International Journal of Mathematical Archive-9(6), 2018, 101-107
Available online through www.ijma.info ISSN 2229 – 5046

Properties of somewhat nano b-continuous and somewhat nano b-open functions

DHANIS ARUL MARY A* & AROCKIARANI I

**Department of Mathematics,
Nirmala College for women, Coimbatore, Tamilnadu, India.**

(Received On: 07-05-18; Revised & Accepted On: 14-06-18)

ABSTRACT

The purpose of this paper is to introduce a new class of functions called somewhat nano b-continuous and somewhat nano b-open functions in nano topological spaces using nano b-open and nano b-closed sets. We obtain their characterizations and their basic properties.

Mathematics Subject Classification: *54C08, 54C10.*

Key words: *Somewhat nano b-continuous functions, somewhat nano b-open functions.*

INTRODUCTION

The concept of somewhat open functions was introduced by Gentry and Hoyle [3]. These ideas are also closely related to the idea of weakly equivalent topologies which was first introduced by Yougslova [8]. D. Andrijevic [1] introduced and studied the concept of b-open sets in topological spaces. D. Santhileela and G. Balasubramanian [7] established the idea of somewhat semi continuous and somewhat semi open functions. In 2013, Lellis Thivagar [5] introduced the notion of nano topological spaces. He also established the weak forms of nano open sets namely nano α -open sets, nano semi open sets and nano pre open sets [5]. In this paper, using the notion of nano b-open sets, the concepts of somewhat nano b-continuous functions and somewhat nano b-open functions are introduced and studied. Also characterizations of these functions are obtained and proved with suitable examples and counterexamples.

PRELIMINARIES

Definition 2.1[9]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space.

Let $X \subseteq U$

1. The lower approximation of X with respect to R is the set of all objects, which can be for certainly classified as X with respect to R and it is denoted by $L_R(X)$. That is
   $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by $x \in U$.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is
   $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) – L_R(X)$.

Corresponding Author: Dhanis Arul Mary A* & Arockiarani I
Department of Mathematics, Nirmala College for women, Coimbatore, Tamilnadu, India.

Definition 2.2[5]: Let U be an universe, R be an equivalence relation on U and $\tau_R(X)$ = {U, ϕ, $L_R(X)$, $U_R(X)$, $B_R(X)$} where X $\subseteq$ U. $\tau_R(X)$ satisfies the following axioms:

(i) U and $\phi \in \tau_R(X)$
(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
(iii) The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$. That is, $\tau_R(X)$ forms a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called nano open sets.

Definition 2.3[5]: Let $(U, \tau_R(X))$ be a nano topological space and A $\subseteq$ U. Then, A is said to be

(i) nano semi-open if A $\subseteq$ $\mathcal{N}$cl(Nint(A))
(ii) nano pre-open if A $\subseteq$ $\mathcal{N}$int($\mathcal{N}$cl(A))
(iii) nano α -open if A $\subseteq$ $\mathcal{N}$int($\mathcal{N}$cl($\mathcal{N}$int(A)))

Definition 2.4[2]: Let $(U, \tau_R(X))$ be a nano topological space and A $\subseteq$ U. Then A is said to be nano b-open if A $\subseteq$ $\mathcal{N}$cl($\mathcal{N}$intA) $\cup$ $\mathcal{N}$int($\mathcal{N}$clA).

Definition 2.5[2]: A function $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is called a nano b-continuous function if the inverse image of every nano closed set in $(V, \tau_{R'}(X))$ is nano b-closed in $(U, \tau_R(X))$.

Definition 2.6[6]: A function $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is called a nano continuous if the inverse image of every nano closed set in $(V, \tau_{R'}(X))$ is nano closed in $(U, \tau_R(X))$.

3. SOMEWHAT NANO b-CONTINUOUS FUNCTIONS

Definition 3.1: Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be any two nano topological spaces. A function $f : (U, \tau_R(X) \rightarrow (V, \tau_{R'}(Y))$ is said to be somewhat nano b-continuous if for every M $\in \tau_{R'}(Y)$ and $f^{-1}(M)$ $\neq \phi$ there exist a nano b-open set N in U such that N $\neq \phi$ and N $\subset f^{-1}(M)$.

Definition 3.2: Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be any two nano topological spaces. A function $(U, \tau_R(X))$ is said to be somewhat nano semi-continuous if for every M $\in \tau_{R'}(Y)$ and $f^{-1}(M) \neq \phi$ there exist a nano semi-open set N in U such that N $\neq \phi$ and N $\subset f^{-1}(M)$.

Definition 3.3: Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be any two nano topological spaces. A function $(U, \tau_R(X))$ is said to be somewhat nano continuous if for every M $\in \tau_{R'}(Y)$ and $f^{-1}(M) \neq \phi$ there exist a nano open set N in U such that N $\neq \phi$ and N $\subset f^{-1}(M)$.

Example 3.4: Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X = {a, b}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {a}, {b, d}, {a, b, d}} . Let V = {x, y, z, w} with V/R' = {{x}, {y, w}, {z}} and Y = {y, w}. Then $\tau_{R'}(Y)$ = {V, ϕ, {y, w}}. Define $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ to be f(a) = z, f(b)= w, f(c) = x, f(d) = y. Then clearly f is somewhat nano b-continuous function.

Theorem 3.5: Every somewhat nano semi-continuous function is somewhat nano b-continuous.

Proof: Let f: U → V be somewhat nano semi continuous function. Let M be any nano open set in V such that $f^{-1}(M) \neq \phi$. Since f is somewhat nano semi continuous, there exists a nano semi open set N in U such that N $\neq \phi$ and N $\subset f^{-1}(M)$. Since every nano semi open set is nano b-open, there exists a nano b-open set N such that N $\neq \phi$ and N $\subset f^{-1}(M)$, which implies that f is somewhat nano b-continuous function.

Remark 3.6: Converse of the above theorem need not be true in general which follows from the following example.

Example 3.7: Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X = {a, b}. Then the nano topology is defined as $\tau_R(X)$= {U, ϕ, {a}, {b, d}, {a, b, d}} . Let V = {x, y, z, w} with V/R' = {{x}, {y, w}, {z}} and Y = {y, w}. Then $\tau_{R'}(Y)$ = {V, ϕ, {y, w}}. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ to be f(a) = z, f(b)= y, f(c) = x, f(d) = x. Then f is somewhat nano b-continuous function but not somewhat nano semi continuous.

Theorem 3.8: Every somewhat nano continuous function is somewhat nano b-continuous.

Proof: follows from the Theorem 3.5.

Remark 3.9: Converse of the above theorem need not be true in general which follows from the following example.

Example 3.10: Let U = {a, b, c, d} with U/R = {{a, c}, {b}, {d}} and X = {a, d}. Then the nano topology is defined as $\tau_R(X)$= {U, ϕ, {d}, {a, c, d}, {a, c}} . Let V = {x, y, z, w} with V/R' = {{x}, {y, w}, {z}} and Y = {y, w}. Then $\tau_{R'}(Y)$ = {V, ϕ, {y, w}}. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ to be f(a) = x, f(b)= x, f(c) = y, f(d) = x. Then f is somewhat nano b-continuous function but not somewhat nano continuous.

Theorem 3.11: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ and $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ be any two functions. If f is somewhat nano b-continuous function and g is nano continuous function, then $g\circ f$ is somewhat nano b-continuous function.

Proof: Let M$\in$ $\tau_{R''}(Z)$. Suppose that $g^{-1}(M)\neq\phi$. Since M$\in$ $\tau_{R''}(Z)$ and g is nano continuous function $g^{-1}(M)\in\tau_{R'}(Y)$. Suppose that $f^{-1}(g^{-1}(M))\neq\phi$. Since by hypothesis f is somewhat nano b-continuous function, there exists a nano b-open set N in U such that N$\neq\phi$ and N$\subset$ $f^{-1}(g^{-1}(M))$. But $f^{-1}(g^{-1}(M))$ = $(g\circ f)^{-1}(M)$, which implies that N$\subset$ $(g\circ f)^{-1}(M)$. Therefore $g\circ f$ is somewhat nano b-continuous function.

Remark 3.12: In the above Theorem 3.11, if f is nano continuous function and g is somewhat nano b-continuous function, then it is not necessarily true that $g\circ f$ is somewhat nano b-continuous function. The following example serves this purpose.

Example 3.13: Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X = {a, b}. Then the nano topology is defined as $\tau_R(X)$= {U, ϕ, {a}, {b, d}, {a, b, d}} . Let V = {x, y, z, w} with V/R' = {{x}, {y, w}, {z}} and Y = {y, w}. Then $\tau_{R'}(Y)$ = {V, ϕ, {y, w}}. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ to be f(a) = x, f(b)= w, f(c) = w, f(d) = y and define $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ as g(a) = w, g(b) = y, g(c) = x, g(d) = z . Then clearly f is nano continuous and g is somewhat nano b-continuous functions but $g\circ f$ is not somewhat nano b- continuous.

Definition 3.14: Let H be a subset of a nano topological space $(U,\tau_R(X))$. Then H is said to be nano b-dense in U if there is no proper nano b-closed set C in U such that H$\subset$C$\subset$U.

Theorem 3.15: Let $f:(U,\tau_R(X)\to(V,\tau_{R'}(Y))$ be a function. Then the following are equivalent:

(i) f is somewhat nano b-continuous function.

(ii) If C is a nano closed subset of Y such that $f^{-1}(C)\neq$ U, then there is a proper nano b-closed subset D of U such that D$\supset$ $f^{-1}(C)$.

(iii) If H is a nano b-dense subset of U then f(H) is a dense subset of V.

Proof:

(i) $\Rightarrow$ (ii): Let C be a nano closed subset of V such that $f^{-1}(C)\neq$ U. Then V − C is a nano open set in V such that $f^{-1}(V-C)$ = U − $f^{-1}(C)\neq\phi$. By hypothesis (i) there exists a nano b-open set N in U such that N$\neq\phi$ and

N$\subset f^{-1}(V-C)$ = U − $f^{-1}(C)$. This means that U − N $\supset f^{-1}(C)$ and U − N = D is a nano b-closed set in U. This proves (ii).

(ii) $\Rightarrow$ (iii): Let H be a nano b-dense set in U. We have to show that f(H) is dense in V. Suppose not, then there exists a proper nano closed set C in V such that f(H) $\subset$C$\subset$V . Clearly, $f^{-1}(C) \neq$ U. Hence by (ii) there exists a proper nano b-closed set D such that H $\subset f^{-1}(C) \subset$D$\subset$U. This contradicts the fact that H is nano b-dense in U.

(iii) $\Rightarrow$ (ii): Suppose that (ii) is not true. This means there exists a nano closed set C in V such that $f^{-1}(C) \neq$ U. But there is no proper nano b-closed set D in U such that $f^{-1}(C) \subset$D. This means that $f^{-1}(C)$ is nano b-dense in U. But by (iii) $f(f^{-1}(C))$ = C must be nano dense in V, which is contradiction to the choice of C.

(ii) $\Rightarrow$ (i): Let M $\in \tau_{R'}(Y)$ and $f^{-1}(M) \neq \phi$. Then V − M is nano closed and $f^{-1}(V-M)$ = U − $f^{-1}(M) \neq \phi$. By hypothesis of (ii) there exists a proper nano b-closed set D such that D$\supset f^{-1}(V-M)$. This implies that U −D $\subset f^{-1}(M)$ and U –D is nano b-open and U − D$\neq \phi$.

Theorem 3.16: Let $(U,\tau_R(X))$ and $(V,\tau_{R'}(Y))$ be any two nano topological spaces, A be a nano open set in U and $f:(A,\tau_R(X)/A)) \rightarrow (V,\tau_{R'}(Y))$ be somewhat nano b-continuous function such that f(A) is nano dense in V . Then any extension F of f is somewhat nano b-continuous function.

Proof: Let M be any nano open set in $(V,\tau_{R'}(Y))$ such that $F^{-1}(M) \neq \phi$. Since f(A) $\subset$V is nano dense in V and M$\cap$ f(A) $\neq \phi$ it follows that $F^{-1}(M) \cap$ A$\neq \phi$. That is $f^{-1}(M) \cap$ A $\neq \phi$. Hence by hypothesis on f, there exists a nano b-open set N in A such that N$\neq \phi$ and N$\subset f^{-1}(M) \subset F^{-1}(M)$ which implies F is somewhat nano b-continuous function.

Theorem 3.15 Let $(U,\tau_R(X))$ and $(V,\tau_{R'}(Y))$ be any two nano topological spaces, U = A$\cup$B where A and B are nano open subsets of U and $f:(U,\tau_R(X) \rightarrow (V,\tau_{R'}(Y))$ be a function such that f/A and f/B are somewhat nano b-continuous functions. Then f is somewhat nano b-continuous function.

Proof: Let M be any nano open set in $(V,\tau_{R'}(Y))$ such that $f^{-1}(M) \neq \phi$. Then $(f/A)^{-1}(M) \neq \phi$ or $(f/B)^{-1}(M)$ or both $(f/A)^{-1}(M) \neq \phi$ and $(f/B)^{-1}(M) \neq \phi$.

Case-1: Suppose $(f/A)^{-1}(M) \neq \phi$
Since f/A is somewhat nano b-continuous, there exists a nano b-open set N in A such that N$\neq \phi$ and N$\subset$ $(f/A)^{-1}(M) \subset f^{-1}(M)$. Since N is nano b-open in A and A is nano open in U, N is nano b-open in U. Thus f is somewhat nano b-continuous function.

Case-2: Suppose $(f/B)^{-1}(M) \neq \phi$
Since f/B is somewhat nano b-continuous, there exists a nano b-open set N in B such that N$\neq \phi$ and N$\subset$ $(f/B)^{-1}(M) \subset f^{-1}(M)$. Since N is nano b-open in B and B is nano open in U, N is nano b-open in U. Thus f is somewhat nano b-continuous function.

Case-3: Suppose $(f/A)^{-1}(M) \neq \phi$ and $(f/B)^{-1}(M) \neq \phi$.
This follows from both the cases 1 and 2. Thus f is somewhat nano b-continuous function.

Definition 3.17 A nano topological space U is said to be nano b-separable if there exists a countable subset B of U which is nano b-dense in U.

Theorem 3.18 If f is somewhat nano b-continuous function from U onto V and if U is nano b-separable, then V is nano separable.

Proof: Let $f:(U,\tau_R(X)\to(V,\tau_{R'}(Y))$ be somewhat nano b-continuous function such that U is nano b-separable. Then by definition there exists a countable subset B of U which is nano b-dense in U. Then by Theorem 3.15, f(B) is nano dense in V. Since B is countable f(B) is also countable which is nano dense in V , which indicates that Y is nano separable.

Definition 3.19: If U is a set and $\tau_R(X)$ and $\tau_{R'}(Y)$ are nano topologies for U, then $\tau_R(X)$ is said to be nano b-weakly equivalent to $\tau_{R'}(Y)$ provided if M$\in\tau_R(X)$ and M$\neq\phi$, then there is a nano b-open set N in $(U,\tau_{R'}(Y))$ such that N$\neq\phi$ and N$\subset$M and if M$\in\tau_{R'}(Y)$ and M$\neq\phi$ then there is a nano b-open set N in $(U,\tau_R(X))$ such that N$\neq\phi$ and N$\subset$M.

Theorem 3.20: Let $f:(U,\tau_R(X)\to(V,\tau_{R'}(Y))$ be somewhat nano continuous function and let $\tau^*_R(X)$ be a nano topology for U, which is nano b-weakly equivalent to $\tau_R(X)$ then the function $f:(U,\tau^*_R(X)\to(V,\tau_{R'}(Y))$ is somewhat nano b-continuous function.

Proof: Let M be any nano open set in $(V,\tau_{R'}(Y))$ such that $f^{-1}(M)\neq\phi$. Since by hypothesis $f:(U,\tau_R(X)\to(V,\tau_{R'}(Y))$ is somewhat nano continuous by definition there exists an nano open set O in $(U,\tau_R(X))$ such that O$\neq\phi$ and O$\subset f^{-1}(M)$. Since O is an nano open set in $(U,\tau_R(X))$ such that O$\neq\phi$ and since by hypothesis $\tau_R(X)$ is nano b-weakly equivalent to $\tau^*_R(X)$ by definition there exists a nano b-open set N in $(U,\tau^*_R(X)$ such that N$\neq\phi$ and N$\subset$O$\subset$ $f^{-1}(M)$. So O$\subset f^{-1}(M)$. Thus for any nano open set M in $(V,\tau_{R'}(Y))$ such that $f^{-1}(M)\neq\phi$ there exist a nano b-open set N in $(U,\tau^*_R(X)$ such that N$\subset f^{-1}(M)$. So $f:(U,\tau^*_R(X)\to(V,\tau_{R'}(Y))$ is somewhat nano b-continuous function.

Theorem 3.21: Let $f:(U,\tau_R(X)\to(V,\tau_{R'}(Y))$ be somewhat nano b-continuous function and let $\tau^*_{R'}(Y)$ be a nano topology for V which is weakly equivalent to $\tau_{R'}(Y)$ .Then $f:(U,\tau_R(X)\to(V,\tau^*_{R'}(Y))$ is somewhat nano b-continuous function.

Proof: Let M be a nano open set in $(V,\tau^*_{R'}(Y))$ such that $f^{-1}(M)\neq\phi$ which implies M$\neq\phi$. Since $\tau_{R'}(Y)$ and $\tau^*_{R'}(Y)$ are weakly equivalent there exists a nano open set W in $(V,\tau_{R'}(Y))$ such that W$\neq\phi$ and W$\subset$M. Now, W is a nano open set such that W$\neq\phi$, which implies $f^{-1}(W)\neq\phi$. Now by hypothesis $f:(U,\tau_R(X)\to(V,\tau_{R'}(Y))$ is somewhat nano b-continuous function. Therefore there exists a nano b-open set N in U, such that N$\subset f^{-1}(W)$. Now W$\subset$M implies $f^{-1}(W)\subset f^{-1}(M)$. So we have N$\subset f^{-1}(M)$, which implies that $f:(U,\tau_R(X)\to(V,\tau^*_{R'}(Y))$ is somewhat nano b-continuous function.

4. SOMEWHAT NANO b-OPEN FUNCTIONS

Definition 4.1: A function $f:(U,\tau_R(X)\to(V,\tau_{R'}(Y))$ is said to be somewhat nano b-open function provided that for M$\in\tau_R(X)$ and M$\neq\phi$ there exists a nano b-open set N in V such that N$\neq\phi$ and N$\subset$f(M).

Example 4.2: Let U = {a, b, c, d} with U/R = {{a, c}, {b}, {d}} and X = {a, d}. Then the nano topology is defined as $\tau_R(X)$= {U, ϕ, {d}, {a, c, d}, {a, c}} . Let V = {x, y, z, w} with V/R' = {{x}, {y, z}, {w}} and Y = {x, z}. Then $\tau_{R'}(Y)$ = {V,ϕ, {x}, {x, y, z}, {y, z}}. Define $f:\left(U,\tau_R(X)\right)\to\left(V,\tau_{R'}(Y)\right)$ to be f(a) = y, f(b)= z, f(c) = x, f(d) = w. Then clearly f is somewhat nano b-open function.

Theorem 4.3: Every somewhat nano semi-continuous function is somewhat nano b-continuous.

Proof: Let f: U → V be somewhat nano semi continuous function. Let M$\in \tau_R(X)$ and M$\neq \phi$. Since f is somewhat nano semi open, there exists a nano semi open set N in V such that N $\neq \phi$ and N$\subset$f(M). But every nano semi open set is nano b-open, there exists a nano b-open set N in V such that N $\neq \phi$ and N$\subset$f(M), which implies that f is somewhat nano b-open function.

Remark 4.4: Converse of the above theorem need not be true in general which follows from the following example.

Example 4.5: Let U = {a, b, c, d} with U/R = {{a, c}, {b}, {d}} and X = {a, d}. Then the nano topology is defined as $\tau_R(X)$= {U, ϕ, {d}, {a, c, d}, {a, c}} . Let V = {x, y, z, w} with V/R' = {{x}, {y, z}, {w}} and Y = {x, z}. Then $\tau_{R'}(Y)$ = {V,ϕ, {x}, {x, y, z}, {y, z}}. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ to be f(a) = y, f(b)= z, f(c) = y, f(d) = w. Then clearly f is somewhat nano b-open function but not somewhat nano semi open function.

Theorem 4.6: Every somewhat nano continuous function is somewhat nano b-continuous.

Proof: follows from the Theorem 4.3.

Remark 4.7: Converse of the above theorem need not be true in general which follows from the following example.

Example 4.8: Let U = {a, b, c, d} with U/R = {{a}, {b, c}, {d}} and X = {a, c}. Then the nano topology is defined as $\tau_R(X)$= {U, ϕ, {a}, {a, b, c}, {b, c}} . Let V = {x, y, z, w} with V/R' = {{x}, {y, w}, {z}} and Y = {y, w}. Then $\tau_{R'}(Y)$ = {V,ϕ, {y, w}}. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ to be f(a) = w, f(b)= y, f(c) = z, f(d) = x. Then clearly f is somewhat nano b-open function but not somewhat nano open function.

Theorem 4.9: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is a nano open function and $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ is somewhat nano b-open function, then $g\circ f$ is somewhat nano b-open function.

Proof: Let M$\in \tau_R(X)$. Suppose that M$\neq \phi$. Since f is a nano open function f(M) is nano open and f(M) $\neq \phi$. Thus f(M)$\in \tau_{R'}(Y)$ and f(M)$\neq \phi$. Since g is somewhat nano b-open function and f(M) $\in \tau_{R'}(Y)$ such that f(M) $\neq \phi$ there exists a nano b-open set N$\in \tau_{R''}(Z)$, N$\subset$g(f(M)), which implies $g\circ f$ is somewhat nano b-open function.

Theorem 4.10: Let $f:(U,\tau_R(X)\to(V,\tau_{R'}(Y))$ is a one-one and onto function, then the following conditions are equivalent:

(i) f is somewhat nano b-open function.
(ii) If C is a nano closed subset of U such that f(C) $\neq$V, then there is a nano b-closed subset D of V such that D $\neq$ V and D$\supset$ f(C).

Proof:
(i) ⇒ (ii): Let C be any nano closed subset of U such that f(C) $\neq$V. Then U − C is a nano open set in U and U-C $\neq \phi$. Since f is somewhat nano b-open function, there exists a nano b-open set N $\neq \phi$ in V such that N$\subset$f(U-C). Put D =V – N. Clearly D is nano b-closed in V and we claim that D $\neq$V. For if D = V, then V = ϕ which is a contradiction. Since N$\subset$f(U-C), D = V – N $\supset$ V- [f(U-C)] = f(C).

(ii) ⇒ (i): Let M be any non-empty nano open set in U. Put C = U – M. Then C is a nano closed subset of U and f(U - M) = f(C) = V- f(M) implies f(C) $\neq \phi$. Therefore by (ii) there is a nano b-closed subset D of V such that D $\neq$V and f(C) $\subset$D. Put N = U – D. Clearly N is a nano b-open set and N $\neq \phi$. Further, N = U - D $\subset$V- f(C) = V- [V-f(M)] = f(M).

Theorem 4.11: Let $f:(U,\tau_R(X)\to(V,\tau_{R'}(Y))$ be somewhat nano b-open function and A be any nano open subset of U. Then $f/A:(A,\tau_R(X)/A))\to(V,\tau_{R'}(Y))$ is somewhat nano b-open function.

Proof: Let M $\in \tau_R(X)/A$ such that M $\neq \phi$. Since M is nano open in A and A is nano open in $(U, \tau_R(X))$, M is nano open in $(U, \tau_R(X))$ and since by hypothesis $f:(U, \tau_R(X) \to (V, \tau_{R'}(Y))$ is somewhat nano b-open function, there exists a nano b-open set N in V, such that N $\subset$ f(M). Thus, for any nano open set M in $(A, \tau_R(X)/A)$ with M $\neq \phi$, there exists a nano b-open set N in V such that N $\subset$ f(M) which implies f/A is somewhat nano b-open function.

Theorem 4.12: Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be any two nano topological spaces, U = A $\cup$ B where A and B are nano open subsets of U and $f:(U, \tau_R(X) \to (V, \tau_{R'}(Y))$ be a function such that f/A and f/B are somewhat nano b-open functions. Then f is also somewhat nano b-open function.

Proof: Let M be any nano open set in $(U, \tau_R(X))$ such that M $\neq \phi$. Since U = A $\cup$ B, either A $\cap$ M $\neq \phi$ or B $\cap$ M $\neq \phi$ or both A $\cap$ M $\neq \phi$ and B $\cap$ M $\neq \phi$. Since M is nano open in $(U, \tau_R(X))$, M is nano open in both $(A, \tau_R(X)/A)$ and $(B, \tau_R(X)/B)$.

Case-(i): Suppose that A $\cap$ M $\neq \phi$ where M $\cap$ A is nano open in $\tau_R(X)/A$. Since by hypothesis f/A is somewhat nano b-open function, there exists a nano b-open set N $\in (V, \tau_{R'}(Y))$ such that N $\subset$ f(M $\cap$ A) $\subset$ f(M), which implies f is somewhat nano b-open function.

Case-(ii): Suppose that B $\cap$ M $\neq \phi$, where M $\cap$ B is nano open in $(B, \tau_R(X)/B)$. Since by hypothesis f/B is somewhat nano b-open function, there exists a nano b-open set N in $(V, \tau_{R'}(Y))$ such that N $\subset$ f(M $\cap$ B) $\subset$ f(M), which implies that f is also somewhat nano b-open function.

Case-(iii): Suppose that both both A $\cap$ M $\neq \phi$ and B $\cap$ M $\neq \phi$. Then obviously f is somewhat nano b-open function from the case (i) and case (ii). Thus f is somewhat nano b-open function.

REFERENCES

1. D. Andrijevic, On b-open sets, Math. Vesnik. 48(1996) No. 1-2, 59-64.
2. Dhanis Arul Mary A and Arockiarani I “On b-open sets and b-continuous functions in nano topological spaces” IJIRS, ISSN 2319-9725, 11(13), (2014)98-116.
3. Gentry, K. R. and Hoyle, H. B., Somewhat continuous functions, Czech. Math. Jl., 21. (1971). No. 1, 5-12.
4. N. Levine, Semi open sets and semi continuity in topological spaces, Amer. Math. Monthly, 70 (1963) 36-41.
5. M. Lellis Thivagar and Carmel Richard, On nano forms of weakly open sets, International Journal of Mathematics and statistics Invention, (2013) 31-37.
6. M. Lellis Thivagar and Carmel Richard "On Nano Continuity", Mathematical theory and modeling, (2013), no.7, 32-37.
7. D. Santhileela and G. Balasubramanian, Somewhat semi continuous and somewhat semi open functions, Bull. Cal. Math. Soc., 94(1) (2002) 41-48.
8. Yougslova A. L., Weakly equivalent topologies, Master’s Thesis (1965) University of Georgia, Georgia.
9. Zadislaw Pawlak (1982) “Rough Sets”, International Journal of Information and Computer Sciences, 11(1982)341-356.

Source of support: Nil, Conflict of interest: None Declared.