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FIXED POINT THEOREM IN FUZZY METRIC SPACE USING COMPATIBLE AND SEQUENCETIALLY CONTINOUS MAPS

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ABSTRACT

The present paper analysis the fixed point theorem in fuzzy Metric space using compatible and sequencetially continuous Maps Our results extend, generalize and fuzzify several fixed point theorems on Metric spaces

Key words: *fixed point, fixed point theorem, fuzzy Metiric space, sequentially continuous Maps.*

1 INTRODUCTION

Fixed point theory one of the Most important tools in analysis. its applications covered wide area of Mathematical analysis, computer science, dynamic problems, physical and Medical science as well as economics. Fixed point results also playa center role to fixed solution for boundary value problems. in real world the complexity generally arises from uncertainly in the for M of ambiguity .the probability theory has been age old and effective tool The concept of Unclear sets was introduced by Zadeh [24]. Following the concept of unclearsets,unclearMetricspace contain been introduced by Kra Mosil and Michalek [14] Geotherwisege and Veera Mani [6] Modified the notion of unclear Metric space with the help of continous t-norm Vasuki [23] investigated some no transform point propositions in unclear Metric space for. pant [15,16] introduced the notion of reciprocal connection of Mappings in Metric SPACE. BalaassistraManiaM, Muralishankar and Pant [15,16] proved the open problem of Rhoades on the existence of a contractive definition which generals a no transform point but does not force the Mapping to be continuous at the no transform point possesses an affirmative answer. In the sequel, Singh and Chauhan introduced the concept of compatible Mappings of Unclear Metric space and proved the otherwise ordinary no transform point proposition Jain *et al.* [15, 16] proved a no transfor M point proposition for six nature Maps in a this space .Using the concept of compatible Maps of class (β), Jain unclear Metric space and Aage and Salunke [1] also prove a result in *et al.* proved a no transform M point proposition in Unclear Metric space. In 2009, Al-Thagafi and Shahzad [6] weakened the concept of compatibility by giving a new notion of occasionally weakly compatible (owc) Maps which is Mother wisee general among the commutatively concepts. Bouhadjera and Thobie also prove no transform point proposition for owc Maps and in [28], weakened the concept of occasionally weak compatibility compatibility reciprocal connection in the form of assist sequential connection respectively and proved some interesting results with these concepts in Metric space. In 2011, Gopal and IMdad [63] studied the concept of assist-compatible Maps in unclear Metricspce.

The Main purpose of this paper is to introducing compatibility and sequential connection in fuzzy Metric space and proves some fixed point results related with these new concepts. We give some definitions and know results which are used in this paper.

2. PRELIMINARY NOTES

Definition 1.1: A binary operation $*: [0,1] \times [0,1] \rightarrow [0,1]$ is continuous aveerage if $*$ is satisfying the following situation:

(i) * is commutative and associative.
(ii) * is continuous.
(iii) $a * 1 = a$ for all $a \in [0,1]$.
(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$,
For all $a, b, c, d \in [0,1]$.

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Definition 4.1.2: A triplet $(X, M, *)$ is said to be a unclear Metric space

i. if X is an arbitrary set, $*$ is a continous $t-$ norm
ii. M is a unclear set on $X^2 \times (0,\infty)$ satisfying the following condition for all $x, y, z, s, t > 0$, $M(x, y, t) > 0$
iii. $M(x, y, t) = 1$ if and only if $x = y$.
iv. $M(x, y, t) = M(y, x, t)$
v. $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$
vi. $M(x, y, \bullet) : (0, \infty) \rightarrow (0,1]$ is continuous .

Then M is called a fuzzy Metric on X. The function $M(x, y, t)$ denote the degree of nearness between x and y with respect to t.

Example 1.3: Let (X, d) be a Metric space. Define $a * b = \text{MinMuM}\ \{a, b\}$ and

$$M(x, y, t) = \frac{t}{t+d(x,y)}$$

for all x, y ∈ X and t > 0. Then (X, M, *) is a Unclear Metric space.

It is called the Unclear Metric space induced by d.

Definition 1.4: Two nature Maps A and B on a unclear Metric space (X, M, *) are said to be compatible if for all t > 0,

$$\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1$$

When ever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for all some $z \in X$

Definition 1.5: Two nature Maps A and B on a unclear Metric space $(X, M, *)$ are said to be weakly compatible (otherwise coincidently commuting) if they commute at their coincidence points i.e. if $At = Bt$ for some $t \in X$ then $ABt = BAt$.

Definition 1.6: Two nature Maps A and B on a set X are said to be owc (occasionally weakly compatible). A coincidence point of A and B at which A and B commute. i.e., there exists a point $x \in X$ such that

i. $Ax = Bx$
ii. $ABx = BAx$.

Definition 1.7: Two nature Maps A and B on a unclear Metric space (X, M, $*$) are said compatible if and only if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z, z \in X \text{ and which satisfy } \lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1. \text{ For } t > 0.$$

Example 1.8: Let $X = [0, \infty)$ with usual Metric d and define

$$M(x, y, t) = \frac{t}{t+d(x,y)}$$

For all x, y ∈ X and t > 0

Define A, B: X→ X as;

$$Ax = \begin{cases} x^2, & x < 1 \\ x + 5, & x \geq 1 \end{cases}$$

and

$$Bx = \begin{cases} 7x - 6, & x < 1 \\ 2x, & x \geq 1 \end{cases}$$

Thus, A and B are assist compatible but A and B are not owc Maps as,

$$A(4) = 8 = B(4)$$

and $$AB(4) = A(8) = 12 \neq BA(4) = B(8) = 16.$$

Definition 4.1.9: Two nature Maps A and B on a unclear Metric space are called reciprocal continuous if

$$\lim_{n\to\infty} Ax_n = At \text{ and}$$
$$\lim_{n\to\infty} Bx_n = Bt$$

for some t ∈ X. whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Ax_n = t \text{ for } t \in X .$$

Definition 1.10: are said to be sequentially continous if and only if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = t$$

For all some t ∈ X and satisfy

$$\lim_{n\to\infty} Ax_n = At \text{ and } \lim_{n\to\infty} Bx_n = Bt.$$

Remark 1.11: If A and S both are continuous otherwise reciprocally continous then they are obviously sequentially continuous.

Example 4.1.12: Let X = R, endowed with usual Metric d and

$$M(x, y, t) = \frac{t}{t+d(x,y)}$$

For all x, y ∈ X and t > 0. Define A, B: X → X as;

$$Ax = \begin{cases} 4, x < 5 \\ x, x \geq 5 \end{cases}$$

and
$$Bx = \begin{cases} 4x - 16, x \leq 5 \\ 5 \quad, x > 5 \end{cases}$$

Consider a sequence $x_n = 5 - \frac{1}{n}$, then

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} A\left(5 - \frac{1}{n}\right) = 4,$$

$$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} B\left(5 - \frac{1}{n}\right)$$

$$= \lim_{n\to\infty}\left[4\left(5 - \frac{1}{n}\right) - 16\right]$$

$$= \lim_{n\to\infty}\left[4 - \frac{4}{n}\right] = 4$$

$$\lim_{n\to\infty} ABx_n = \lim_{n\to\infty} A\left(4 - \frac{4}{n}\right) = 4 = A(4),$$

$$\lim_{n\to\infty} BAx_n = B(4) = 0 = B(4)$$

Thus A and B are not reciprocally continous but if we

Consider a sequence $x_n = 5 + \frac{1}{n}$, then

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} A\left(5 + \frac{1}{n}\right) = 5,$$

$$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} B\left(5 + \frac{1}{n}\right) = 5$$

$$\lim_{n\to\infty} ABx_n = A(5) = 5 = A(5),$$

$$\lim_{n\to\infty} BAx_n = \lim_{n\to\infty} B\left(5 + \frac{1}{n}\right) = 5 = B(5).$$

Therefore, A and B are sequentially continuous.

2. MAIN RESULTS

Proposition 2.1: Let A, B, S and T be four nature Mappings of a vague Metric space (X, M,∗). If the pairs (A, S) and (B, T) are compatible and sequentially continous, then

2.1(I) A and S contain a point of coincidence.

2.1(II) B and T contain a point of coincidence.

2.1(III) $\phi[\text{MINMUM } M(Ax, By, t)M(Sx, Ty, t), M(Sx, Ax, t), M(By, Ty, t)M(Ax, Ax, t)M(By, By, t)] \geq 0$

For all x, y ∈ X and t > 0 where $\phi: [0,1] \to [0,1]$ is a continuous function with $\phi(s) > s$ for each $0 < s < 1$. Then A, B, S and T have a unique common fixed point in X.

Proof: since the pairs (A, S) and (B, T) are compatible and sequentially since the pairs (A,S) and (B,T) continuous therefore, there are two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = u$$

For some u ∈ X and which satisfy

$$\lim_{n\to\infty} M, SAx_n, t) = M(Au, Su, t) = 1,$$

$$\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = v$$

for some u ∈ X and which satisfy

$$\lim_{n\to\infty} BTx_n, TBx_n, t) = M(Bv, Tv, t) = 1.$$

For Au = Su and Bv = Tv.

i.e. u is the coincidence point of A and S and v is the coincidence point of B and T. Now using 2.1(III) for x= x_n and y = y_n, we get

$$\phi[\mathrm{MinMuM}\ \{M\ (Ax_n, By_n, t)\ M(Sx_n, Ty_n, t), M(Sx_n, Ax_n, t), M(Byn, Ty_n, t)$$
$$M(Ax_n, Ax_n, t)M(By_n, By_n, t)\}] \geq 0$$

On taking limit as $n \to \infty$

$$M(u,v,t) \geq \varphi[\mathrm{MinMuM}\ \{M(u,v,t), M(u,u,t), M(v,v,t)M(u,u,t)M(v,v,t)\})$$
$$\geq \varphi(\mathrm{MinMuM}\ \{M(u,v,t),1,1,1,1\}]$$

i.e. $M\ (u,v,t) = \varphi[\mathrm{MinMuM}\ \{M(u,v,t)\}] > M(u,v,t)$

which contradiction. hence U = V.

Again using 2.1(III) for $x = u, y = y_n$ we obtain

$$\Phi\begin{bmatrix}\mathrm{minmum}\ \{m(au, by_n, t)m(su, ty_n, t), m(su, au, t), m(by_n, ty_n, t)\} \\ m(au, au, t)m(by_n, by_n, t)\end{bmatrix} \geq 0$$

On taking limit as $n \to \infty$

$$\varphi[\mathrm{minmum}\ \{mau, v, t)\ m(su, v, t), m(su, au, t), m(v, v, t)$$
$$m(u,u,t)m(v,v,t)\}] \geq 0$$
$$\geq \Phi[\mathrm{minmum}\ \{M(su, v, t), 1,1,1,1\}]$$
$$\geq \Phi[\mathrm{minmum}\ \{M(su, v, t)\}]$$
$$\geq \Phi[\mathrm{minmum}\ \{M(au, v, t)\}]$$

since $au = su$ i.e.

$$m(au, v, t) = \varphi[\{m(au, v, t)\}]$$
$$> M(Au, v, t),$$

Which yields $Au = v = u$.

There for $u = v$ is common fixed point of A, B, S and T.

Uniqueness: Let $w \neq u$ be another no for otherwise point of A, B, S and T. Then on otherwise after 2.1(III) we contain

$$\varphi[\mathrm{MinMuM}\ \{M(au, bw, t)M(su, tw, t), M(su, au, t), M(au, au, t)M(bw, bw, t\}] \geq 0$$
$$\varphi[\mathrm{minmum}\ \{m(au, bw, t),1,1,1,1\}] \geq 0$$
$$\varphi[\mathrm{minmum}\ \{m(au, bw, t)\}] \geq 0$$
$$\varphi[\mathrm{minmum}\ \{m(au, bw, t)\}] \geq 0$$
$$\phi\ [\{m(au, bw, t), M(au, bw, t)\}]\ > 0$$

which yields w = u and therefore uniqueness follows.

If we put A = B and S = T in Proposition.2.1, we get the following result.

Corollary 2.2: Let A and S be two nature Mappings of a unclear Metric space $(X, M,*)$. If the pairs (A, S) is compatible and sequentially continous, then

2.2(I) A and S contain a point of coincidence.

Further, if

2.2(II) $\phi[\mathrm{MinMuM}\ \{M(Ax, Ay, t)\ M(Sx, Sy, t), M(Sx, Ax, t), M(Ay, Sy, t)$
$M(Ax, Ay, t)M(By, By, t)\}] \geq 0$

For all $x, y \in X$ and $t > 0$ where ϕ:[0,1] →[0,1] is a continous function with $\phi(s) > s$ for each $0 < s < 1$.Then A, and S contain a unique no for point in X. If we put S = T in Proposition- 2.1, we get the following result.

Corollary 2.3: Let A, B and s be three nature Mappings of a unclear Metric space $(x, M,*)$. if the pairs (A, S) and (B, S) are compatible and sequentially continous, then let a, b and s be three nature Mappings of a unclear Metric space $(x, M,*)$. if the pairs (a, s) and (b, s) are compatible and sequentially continous, then

2.3(I) A and S contain a point of coincidence.

2.3(II) B and S contain a point of coincidence.

Further, if

2.3(III)

$$\phi[\mathrm{MINMUM}\ M\ (Ax, By\ , t)M(Sx, Ty, t), M(Sx, Ax, t), M(By, Ty, t)M(Ax, Ax, t)M(By, By, t)] \geq 0$$

for all $x, y \in X$ and $t > 0$ where $\phi: [0,1] \to [0,1]$ is a continous function with $\phi(s) > s$ for each $0 < s < 1$. Let A, B and S be three nature Mappings of a unclear sequentially continous, then A, B and S contain a unique no transform point in X.

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