}_\beta^L-t/\tilde{x}_\beta^L>0\right)\right\}dx} \leq \frac{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{y}_\beta^L\right), \min_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{y}_\beta^U\right)\right\}dy}{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{y}_\beta^L\right), \min_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{y}_\beta^U\right)\right\}dy}$$

and

$$\frac{\int_x^\infty max\left\{\max_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{x}_\beta^L-t/\tilde{x}_\beta^L>0\right), \max_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{x}_\beta^L-t/\tilde{x}_\beta^L>0\right)\right\}dx}{\int_x^\infty max\left\{\max_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{x}_\beta^L-t/\tilde{x}_\beta^L>0\right), \max_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{x}_\beta^L-t/\tilde{x}_\beta^L>0\right)\right\}dx} \leq \frac{\int_x^\infty min\left\{\max_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{y}_\beta^L\right), \max_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{y}_\beta^U\right)\right\}dy}{\int_x^\infty min\left\{\max_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{y}_\beta^L\right), m_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{y}_\beta^U\right)\right\}dy}$$

increases in x over $\{x: \int_x^\infty \bar{F}(x)dx, \int_x^\infty \bar{F}(x)dx > o\}$. For each α,β Є (0,1]∩Q, where $\bar{F}, \bar{G}$ are the survival functions of X and Y respectively.

4.4. DOWN PROPORTIONAL FUZZY MEAN RESIDUAL LIFE ORDER ORDER:

Let X and Y are two non negative fuzzy random variables with continuous distribution functions and with fuzzy mean residual life order functions $\bar{F}(\tilde{x})$ and$\bar{G}$ $(\tilde{x})$ respectively, then X is smaller than Y in Mean residual life order. Denoted as $X \leq_{DPFMLR} Y$. If

$$\frac{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}dx} \leq \frac{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{y}_\beta^L-t/\tilde{y}_\beta^L>0\right), \min_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{y}_\beta^L-t/\tilde{y}_\beta^L>0\right)\right\}dy}{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{y}_\beta^L-t/\tilde{y}_\beta^L>0\right), \min_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{y}_\beta^L-t/\tilde{y}_\beta^L>0\right)\right\}dy}$$

and

$$\frac{\int_x^\infty max\left\{\max_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{x}_\beta^L\right), \max_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty max\left\{\max_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{x}_\beta^L\right), \max_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}dx} \leq \frac{\int_x^\infty max\left\{\max_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{y}_\beta^L-t/\tilde{y}_\beta^L>0\right), \max_{\alpha\leq\beta\leq1}\bar{G}\left(\tilde{y}_\beta^L-t/\tilde{y}_\beta^L>0\right)\right\}dy}{\int_x^\infty max\left\{\max_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{y}_\beta^L-t/\tilde{y}_\beta^L>0\right), \max_{\alpha\leq\beta\leq1}\bar{F}\left(\tilde{y}_\beta^L-t/\tilde{y}_\beta^L>0\right)\right\}dy}$$

increases in x over $\{x: \int_x^\infty \bar{F}(x)dx, \int_x^\infty \bar{F}(x)dx > o\}$. For each α,β Є (0,1]∩Q, where $\bar{F}, \bar{G}$ are the survival functions of X and Y respectively.

4.5. SHIFTED FUZZY MEAN RESIDUAL LIFE ORDER ORDER:
Let X and Y are two non negative fuzzy random variables with continuous distribution functions and with fuzzy mean residual life order functions $\bar{F}(\tilde{x})$ and$\bar{G}(\tilde{x})$ respectively, then X is smaller than Y in Mean residual life order. Denoted as $X \le_{FMLR\uparrow} Y$. If

$$\frac{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{min}\bar{G}\left(a+\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}\bar{G}\left(\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{min}\bar{F}\left(a+\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}dx} \le \frac{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{min}\bar{G}\left(\tilde{y}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}\bar{G}\left(a+\tilde{y}_\beta^U\right)\right\}dy}{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{min}\bar{F}\left(\tilde{y}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}\bar{F}\left(a+\tilde{y}_\beta^U\right)\right\}dy}$$

and

$$\frac{\int_x^\infty max\left\{\underset{\alpha\le\beta\le1}{max}\bar{G}\left(a+\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{max}\bar{G}\left(\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty max\left\{\underset{\alpha\le\beta\le1}{max}\bar{F}\left(a+\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{max}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}dx} \le \frac{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{max}\bar{G}\left(\tilde{y}_\beta^L\right),\underset{\alpha\le\beta\le1}{max}\bar{G}\left(a+\tilde{y}_\beta^U\right)\right\}dy}{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{max}\bar{F}\left(\tilde{y}_\beta^L\right),\underset{\alpha\le\beta\le1}{m}\bar{F}\left(a+\tilde{y}_\beta^U\right)\right\}dy},$$

increases in x over $\{x: \int_x^\infty \bar{F}(x)dx, \int_x^\infty \bar{F}(x)dx > o\}$. For each α,β Є (0,1]∩Q, where $\bar{F}, \bar{G}$ are the survival functions of X and Y respectively.

4.6. SHIFTED PROPORTIONAL FUZZY MEAN RESIDUAL LIFE ORDER ORDER:
Let X and Y are two non negative fuzzy random variables with continuous distribution functions and with fuzzy mean residual life order functions $\bar{F}(\tilde{x})$ and$\bar{G}$ $(\tilde{x})$ respectively, then X is smaller than Y in Mean residual life order. Denoted as $X \le_{FMLR\uparrow} Y$. If

$$\frac{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{min}\bar{G}\left(a+\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}\bar{G}\left(\lambda\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{min}\bar{F}\left(a+\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}\bar{F}\left(\lambda\tilde{x}_\beta^U\right)\right\}dx} \le \frac{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{min}\bar{G}\left(\lambda\tilde{y}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}\bar{G}\left(a+\tilde{y}_\beta^U\right)\right\}dy}{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{min}\bar{F}\left(\lambda\tilde{y}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}\bar{F}\left(a+\tilde{y}_\beta^U\right)\right\}dy}$$

and

$$\frac{\int_x^\infty max\left\{\underset{\alpha\le\beta\le1}{max}\bar{G}\left(a+\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{max}\bar{G}\left(\lambda\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty max\left\{\underset{\alpha\le\beta\le1}{max}\bar{F}\left(a+\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{max}\bar{F}\left(\lambda\tilde{x}_\beta^U\right)\right\}dx} \le \frac{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{max}\bar{G}\left(\lambda\tilde{y}_\beta^L\right),\underset{\alpha\le\beta\le1}{max}\bar{G}\left(a+\tilde{y}_\beta^U\right)\right\}dy}{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{max}\bar{F}\left(\lambda\tilde{y}_\beta^L\right),\underset{\alpha\le\beta\le1}{m}\bar{F}\left(a+\tilde{y}_\beta^U\right)\right\}dy}$$

increases in x over $\{x: \int_x^\infty \bar{F}(x)dx, \int_x^\infty \bar{F}(x)dx > o\}$. For each α, β Є (0,1]∩Q, where $\bar{F}, \bar{G}$ are the survival functions of X and Y respectively.

4.7. INCREASING FUZZY PROPORTIONAL MEAN RESIDUAL LIFE ORDER PROPERTY:

1. Definition: X has the increasing fuzzy proportional Mean residual life order property, $X \in_{IFMLR}$if

$$\frac{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{min}\bar{G}\left(\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}\bar{G}\left(\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{min}\bar{F}\left(\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}dx} \le \frac{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{min}\bar{G}\left(\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}\bar{G}\left(\tilde{x}_\beta^U\right)\right\}dy}{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{min}\bar{F}\left(\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}dy}$$

and

$$\frac{\int_x^\infty max\left\{\underset{\alpha\le\beta\le1}{max}\bar{G}\left(\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{max}\bar{G}\left(\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty max\left\{\underset{\alpha\le\beta\le1}{max}\bar{F}\left(\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{max}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}dx} \le \frac{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{max}\bar{G}\left(\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{max}\bar{G}\left(\tilde{x}_\beta^U\right)\right\}dy}{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{max}\bar{F}\left(\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{m}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}dy}$$

2. Definition: continuous non negative fuzzy random variable X admits up increasing fuzzy proportional Mean residual life order property denoted by$X$ $\in_{UIFPMR}$ if,

$$\frac{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{min}\bar{G}\left(\tilde{x}_\beta^L-t/\tilde{x}_\beta^L>0\right),\underset{\alpha\le\beta\le1}{min}\bar{G}\left(\tilde{x}_\beta^L-t/\tilde{x}_\beta^L>0\right)\right\}dx}{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{min}\bar{F}\left(\tilde{x}_\beta^L-t/\tilde{x}_\beta^L>0\right),\underset{\alpha\le\beta\le1}{min}\bar{F}\left(\tilde{x}_\beta^L-t/\tilde{x}_\beta^L>0\right)\right\}dx} \le \frac{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{min}\bar{G}\left(\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}\bar{G}\left(\tilde{x}_\beta^U\right)\right\}dy}{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{min}\bar{F}\left(\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}dy}$$

and

$$\frac{\int_x^\infty max\left\{\underset{\alpha\le\beta\le1}{max}\bar{G}\left(\tilde{x}_\beta^L-t/\tilde{x}_\beta^L>0\right),\underset{\alpha\le\beta\le1}{max}\bar{G}\left(\tilde{x}_\beta^L-t/\tilde{x}_\beta^L>0\right)\right\}dx}{\int_x^\infty max\left\{\underset{\alpha\le\beta\le1}{max}\bar{F}\left(\tilde{x}_\beta^L-t/\tilde{x}_\beta^L>0\right),\underset{\alpha\le\beta\le1}{max}\bar{F}\left(\tilde{x}_\beta^L-t/\tilde{x}_\beta^L>0\right)\right\}dx} \le \frac{\int_x^\infty max\left\{\underset{\alpha\le\beta\le1}{max}\bar{G}\left(\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{max}\bar{G}\left(\tilde{x}_\beta^U\right)\right\}dy}{\int_x^\infty max\left\{\underset{\alpha\le\beta\le1}{max}\bar{F}\left(\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{max}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}dy}$$

4.8. Theorem: If X and Y are two fuzzy random variables such that,

$$\frac{min\left\{\underset{\alpha\le\beta\le1}{min}f\left(\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}f\left(\tilde{x}_\beta^U\right)\right\}}{min\left\{\underset{\alpha\le\beta\le1}{min}\bar{F}\left(\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}} \le \frac{min\left\{\underset{\alpha\le\beta\le1}{min}g\left(\tilde{y}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}g\left(\tilde{y}_\beta^U\right)\right\}}{min\left\{\underset{\alpha\le\beta\le1}{min}\bar{G}\left(\tilde{y}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}\bar{G}\left(\tilde{y}_\beta^U\right)\right\}}$$

and

$$\frac{max\left\{\underset{\alpha\le\beta\le1}{max}f\left(\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{max}f\left(\tilde{x}_\beta^U\right)\right\}}{max\left\{\underset{\alpha\le\beta\le1}{max}\bar{F}\left(\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{max}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}} \le \frac{max\left\{\underset{\alpha\le\beta\le1}{max}g\left(\tilde{y}_\beta^L\right),\underset{\alpha\le\beta\le1}{max}g\left(\tilde{y}_\beta^U\right)\right\}}{max\left\{\underset{\alpha\le\beta\le1}{max}\bar{G}\left(\tilde{y}_\beta^L\right),\underset{\alpha\le\beta\le1}{max}\bar{G}\left(\tilde{y}_\beta^U\right)\right\}},$$

Then

$$\frac{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{min}\bar{G}\left(\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}\bar{G}\left(\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{min}\bar{F}\left(\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}dx} \le \frac{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{min}\bar{G}\left(\tilde{y}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}\bar{G}\left(\tilde{y}_\beta^U\right)\right\}dy}{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{min}\bar{F}\left(\tilde{y}_\beta^L\right),\underset{\alpha\le\beta\le1}{min}\bar{F}\left(\tilde{y}_\beta^U\right)\right\}dy}$$

and

$$\frac{\int_x^\infty max\left\{\underset{\alpha\le\beta\le1}{max}\bar{G}\left(\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{max}\bar{G}\left(\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty max\left\{\underset{\alpha\le\beta\le1}{max}\bar{F}\left(\tilde{x}_\beta^L\right),\underset{\alpha\le\beta\le1}{max}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}dx} \le \frac{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{max}\bar{G}\left(\tilde{y}_\beta^L\right),\underset{\alpha\le\beta\le1}{max}\bar{G}\left(\tilde{y}_\beta^U\right)\right\}dy}{\int_x^\infty min\left\{\underset{\alpha\le\beta\le1}{max}\bar{F}\left(\tilde{y}_\beta^L\right),\underset{\alpha\le\beta\le1}{m}\bar{F}\left(\tilde{y}_\beta^U\right)\right\}dy}$$

4.9. Note: Neither of the orders stochastic and fuzzy mean residual life order implies others

4.10. Theorem: Let X and Y are two non negative fuzzy random variables with mean residual life functions $\bar{F}_X$and $\bar{G}_Y$ respectively. Suppose that $\frac{\bar{F}_X(\tilde{x})}{\bar{G}_Y(\tilde{x})}$ is increasing in $x \geq 0$ and for each $\alpha\epsilon(0,1]$. Then

$$\frac{min\left\{\min_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^U\right)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^U\right)\right\}} \leq \frac{min\left\{\min_{\alpha\leq\beta\leq1} g\left(\tilde{y}_\beta^L\right), \min_{\alpha\leq\beta\leq1} g\left(\tilde{y}_\beta^U\right)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^U\right)\right\}}$$

and

$$\frac{max\left\{\max_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^L\right), \max_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^U\right)\right\}}{max\left\{\max_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^L\right), \max_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^U\right)\right\}} \leq \frac{max\left\{\max_{\alpha\leq\beta\leq1} g\left(\tilde{y}_\beta^L\right), \max_{\alpha\leq\beta\leq1} g\left(\tilde{y}_\beta^U\right)\right\}}{max\left\{\max_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^L\right), \max_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^U\right)\right\}}$$

If

$$\frac{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^U\right)\right\}dx} \leq \frac{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^U\right)\right\}dy}{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{y}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{y}_\beta^U\right)\right\}dy}$$

and

$$\frac{\int_x^\infty max\left\{\max_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{x}_\beta^L\right), \max_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty max\left\{\max_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^L\right), \max_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^U\right)\right\}dx} \leq \frac{\int_x^\infty min\left\{\max_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^L\right), \max_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^U\right)\right\}dy}{\int_x^\infty min\left\{\max_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{y}_\beta^L\right), m_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{y}_\beta^U\right)\right\}dy}$$

Proof: It is not hard to verify that $\bar{F}_X(\tilde{x})$ is differentiable over $\{\tilde{x}: P\{X > \tilde{x}\} > 0\}$ and that if X has the fuzzy Hazard rate function $f(x)$, then

$$f(x) = \frac{\frac{min\left\{\min_{\alpha\leq\beta\leq1} f\left(a+\tilde{x}_\alpha^L\right)g\left(\tilde{x}_\alpha^U\right), \min_{\alpha\leq\beta\leq1} f\left(a+\tilde{x}_\beta^U\right)g\left(\tilde{x}_\alpha^L\right)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}\left(a+\tilde{x}_\alpha^L\right)\bar{G}\left(\tilde{x}_\alpha^U\right), \min_{\alpha\leq\beta\leq1} \bar{F}\left(a+\tilde{x}_\beta^U\right)\bar{G}\left(\tilde{x}_\alpha^L\right)\right\}}+1}{\frac{min\left\{\min_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^U\right)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^U\right)\right\}}}$$

Where $f(a + \tilde{x}_\alpha^L)$ denotes the shifted of $f(x)$.

If Y has the fuzzy Hazard rate function $f(y)$, then

$$f(y) = \frac{\frac{min\left\{\min_{\alpha\leq\beta\leq1} f\left(a+\tilde{y}_\alpha^L\right)g\left(\tilde{y}_\alpha^U\right), \min_{\alpha\leq\beta\leq1} f\left(a+\tilde{y}_\beta^U\right)g\left(\tilde{y}_\alpha^L\right)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}\left(a+\tilde{y}_\alpha^L\right)\bar{G}\left(\tilde{y}_\alpha^U\right), \min_{\alpha\leq\beta\leq1} \bar{F}\left(a+\tilde{y}_\beta^U\right)\bar{G}\left(\tilde{y}_\alpha^L\right)\right\}}+1}{\frac{min\left\{\min_{\alpha\leq\beta\leq1} f\left(\tilde{y}_\beta^L\right), \min_{\alpha\leq\beta\leq1} f\left(\tilde{y}_\beta^U\right)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{y}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{y}_\beta^U\right)\right\}}}$$

The monotonicity of $\frac{\bar{F}_X(\tilde{x})}{\bar{G}_Y(\tilde{x})}$, implies that,

$$f(x) = \frac{\frac{min\left\{\min_{\alpha\leq\beta\leq1} f\left(a+\tilde{x}_\alpha^L\right)g\left(\tilde{x}_\alpha^U\right), \min_{\alpha\leq\beta\leq1} f\left(a+\tilde{x}_\beta^U\right)g\left(\tilde{x}_\alpha^L\right)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}\left(a+\tilde{x}_\alpha^L\right)\bar{G}\left(\tilde{x}_\alpha^U\right), \min_{\alpha\leq\beta\leq1} \bar{F}\left(a+\tilde{x}_\beta^U\right)\bar{G}\left(\tilde{x}_\alpha^L\right)\right\}}+1}{\frac{min\left\{\min_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^U\right)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^U\right)\right\}}} \geq$$

$$f(y) = \frac{\frac{min\left\{\min_{\alpha\leq\beta\leq1} f\left(a+\tilde{y}_\alpha^L\right)g\left(\tilde{y}_\alpha^U\right), \min_{\alpha\leq\beta\leq1} f\left(a+\tilde{y}_\beta^U\right)g\left(\tilde{y}_\alpha^L\right)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}\left(a+\tilde{y}_\alpha^L\right)\bar{G}\left(\tilde{y}_\alpha^U\right), \min_{\alpha\leq\beta\leq1} \bar{F}\left(a+\tilde{y}_\beta^U\right)\bar{G}\left(\tilde{y}_\alpha^L\right)\right\}}+1}{\frac{min\left\{\min_{\alpha\leq\beta\leq1} f\left(\tilde{y}_\beta^L\right), \min_{\alpha\leq\beta\leq1} f\left(\tilde{y}_\beta^U\right)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{y}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{y}_\beta^U\right)\right\}}}$$

That is,

$$\frac{min\left\{\min_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^U\right)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^U\right)\right\}} \leq \frac{min\left\{\min_{\alpha\leq\beta\leq1} g\left(\tilde{y}_\beta^L\right), \min_{\alpha\leq\beta\leq1} g\left(\tilde{y}_\beta^U\right)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^U\right)\right\}}$$

Similarly, we can apply this method for maximum also.

4.11. Theorem: Let X and Y be two non negative fuzzy random variables with Mean residual life orders $\bar{F}_X(\tilde{x})$ and $\bar{G}_Y(\tilde{x})$, respectively. Suppose that $\frac{\bar{F}_X(\tilde{x})}{\bar{G}_Y(\tilde{x})} \geq \frac{\bar{F}_X(0)}{\bar{G}_Y(0)}$, $x \geq 0$. If ,

$$\frac{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^U\right)\right\}dx} \leq \frac{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^U\right)\right\}dy}{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{y}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{y}_\beta^U\right)\right\}dy}$$

and

$$\frac{\int_x^\infty max\left\{\max_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{x}_\beta^L\right), \max_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty max\left\{\max_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^L\right), \max_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^U\right)\right\}dx} \leq \frac{\int_x^\infty min\left\{\max_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^L\right), \max_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^U\right)\right\}dy}{\int_x^\infty min\left\{\max_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{y}_\beta^L\right), m_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{y}_\beta^U\right)\right\}dy}$$

Then X is stochastically larger than Y.

Proof: Let $\bar{F}_X$ be the survival function of X. It is not hard to verify that

$$\bar{F}_X(\tilde{x}) = \frac{EX}{\frac{min\left\{\min_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^U\right)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^U\right)\right\}}} \exp\left\{-\int_0^x \frac{1}{\frac{min\left\{\min_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^U\right)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^U\right)\right\}}}\right\}$$

over $\{x: P\{X > x\} > 0\}$

The survival function of Y can be expressed as

$$\bar{G}_Y(\tilde{x}) = \frac{EY}{\frac{min\{\underset{\alpha\le\beta\le1}{min} f(\tilde{y}_\beta^L),\underset{\alpha\le\beta\le1}{min} f(\tilde{y}_\beta^U)\}}{min\{\underset{\alpha\le\beta\le1}{min} \bar{F}(\tilde{y}_\beta^L),\underset{\alpha\le\beta\le1}{min} \bar{F}(\tilde{y}_\beta^U)\}}} \exp\{-\int_0^x \frac{1}{\frac{min\{\underset{\alpha\le\beta\le1}{min} f(\tilde{y}_\beta^L),\underset{\alpha\le\beta\le1}{min} f(\tilde{y}_\beta^U)\}}{min\{\underset{\alpha\le\beta\le1}{min} \bar{F}(\tilde{y}_\beta^L),\underset{\alpha\le\beta\le1}{min} \bar{F}(\tilde{y}_\beta^U)\}}}\}$$

over $\{x: P\{Y > x\} > 0\}$

Therefore, under the assumptions of the theorem, it is seen that $\frac{\bar{F}_X(\tilde{x})}{\bar{G}_Y(\tilde{x})} \ge 1$.

Similarly, we can apply this method for maximum also.

The mean residual life order can be characterized by means of the hazard rate order and the appropriate equilibrium age variables that for non- negative fuzzy random variables X and Y with finite means we denote by and the corresponding asymptotic equilibrium ages.

4.12. Theorem: For non negative fuzzy random variables X and Y with finite mean we have

$$\frac{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{G}(\tilde{x}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{G}(\tilde{x}_\beta^U)\}dx}{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{F}(\tilde{x}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{F}(\tilde{x}_\beta^U)\}dx} \le \frac{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{G}(\tilde{y}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{G}(\tilde{y}_\beta^U)\}dx}{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{F}(\tilde{y}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{F}(\tilde{y}_\beta^U)\}dx}$$

and

$$\frac{\int_x^\infty max\{\underset{\alpha\le\beta\le1}{max}\bar{G}(\tilde{x}_\beta^L),\underset{\alpha\le\beta\le1}{max}\bar{G}(\tilde{x}_\beta^U)\}dx}{\int_x^\infty max\{\underset{\alpha\le\beta\le1}{max}\bar{F}(\tilde{x}_\beta^L),\underset{\alpha\le\beta\le1}{max}\bar{F}(\tilde{x}_\beta^U)\}dx} \le \frac{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{max}\bar{G}(\tilde{y}_\beta^L),\underset{\alpha\le\beta\le1}{max}\bar{G}(\tilde{y}_\beta^U)\}dx}{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{max}\bar{F}(\tilde{y}_\beta^L),\underset{\alpha\le\beta\le1}{m}\bar{F}(\tilde{y}_\beta^U)\}dx}$$ if and only if

$$\frac{min\{\underset{\alpha\le\beta\le1}{min} f(a+\tilde{x}_\alpha^L)g(\lambda\tilde{x}_\alpha^U),\underset{\alpha\le\beta\le1}{min} f(a+\tilde{x}_\beta^U)g(\widetilde{\lambda x}_\alpha^L)\}}{min\{\underset{\alpha\le\beta\le1}{min}\bar{F}(a+\tilde{x}_\alpha^L)\bar{G}(\lambda\tilde{x}_\alpha^U),\underset{\alpha\le\beta\le1}{min}\bar{F}(a+\tilde{x}_\beta^U)\bar{G}(\widetilde{\lambda x}_\alpha^L)\}} \ge \frac{min\{\underset{\alpha\le\beta\le1}{min} g(a+\tilde{x}_\alpha^U)f(\lambda\tilde{x}_\alpha^L),\underset{\alpha\le\beta\le1}{min} g(a+\tilde{x}_\alpha^L)f(\lambda\tilde{x}_\alpha^U)\}}{min\{\underset{\alpha\le\beta\le1}{min}\bar{G}(a+\tilde{x}_\alpha^U)\bar{F}(\lambda\tilde{x}_\alpha^L),\underset{\alpha\le\beta\le1}{min}\bar{G}(a+\tilde{x}_\beta^L)\bar{F}(\lambda\tilde{x}_\alpha^U)\}}$$

and

$$\frac{max\{\underset{\alpha\le\beta\le1}{max} f(a+\tilde{x}_\alpha^L)g(\widetilde{\lambda x}_\alpha^U),\underset{\alpha\le\beta\le1}{max} f(a+\tilde{x}_\beta^U)g(\lambda\tilde{x}_\alpha^L)\}}{max\{\underset{\alpha\le\beta\le1}{max}\bar{F}(a+\tilde{x}_\alpha^L)\bar{G}(\lambda\tilde{x}_\alpha^U),\underset{\alpha\le\beta\le1}{max}\bar{F}(a+\tilde{x}_\beta^U)\bar{G}(\lambda\tilde{x}_\alpha^L)\}} \ge \frac{max\{\underset{\alpha\le\beta\le1}{max} g(a+\tilde{x}_\alpha^U)f(\lambda\tilde{x}_\alpha^L),\underset{\alpha\le\beta\le1}{max} g(a+\tilde{x}_\alpha^L)f(\lambda\tilde{x}_\alpha^U)\}}{max\{\underset{\alpha\le\beta\le1}{max}\bar{G}(a+\tilde{x}_\alpha^U)\bar{F}(\lambda\tilde{x}_\alpha^L),\underset{\alpha\le\beta\le1}{max}\bar{G}(a+\tilde{x}_\beta^L)\bar{F}(\lambda\tilde{x}_\alpha^U)\}}$$

5. SOME CLOSURE PROPERTIES

If $X_1 \le Y_1 \implies \frac{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{G}(\widetilde{x1}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{G}(\widetilde{x1}_\beta^U)\}dx}{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{F}(\widetilde{x1}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{F}(\widetilde{x1}_\beta^U)\}dx} \le \frac{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{G}(\widetilde{y1}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{G}(\widetilde{y1}_\beta^U)\}dx}{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{F}(\widetilde{y1}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{F}(\widetilde{y1}_\beta^U)\}dx}$ and

$$\frac{\int_x^\infty max\{\underset{\alpha\le\beta\le1}{max}\bar{G}(\widetilde{x1}_\beta^L),\underset{\alpha\le\beta\le1}{max}\bar{G}(\widetilde{x1}_\beta^U)\}dx}{\int_x^\infty max\{\underset{\alpha\le\beta\le1}{max}\bar{F}(\widetilde{x1}_\beta^L),\underset{\alpha\le\beta\le1}{max}\bar{F}(\widetilde{x1}_\beta^U)\}dx} \le \frac{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{max}\bar{G}(\widetilde{y1}_\beta^L),\underset{\alpha\le\beta\le1}{max}\bar{G}(\widetilde{y1}_\beta^U)\}dx}{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{max}\bar{F}(\widetilde{y1}_\beta^L),\underset{\alpha\le\beta\le1}{m}\bar{F}(\widetilde{y1}_\beta^U)\}dx}$$

AND

If $X_2 \le Y_2 \implies \frac{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{G}(\widetilde{x2}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{G}(\widetilde{x2}_\beta^U)\}dx}{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{F}(\widetilde{x2}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{F}(\widetilde{x2}_\beta^U)\}dx} \le \frac{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{G}(\widetilde{y2}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{G}(\widetilde{y2}_\beta^U)\}dx}{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{F}(\widetilde{y2}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{F}(\widetilde{y2}_\beta^U)\}dx}$ and

$$\frac{\int_x^\infty max\{\underset{\alpha\le\beta\le1}{max}\bar{G}(\widetilde{x2}_\beta^L),\underset{\alpha\le\beta\le1}{max}\bar{G}(\widetilde{x2}_\beta^U)\}dx}{\int_x^\infty max\{\underset{\alpha\le\beta\le1}{max}\bar{F}(\widetilde{x2}_\beta^L),\underset{\alpha\le\beta\le1}{max}\bar{F}(\widetilde{x2}_\beta^U)\}dx} \le \frac{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{max}\bar{G}(\widetilde{y2}_\beta^L),\underset{\alpha\le\beta\le1}{max}\bar{G}(\widetilde{y2}_\beta^U)\}dx}{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{max}\bar{F}(\widetilde{y2}_\beta^L),\underset{\alpha\le\beta\le1}{m}\bar{F}(\widetilde{y2}_\beta^U)\}dx}$$

X_1, X_2, Y_1, Y_2 are independant fuzzy random variables. Then it is not necessarily true that $\{X_1+X_2\} \le \{Y_1+Y_2\} \implies$

$$\frac{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{G}(\widetilde{x1+x2}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{G}(\widetilde{x1+x2}_\beta^U)\}dx}{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{F}(\widetilde{x1+x2}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{F}(\widetilde{x1+x2}_\beta^U)\}dx} \le \frac{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{G}(\widetilde{y1+y2}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{G}(\widetilde{y1+y2}_\beta^U)\}dx}{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{F}(\widetilde{y1+y2}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{F}(\widetilde{y1+y2}_\beta^U)\}dx}$$ and

$$\frac{\int_x^\infty max\{\underset{\alpha\le\beta\le1}{max}\bar{G}(\widetilde{x1+x2}_\beta^L),\underset{\alpha\le\beta\le1}{max}\bar{G}(\widetilde{x1+x2}_\beta^U)\}dx}{\int_x^\infty max\{\underset{\alpha\le\beta\le1}{max}\bar{F}(\widetilde{x1+x2}_\beta^L),\underset{\alpha\le\beta\le1}{max}\bar{F}(\widetilde{x1+x2}_\beta^U)\}dx} \le \frac{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{max}\bar{G}(\widetilde{y1+y2}_\beta^L),\underset{\alpha\le\beta\le1}{max}\bar{G}(\widetilde{y1+y2}_\beta^U)\}dx}{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{max}\bar{F}(\widetilde{y1+y2}_\beta^L),\underset{\alpha\le\beta\le1}{m}\bar{F}(\widetilde{y1+y2}_\beta^U)\}dx}$$

5.1. Lemma: If the random variables X and Y are such that

$$\frac{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{G}(\tilde{x}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{G}(\tilde{x}_\beta^U)\}dx}{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{F}(\tilde{x}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{F}(\tilde{x}_\beta^U)\}dx} \le \frac{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{G}(\tilde{y}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{G}(\tilde{y}_\beta^U)\}dy}{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{F}(\tilde{y}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{F}(\tilde{y}_\beta^U)\}dy}$$

and

$$\frac{\int_x^\infty max\{\underset{\alpha\le\beta\le1}{max}\bar{G}(\tilde{x}_\beta^L),\underset{\alpha\le\beta\le1}{max}\bar{G}(\tilde{x}_\beta^U)\}dx}{\int_x^\infty max\{\underset{\alpha\le\beta\le1}{max}\bar{F}(\tilde{x}_\beta^L),\underset{\alpha\le\beta\le1}{max}\bar{F}(\tilde{x}_\beta^U)\}dx} \le \frac{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{max}\bar{G}(\tilde{y}_\beta^L),\underset{\alpha\le\beta\le1}{max}\bar{G}(\tilde{y}_\beta^U)\}dy}{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{max}\bar{F}(\tilde{y}_\beta^L),\underset{\alpha\le\beta\le1}{m}\bar{F}(\tilde{y}_\beta^U)\}dy}$$, and if Z is an IFR random variables which is independant of X and Y, then

$$\frac{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{G}(\widetilde{x+z}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{G}(\widetilde{x+z}_\beta^U)\}d(xz)}{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{F}(\widetilde{x+z}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{F}(\widetilde{x+z}_\beta^U)\}d(xz)} \le \frac{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{G}(\widetilde{y+z}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{G}(\widetilde{y+z}_\beta^U)\}d(yz)}{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{min}\bar{F}(\widetilde{y+z}_\beta^L),\underset{\alpha\le\beta\le1}{min}\bar{F}(\widetilde{y+z}_\beta^U)\}d(yz)}$$

and

$$\frac{\int_x^\infty max\{\underset{\alpha\le\beta\le1}{max}\bar{G}(\widetilde{x+z}_\beta^L),\underset{\alpha\le\beta\le1}{max}\bar{G}(\widetilde{x+z}_\beta^U)\}d(xz)}{\int_x^\infty max\{\underset{\alpha\le\beta\le1}{max}\bar{F}(\widetilde{x+z}_\beta^L),\underset{\alpha\le\beta\le1}{max}\bar{F}(\widetilde{x+z}_\beta^U)\}d(xz)} \le \frac{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{max}\bar{G}(\widetilde{y+z}_\beta^L),\underset{\alpha\le\beta\le1}{max}\bar{G}(\widetilde{y+z}_\beta^U)\}d(yz)}{\int_x^\infty min\{\underset{\alpha\le\beta\le1}{max}\bar{F}(\widetilde{y+z}_\beta^L),\underset{\alpha\le\beta\le1}{m}\bar{F}(\widetilde{y+z}_\beta^U)\}d(yz)}$$

Proof: Denote by f_w and $\bar{F}_w$the density function and the survival function of random variables X and g_w and $\bar{G}_w$ are survival and density functions of Y. Note that

$$\int_{x=s}^{\infty}\bar{F}_w\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\widetilde{x+z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\widetilde{x+z}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\widetilde{x+z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\widetilde{x+z}_\beta^U\right)\right\}dx}\right\}dx =$$

$$\int_{-\infty}^{+\infty}\bar{F}_w\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}dx}\right\}(u)\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^U\right)\right\}dx}\right\}(s-u)du,\ \forall\ s.$$

Now, for $s\ \le t$, compute

$$\int_{x=s}^{\infty}\bar{F}_w\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\widetilde{x+z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\widetilde{x+z}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\widetilde{x+z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\widetilde{x+z}_\beta^U\right)\right\}dx}\right\}dx\int_{y=t}^{\infty}\bar{F}_w\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\widetilde{y+z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\widetilde{y+z}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\widetilde{y+z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\widetilde{y+z}_\beta^U\right)\right\}dx}\right\}dx(-)$$

$$\int_{x=t}^{\infty}\bar{F}_w\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\widetilde{x+z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\widetilde{x+z}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\widetilde{x+z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\widetilde{x+z}_\beta^U\right)\right\}dx}\right\}dx\int_{y=s}^{\infty}\bar{F}_w\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\widetilde{y+z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\widetilde{y+z}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\widetilde{y+z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\widetilde{y+z}_\beta^U\right)\right\}dx}\right\}dx$$

which is equal to

$$\int\ \{\int_{u\ge v}^{+\infty}\bar{F}_w\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}dx}\right\}(u)\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^U\right)\right\}dx}\right\}(s-u)$$

$$\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{y}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{y}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{y}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{y}_\beta^U\right)\right\}dx}(v)\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^U\right)\right\}dx}\right\}(t-v)\ (+)$$

$$\int\int_{u\ge v}^{+\infty}\bar{F}_w\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}dx}\right\}(v)\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^U\right)\right\}dx}\right\}(s-v)$$

$$\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{y}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{y}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{y}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{y}_\beta^U\right)\right\}dx}(u)\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^U\right)\right\}dx}\right\}(s-u)\}dudv.\ (-)$$

$$\int\ \{\int_{u\ge v}^{+\infty}\bar{F}_w\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}dx}\right\}(u)\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^U\right)\right\}dx}\right\}(t-u)$$

$$\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{y}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{y}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{y}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{y}_\beta^U\right)\right\}dx}(v)\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^U\right)\right\}dx}\right\}(t-v)\ (+)$$

$$\int\int_{u\ge v}^{+\infty}\bar{F}_w\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}dx}\right\}(v)\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^U\right)\right\}dx}\right\}(t-v)$$

$$\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{y}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{y}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{y}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{y}_\beta^U\right)\right\}dx}(u)\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^U\right)\right\}dx}\right\}(s-u)\}dudv.$$

$$=\int\int_{x=u}^{+\infty}\overline{\{F_w}\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}dx}\right\}(x)dx\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{y}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{y}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{y}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{y}_\beta^U\right)\right\}dx}\right\}(v)(-)$$

$$\int\int_{x=u}^{+\infty}\bar{F}_w\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_\beta^U\right)\right\}dx}\right\}(v)dx\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{y}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{y}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{y}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{y}_\beta^U\right)\right\}dx}\right\}(x)\}$$

which is multiplied in to

$$\{\bar{f}_z(s-u)[\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^U\right)\right\}dx}\right\}(t-v)(-)\ \bar{f}_z(s-u)$$

$$\left\{\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{z}_\beta^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{z}_\beta^U\right)\right\}dx}\right\}]dudv$$

Similarly we proof maximum function also.

5.2. Theorem: Let (X_i, Y_i) , $i=1,2,\ldots,m$, be independent pairs of fuzzy random variables such that

$$\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_{i\beta}^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_{i\beta}^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_{i\beta}^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_{i\beta}^U\right)\right\}dx}\le\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{y}_{i\beta}^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{y}_{i\beta}^U\right)\right\}dy}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{y}_{i\beta}^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{y}_{i\beta}^U\right)\right\}dy}\text{, for all } i=1,2,\ldots,m.$$

and

$$\frac{\int_x^\infty max\left\{\max_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_{i\beta}^L\right),\max_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_{i\beta}^U\right)\right\}dx}{\int_x^\infty max\left\{\max_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_{i\beta}^L\right),\max_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_{i\beta}^U\right)\right\}dx}\le\frac{\int_x^\infty max\left\{\max_{\alpha\le\beta\le1}\bar{G}\left(\tilde{y}_{i\beta}^L\right),\max_{\alpha\le\beta\le1}\bar{G}\left(\tilde{y}_{i\beta}^U\right)\right\}dy}{\int_x^\infty max\left\{\max_{\alpha\le\beta\le1}\bar{F}\left(\tilde{y}_{i\beta}^L\right),m_{\alpha\le\beta\le1}\bar{F}\left(\tilde{y}_{i\beta}^U\right)\right\}dy}\text{, for all } i=1,2,\ldots,m.$$

If (X_i, Y_i) , $i=1,2,\ldots,m$, are all IFR, then

$$\sum_{i=1}^m\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_{i\beta}^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_{i\beta}^U\right)\right\}dx}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_{i\beta}^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_{i\beta}^U\right)\right\}dx}\le\sum_{i=1}^m\frac{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{y}_{i\beta}^L\right),\min_{\alpha\le\beta\le1}\bar{G}\left(\tilde{y}_{i\beta}^U\right)\right\}dy}{\int_x^\infty min\left\{\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{y}_{i\beta}^L\right),\min_{\alpha\le\beta\le1}\bar{F}\left(\tilde{y}_{i\beta}^U\right)\right\}dy}$$

and

$$\sum_{i=1}^m\frac{\int_x^\infty max\left\{\max_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_{i\beta}^L\right),\max_{\alpha\le\beta\le1}\bar{G}\left(\tilde{x}_{i\beta}^U\right)\right\}dx}{\int_x^\infty max\left\{\max_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_{i\beta}^L\right),\max_{\alpha\le\beta\le1}\bar{F}\left(\tilde{x}_{i\beta}^U\right)\right\}dx}\le\sum_{i=1}^m\frac{\int_x^\infty max\left\{\max_{\alpha\le\beta\le1}\bar{G}\left(\tilde{y}_{i\beta}^L\right),\max_{\alpha\le\beta\le1}\bar{G}\left(\tilde{y}_{i\beta}^U\right)\right\}dy}{\int_x^\infty max\left\{\max_{\alpha\le\beta\le1}\bar{F}\left(\tilde{y}_{i\beta}^L\right),m_{\alpha\le\beta\le1}\bar{F}\left(\tilde{y}_{i\beta}^U\right)\right\}dy}$$

Proof: We applying the above Closure property, we will get the result.

5.3. Lemma: If the fuzzy random variables X and Y are such that,

$$\frac{min\left\{\min_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^U\right)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^U\right)\right\}} \leq \frac{min\left\{\min_{\alpha\leq\beta\leq1} g\left(\tilde{y}_\beta^L\right), \min_{\alpha\leq\beta\leq1} g\left(\tilde{y}_\beta^U\right)\right\}}{min\left\{\min_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^U\right)\right\}},$$

and

$$\frac{max\left\{\max_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^L\right), \max_{\alpha\leq\beta\leq1} f\left(\tilde{x}_\beta^U\right)\right\}}{max\left\{\max_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^L\right), \max_{\alpha\leq\beta\leq1} \bar{F}\left(\tilde{x}_\beta^U\right)\right\}} \leq \frac{max\left\{\max_{\alpha\leq\beta\leq1} g\left(\tilde{y}_\beta^L\right), \max_{\alpha\leq\beta\leq1} g\left(\tilde{y}_\beta^U\right)\right\}}{max\left\{\max_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^L\right), \max_{\alpha\leq\beta\leq1} \bar{G}\left(\tilde{y}_\beta^U\right)\right\}}$$

and if Z is $DFMLR$ fuzzy random variables independent of X and Y, then

$$\frac{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1} \bar{G}\left(\widetilde{x+z}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{G}\left(\widetilde{x+z}_\beta^U\right)\right\} d(xz)}{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}\left(\widetilde{x+z}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{F}\left(\widetilde{x+z}_\beta^U\right)\right\} d(xz)} \leq \frac{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1} \bar{G}\left(\widetilde{y+z}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{G}\left(\widetilde{y+z}_\beta^U\right)\right\} d(yz)}{\int_x^\infty min\left\{\min_{\alpha\leq\beta\leq1} \bar{F}\left(\widetilde{y+z}_\beta^L\right), \min_{\alpha\leq\beta\leq1} \bar{F}\left(\widetilde{y+z}_\beta^U\right)\right\} d(yz)}$$

and

$$\frac{\int_x^\infty max\left\{\max_{\alpha\leq\beta\leq1} \bar{G}\left(\widetilde{x+z}_\beta^L\right), \max_{\alpha\leq\beta\leq1} \bar{G}\left(\widetilde{x1+x2}_\beta^U\right)\right\} d(xz)}{\int_x^\infty max\left\{\max_{\alpha\leq\beta\leq1} \bar{F}\left(\widetilde{x+z}_\beta^L\right), \max_{\alpha\leq\beta\leq1} \bar{F}\left(\widetilde{x+z}_\beta^U\right)\right\} d(xz)} \leq \frac{\int_x^\infty min\left\{\max_{\alpha\leq\beta\leq1} \bar{G}\left(\widetilde{y+z}_\beta^L\right), \max_{\alpha\leq\beta\leq1} \bar{G}\left(\widetilde{y+z}_\beta^U\right)\right\} d(yz)}{\int_x^\infty min\left\{\max_{\alpha\leq\beta\leq1} \bar{F}\left(\widetilde{y+z}_\beta^L\right), m_{\alpha\leq\beta\leq1} \bar{F}\left(\widetilde{y+z}_\beta^U\right)\right\} d(yz)}$$

Proof: We applying the above Closure property, definition, we will get the result.

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