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PROPERTIES OF A NEW CLASS OF ANALYTIC FUNCTIONS DEFINED BY A DERRERENTIAL OPERATOR

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ABSTRACT

A *new class of analytic functions $US_\lambda(k,b)$ is introduced by applying Salagean Operators. Some properties such as the coefficient limits and growth and distortion theorems for this class are found. A wide class of time series models may be expediently represented in a state-space framework; the state space technique is particularly useful when data complications like missing observations, aggregation, and errors-in-variables are present. Analytic first and second derivatives for the recursive prediction error algorithm's log likelihood function.* .

Keywords: *Analytic functions, Salagean operators, Cofficient bounds, and Distortion Theorems.*

INTRODUCTION

Let the S class of functions $f(z)$ in the form $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ (1)

Which are normalized, analytic and univalent in the unit disk $U = \{z: |z| < 1\}$. Let S^* to be the class of starlike functions and K to be the classes of convex functions. If $f(z)$ is in K (or S^*) and has the property that for all circular arcs γ with the center ξ in the unit disk, if the arc $f(z)$ is starlike convex with respect to $f(\xi)$, then the function $f(z)$ is uniformly uniformly convex in U. Letting UCV denote class of uniformly convex functions and S_P dente the class of uniformly convex starlike functions we find that from maxima and minima as follows:

$$f \in S_P \Leftrightarrow Re\left\{\frac{zf'(z)}{f(z)}\right\} \geq \left|\frac{zf'(z)}{f(z)} - 1\right| and\ f \in UCV \Leftrightarrow Re\{1 + \frac{zf''(z)}{f'(z)}\} \geq \left|\frac{zf''(z)}{f'(z)}\right|$$

It is obvious that $f(z) \in UCV$ if and only if $zf'(z) \in S_P$. The definition of the function $f \in S$ to be K- uniformly convex K-uniformly convex if for $0 \leq k < \infty$ the image of every circular γ with the center ξ where $\xi \leq k$ in the unit disk is convex.

The definition of the class $k - S_P(\propto)$ where $Re\left\{\frac{z\{\Omega^\lambda f(z)\}'}{\Omega^\lambda f(z)}\right\} \geq k\left|\frac{z\{\Omega^\lambda f(z)\}'}{\Omega^\lambda f(z)} - 1\right| + \propto$ with $0 \leq k < \infty, 0 \leq \propto < \infty,$ $and\ 0 \leq \lambda < 1$. The operator $\Omega^\lambda f(z)$ is a fractional operator.

Definition: The definition classes of analytic functions using differential operators such as the Ruscheweyh derivative operator defined by Rusheweyh and the Salagean differential operator defined by Salagean. To develop by using Salagean operator we will define the class $US_\lambda(k,b)$ where functions in this class satisfy
$Re\left\{1 + \frac{1}{b}\left[\frac{z(D^\lambda f(z))'}{(D^\lambda f(z)} - 1\right]\right\} \geq k\left|\frac{1}{b}\left[\frac{z(D^\lambda f(z))'}{(D^\lambda f(z)} - 1\right]\right|$. Where b is a nonzero complex number and $D^\lambda f$ with $\lambda \in N$ is the Salagean differential operator defined as $D^\lambda f = z + \sum_{n=2}^{\infty} n^\lambda a_n z^n$. Notice that US_λ is the known class which was introduced by maxima and minima. For the class $US_\lambda(k,b)$ we will obtain the coefficient bounds theorem, growth theorem and distortion theorem.

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MAIN RESUSLTS

Theorem 1: Let f be analytic and univalent function given by (1, 1). $if\ f \in US_\lambda(k,b)$ then $\sum_{n=2}^{\infty} n^\lambda[(k+1)(n-1)+|b|a_n \le |b|$.

Proof: suppose $f \in US_\lambda(k,b)$ then $Re\ \{1+\frac{1}{b}\left[\frac{z(D^\lambda f(z))'}{(D^\lambda f(z)}-1\right]\} \ge k\left|\frac{1}{b}\left[\frac{z(D^\lambda f(z))'}{(D^\lambda f(z)}-1\right]\right|$ we obtain that

$$k\left|\frac{1}{b}\left[\frac{z(D^\lambda f(z))'}{(D^\lambda f(z)}-1\right]\right| - Re\ \{1+\frac{1}{b}\left[\frac{z(D^\lambda f(z))'}{(D^\lambda f(z)}-1\right] \le 1$$

we have $k\left|\frac{1}{b}\left[\frac{z(D^\lambda f(z))'}{(D^\lambda f(z)}-1\right]\right| - Re\ \{1+\frac{1}{b}\left[\frac{z(D^\lambda f(z))'}{(D^\lambda f(z)}-1\right] \le (k+1)\frac{1}{b}\left[\frac{z(D^\lambda f(z))'}{(D^\lambda f(z)}-1\right] = \frac{(k+1)}{|b|}\left|\frac{\sum_{n=2}^{\infty} n^\lambda (n-1)a_n z^{n-1}}{1+\sum_{n=2}^{\infty} n^\lambda a_n z^{n-1}}\right| \le \frac{(k+1)}{|b|}\left|\frac{\sum_{n=2}^{\infty} n^\lambda (n-1)a_n z^{n-1}}{1-\sum_{n=2}^{\infty} n^\lambda a_n z^{n-1}}\right|$. Now, if $|z|$ approaches the left on the real axis, then $k\left|\frac{1}{b}\left[\frac{z(D^\lambda f(z))'}{(D^\lambda f(z)}-1\right]\right| - Re\ \{1+\frac{1}{b}\left[\frac{z(D^\lambda f(z))'}{(D^\lambda f(z)}-1\right] \le \frac{(k+1)}{|b|}\left|\frac{\sum_{n=2}^{\infty} n^\lambda (n-1)a_n z^{n-1}}{1-\sum_{n=2}^{\infty} n^\lambda a_n z^{n-1}}\right|$

Notice that the expression above is bounded by the hypothesis if
$\sum_{n=2}^{\infty} n^\lambda[(k+1)(n-1)a_n \le |b|[1-\sum_{n=2}^{\infty} n^\lambda |a_n|$, which can be written as
$\sum_{n=2}^{\infty} n^\lambda[(k+1)(n-1)+|b|a_n \le |b|$. The growth and distortion theorems for the class $f \in US_\lambda(k,b)$.

Theorem 2: If $f \in US_\lambda(k,b)$ then $r-\frac{|b|}{2^\lambda[(k+1)+|b|}r^2 \le |f(z)| \le r+\frac{|b|}{2^\lambda[(k+1)+|b|}r^2$ equality occurs for $f(z)=z+\frac{|b|}{2^\lambda[(k+1)+|b|}r^2\ (|z|=r)$.

Proof: since $f(z) \in US_\lambda(k,b)$ it is obtained that
$2^\lambda[(k+1)(n-1)+|b|\sum_{n=2}^{\infty}|a_n| \le \sum_{n=2}^{\infty} n^\lambda[(k+1)(n-1)+|b|]\ a_n \le |b|$ which immediately yields

$$\sum_{n=2}^{\infty}|a_n| \le \frac{|b|}{2^\lambda[(k+1)+|b|}$$

We have $|f(z)| \le |z|+\sum_{n=2}^{\infty}|a_n|\ |z|^n \le |z|+\frac{|b|}{2^\lambda[(k+1)+|b|}|z|^2$
We get $|f(z)| \ge |z|-\sum_{n=2}^{\infty}|a_n|\ |z|^n \ge |z|+\frac{|b|}{2^\lambda[(k+1)+|b|}|z|^2$

Theorem 3: If $f(z) \in US_\lambda(k,b)$ then $1-\frac{2|b|}{2^\lambda[(k+1)+|b|}r \le |f'(z)| \le 1+\frac{2|b|}{2^\lambda[(k+1)+|b|}r$ with equality for $f(z)=z+\frac{|b|}{2^\lambda[(k+1)+|b|}|z|^2\ \ (|z|=r\ )$

Proof: We obtained $|f(z)| \le |z|+\frac{|b|}{2^\lambda[(k+1)+|b|}|z|^2$ i.e., $|f'(z)| \le 1+\frac{2|b|}{2^\lambda[(k+1)+|b|}|z|$
We have $|f(z)| \ge |z|-\frac{|b|}{2^\lambda[(k+1)+|b|}|z|^2$ we find that $|f'(z)| \ge 1-\frac{2|b|}{2^\lambda[(k+1)+|b|}|z|$.

II. FIRST DERIVATES

Taking the derivative of L with respect to a typical element $\alpha\ of\ \theta$ gives $\frac{\partial L}{\partial\alpha}=T\frac{\partial ln|\Sigma_x|}{\partial\alpha}+\frac{\partial}{\partial\alpha}\sum_{t=1}^{T}\delta_t'\sum_x^{-1}\delta_t$ after multiplying by the constant factor-2. The two terms of will be treated separately. The first derivative term applying a matrix derivative identity and then expanding yields . . . .

- ***The first derivative Term***:

Appling a matrix derivative identity and then expanding yields $T\frac{\partial ln|\Sigma_x|}{\partial\alpha}=T\ tr(\sum_x^{-1}\frac{\partial\Sigma_x}{\partial\alpha}=T\ tr\ \left(\sum_x^{-1}\frac{\partial(H\sum H'+R)}{\partial\alpha}\right)=$ *$T\ tr\ (x-1[\partial H\partial\alpha H'+H\partial\Sigma\partial\alpha H'+H\Sigma\partial H'\partial\alpha+\partial R\partial\alpha])$.* It is consists of derivatives of system matrices and *$(\partial\Sigma x\partial\alpha)$*. The latter term may be obtained by partially differentiating both sides of (3) with respect to $\propto$ is as follows:

$$\frac{\partial\Sigma}{\partial\alpha}=\frac{\partial F}{\partial\alpha}\Sigma F'+F\frac{\partial\Sigma}{\partial\alpha}F'+F\Sigma\frac{\partial F'}{\partial\alpha}+\frac{\partial(GQG')}{\partial\alpha}-\left[\frac{\partial F}{\partial\alpha}\Sigma F'+F\frac{\partial\Sigma}{\partial\alpha}F'+F\Sigma\frac{\partial F'}{\partial\alpha}+\frac{\partial(GW)}{\partial\alpha}\right]K'$$
$$+K[\frac{\partial H}{\partial\alpha}\sum H'+H\frac{\partial\Sigma}{\partial\alpha}H'+H\Sigma\frac{\partial H'}{\partial\alpha}+\frac{\partial R}{\partial\alpha}])\,K'-K[\frac{\partial F}{\partial\alpha}\Sigma F'+F\frac{\partial\Sigma}{\partial\alpha}F'+F\Sigma\frac{\partial F'}{\partial\alpha}+\frac{\partial(GW)'}{\partial\alpha}$$
$$=\bar{F}\frac{\partial\Sigma}{\partial\alpha}\bar{F}'+\widetilde{\Omega}_\alpha+\widetilde{\Omega}'_\alpha$$

where $\bar{F}=F-KH, and\ \widetilde{\Omega}_\alpha=\frac{\partial\Sigma}{\partial\alpha}\Sigma\bar{F}'-\bar{F}\Sigma\frac{\partial H'}{\partial\alpha}K'-\frac{\partial(GW)}{\partial\alpha}K'+1/2\left[K\frac{\partial R}{\partial\alpha}K'+\frac{\partial(GQG')}{\partial\alpha}\right]$.

Equation (7) is a discrete Lyapunov equation which may be solved by any of several standard methods and $\bar{F}\ and\ \widetilde{\Omega}_\alpha$ are solely functions of the system matrices and their first derivatives.

➢ ***The second First Derivative Term:***

The second term of the derivative of the likelihood function involves functions of both the data and system matrices, as can be seen from the innovations representation (4). Writing out the terms separately as follows:

$$\frac{\partial}{\partial\alpha}\sum_{t=1}^{T}\delta_t'\sum_x^{-1}\delta_t=\sum_{t=1}^{T}\frac{\partial\delta_t'}{\partial\alpha}\sum_x^{-1}\delta_t+\sum_{t=1}^{T}\delta_t'\frac{\partial\sum_x^{-1}\delta_t}{\partial\alpha}+\sum_{t=1}^{T}\delta_t'\sum_x^{-1}\frac{\partial\delta_t}{\partial\alpha}$$

The middle term of (9) involves δ_t and $(\frac{\partial\sum_x^{-1}\delta_t}{\partial\alpha})$, which are known.

III. SECOND DERIVATIVES

➢ The first second derivative term:

The derivative of (6) with respect to β, a second typical element of θ, may be expressed as

$$T\frac{\partial^2 ln|\Sigma_x|}{\partial\alpha\partial\beta}=T\frac{\partial tr(\Sigma_x^{-1}\frac{\partial\Sigma_x}{\partial\alpha})}{\partial\beta}=T\,tr\frac{\partial(\Sigma_x^{-1}\frac{\partial\Sigma_x}{\partial\alpha})}{\partial\beta}$$

$$=T\,tr\,[\frac{\partial\sum_x^{-1}\partial\sum x}{\partial\beta\partial\alpha}+$$

$$\sum_x^{-1}\frac{\partial^2\sum x}{\partial\alpha\partial\beta}]=T\,tr\,[-\sum_x^{-1}\frac{\partial\sum x}{\partial\beta}\sum_x^{-1}\frac{\partial\sum x}{\partial\alpha}+\ \sum_x^{-1}\frac{\partial^2\sum x}{\partial\alpha\partial\beta}]=T\,tr\,[-\sum_x^{-1}\frac{\partial(H\sum H'+R)}{\partial\beta}\sum_x^{-1}\frac{\partial(H\sum H'+R)}{\partial\alpha}+\sum_x^{-1}\frac{\partial(H\sum H'+R)}{\partial\alpha\partial\beta}].$$

The RHS (12) involves objects known and calculable expect for $(\partial^2\Sigma/_{\partial\alpha\partial\beta})$ term. $(\frac{\partial\Sigma}{\partial\beta})$ can be found the same way as $(\partial\Sigma/\partial\alpha)$.

Using $(\frac{\partial\Sigma}{\partial\beta})$ = F $\left(\frac{\partial\Sigma}{\partial\beta}\right)\bar{F}'+\widetilde{\Omega}_\beta+\widetilde{\Omega}'_\beta$

Now we obtain $\left(\partial^2\Sigma/_{\partial\alpha\partial\beta}\right)=\bar{F}\ \left(\partial^2\Sigma/_{\partial\alpha\partial\beta}\right)\bar{F}'+\Gamma_{\alpha\beta}+\Gamma'_{\alpha\beta}$.

Here $\Gamma_{\alpha\beta}=\ \frac{\partial^2F}{\partial\alpha\partial\beta}\Sigma F'+\frac{\partial F}{\partial\alpha}\frac{\partial\Sigma}{\partial\beta}F'+\frac{\partial F}{\partial\alpha}\Sigma\frac{\partial F\prime}{\partial\beta}+\frac{\partial F}{\partial\beta}\frac{\partial\Sigma}{\partial\alpha}F'-[\frac{\partial}{\partial\beta}\left(\frac{\partial F}{\partial\alpha}\Sigma H'\right)+\frac{\partial F}{\partial\beta}\frac{\partial\Sigma}{\partial\alpha}F'+F\frac{\partial\Sigma}{\partial\alpha}\frac{\partial H'}{\partial\beta}+\frac{\partial}{\partial\beta}(F\Sigma\frac{\partial H'}{\partial\alpha})$
$+\frac{\partial^2(GW)}{\partial\alpha\partial\beta}]K'-\frac{\partial(F\Sigma H'+GW)}{\partial\alpha}\sum_x^{-1}\frac{\partial(F\Sigma H'+GW)\prime}{\partial\beta}+\sum_x^{-1}\frac{\partial\Sigma_z}{\partial\alpha}K'-K\frac{\partial\Sigma_z}{\partial\beta}\sum_x^{-1}\frac{\partial\Sigma_z}{\partial\alpha}K'+1/2\{\frac{\partial^2(GQG')}{\partial\alpha\partial\beta}$
$+K\left[\frac{\partial}{\partial\beta}\left(\frac{\partial F}{\partial\alpha}\Sigma H'\right)+\frac{\partial H}{\partial\beta}\frac{\partial\Sigma}{\partial\alpha}H'+H\frac{\partial\Sigma}{\partial\alpha}\frac{\partial H'}{\partial\beta}+\frac{\partial}{\partial\beta}\left(H\Sigma\frac{\partial H'}{\partial\alpha}\right)+\frac{\partial^2R}{\partial\alpha\partial\beta}\right]K'\}$.

Thus $(\partial^2\Sigma/_{\partial\alpha\partial\beta})$ too satisfies a discrete Laypnouv equation.

➢ ***The Second Second Derivative Term***:

It remains to compute the equation to developed Second Derivate Term as follows:

$$\partial^2\Sigma/_{\partial\alpha\partial\beta}=\sum_{t=1}^{T}\delta_t'\sum_x^{-1}\delta_t=\frac{\partial}{\partial\beta}[\sum_{t=1}^{T}\frac{\partial\delta_t'}{\partial\alpha}\sum_x^{-1}\delta_t+\sum_{t=1}^{T}\delta_t'\frac{\partial\sum_x^{-1}\delta_t}{\partial\alpha}+\ \sum_{t=1}^{T}\delta_t'\sum_x^{-1}\frac{\partial\delta_t}{\partial\alpha}$$

$$=\sum_x^{t=1}\frac{\partial^2\delta_t'}{\partial\alpha\partial\beta}\sum_x^{-1}\delta_t+\sum_{t=1}^{T}\frac{\partial\delta_t'}{\partial\alpha}\frac{\partial\sum_x^{-1}\delta_t}{\partial\beta}\sum_{t=1}^{T}\delta_t'\frac{\partial^2\sum_x^{-1}\delta_t}{\partial\alpha\partial\beta}+\sum_{t=1}^{T}\delta_t'\frac{\partial\sum_x^{-1}\partial\delta_t}{\partial\alpha\partial\beta}+\sum_{t=1}^{T}\frac{\partial\delta_t'}{\partial\beta}\sum_x^{-1}\frac{\partial\delta_t}{\partial\alpha}$$

$$+\sum_{t=1}^{T}\delta_t'\frac{\partial\sum_x^{-1}\partial\delta_t}{\partial\beta\partial\alpha}+\sum_{t=1}^{T}\delta_t'\sum_x^{-1}\frac{\partial^2\delta_t}{\partial\alpha\partial\beta}\quad.$$

All of the terms of above have been computed above or have direct analogies with respect to, with the exception of $\sum_x^{t=1}\frac{\partial^2\delta_t'}{\partial\alpha\partial\beta}\sum_x^{-1}\delta_t$ and its transpose.

$\frac{\partial^2\delta_t'}{\partial\alpha\partial\beta}=-\frac{\partial}{\partial\beta}\left[\frac{\partial H}{\partial\propto}\hat{x}_t+H\frac{\partial\hat{x}_t}{\partial\alpha}\right]=-[\frac{\partial^2H}{\partial\alpha\partial\beta}\hat{x}_t+\frac{\partial H}{\partial\propto}\frac{\partial\hat{x}_t}{\partial\beta}+\frac{\partial H}{\partial\beta}\frac{\partial\hat{x}_t}{\partial\alpha}+H\frac{\partial^2\hat{x}_t}{\partial\alpha\partial\beta}]$ the only unknown object is $[\frac{\partial^2\hat{x}_t}{\partial\alpha\partial\beta}]$. For t = 0, $\left[\frac{\partial^2\hat{x}_t}{\partial\alpha\partial\beta}\right]=\ 0$, while for $t\geq 1$. $[\frac{\partial^2\hat{x}_t}{\partial\alpha\partial\beta}]$ is defined recursively in a way analogous.

$$\left[\frac{\partial^2\hat{x}_t}{\partial\alpha\partial\beta}\right]=\ \frac{\partial^2[(F-KH)\hat{x}_{t-1}+Kz_{t-1}]}{\partial\alpha\partial\beta}=\frac{\partial^2(F-KH)}{\partial\alpha\partial\beta}\hat{x}_{t-1}+\frac{\partial(F-KH)\partial\hat{x}_{t-1}}{\partial\alpha\partial\beta}+\frac{\partial(F-KH)\partial\hat{x}_{t-1}}{\partial\beta\partial\alpha}+(F-KH)\frac{\partial^2\hat{x}_{t-1}}{\partial\alpha\partial\beta}+\frac{\partial^2K}{\partial\alpha\partial\beta}z_{t-1.}\ .$$

CONCLUSION

This note improves on a method for computing analytic derivatives of the like hood function of a discrete, linear, time invariant Gaussian state-space system and extends it to handle correlation between the state transition and measurement noise terms and to compute the analytic Hessian matrix.We remark that several subclasses of analytic univalent functions can be derived using the operators and studied their properties, In this paper we developed the calculation of log-likelihood function, its gradient vector, and its Hessian matrix may be achieved and first and second derivatives of store data input the system matrices F,G, H, R, and W. Choose an initial condition and generate function. Computation of first and second order derivatives i.e., $(\partial^2\Sigma/_{\partial\alpha\partial\beta})$ and $(\frac{\partial^2\delta_t'}{\partial\alpha\partial\beta})$.

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