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(ψ*g*)*- CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

*The aim of this paper is to introduce and study a new class of generalized closed sets called **(ψ*g*)*-closed sets** in topological spaces. This class was obtained by generalizing closed sets via **ψ*g*** open sets which was introduced by N. Balamani, A. Parvathi [3]. This new class falls strictly between the class of closed sets and ψg -closed sets. Also some of their properties have been investigated*

***Keywords**: (ψ*g*)*-closed sets and (ψ*g*)* -open sets*

I. INTRODUCTION

Levine [8] introduced the concepts of generalized closed sets denoted by g-closed sets in topological spaces and studied their basic properties. Veerakumar [22] introduced and analysed ψ -closed sets in topological spaces. Veerakumar [26] introduced the concepts of g *ψ -closed sets using g-closed sets in topological spaces. Ramya and Parvathi [18] introduced a new concept of generalized closed sets called $\hat{\psi}$g -closed sets and ψg -closed sets in topological spaces. N. Balamani, A. Parvathi [3] introduce a new class of generalized closed sets called ψ*g*- Closed Sets in Topological Spaces.

The purpose of this paper is to introduce a new class of generalized closed sets called (ψ*g*)*-closed sets in topological spaces and study some properties.

II. PRELIMINARIES

Throughout this paper (X, τ) represents non-empty topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure of A and the interior of A respectively.

Definition 2.1: A subset A of a topological space (X, τ) is called
- (i) Regular open set [20] if A=int(cl(A))
- (ii) Semi-open set [7] if A⊆cl(int(A))
- (iii) α -open set[16] if A⊆int(cl(int(A)))
- (iv) Pre-open set [13] if A⊆int(cl(A))
- (v) semi pre-open set [2] if A⊆cl(int(clA)))

The complements of the above mentioned sets are called regular closed, semi closed, α -closed, pre-closed and semi pre-closed sets respectively.

The intersection of all regular closed subsets of (X, τ) containing A is called the regular closure of A and is denoted by rcl(A). Similarly scl(A)-semi closure of A, αcl(A) - α closure of A, pcl(A)-pre closure of A and spcl(A)-semi pre closure of A are defined.

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Definition 2.2: A subset A of a topological space (X, τ) is called

(a) generalized closed set (g-closed) [8] if cl(A)⊆U whenever A⊆U and U is open in (X, τ).
(b) semi-generalized closed set (sg-closed) [4] if scl(A)⊆U whenever A⊆U and U is semi-open in (X, τ).
(c) generalized α-closed set(gα-closed) [9] if αcl(A)⊆U whenever A⊆U and U is α-open in (X, τ).
(d) α-generalized closed set (αg-closed) [10] if αcl(A)⊆U whenever A⊆U and U is open in (X, τ).
(e) generalized semi-pre -closed set (gsp-closed) [6] if spcl(A)⊆U whenever A⊆U and U is open in (X, τ).
(f) g* -closed set[23] if cl(A)⊆U whenever A⊆U and U is g-open in (X, τ).
(g) g^-closed set [24] if cl(A)⊆U whenever A⊆U and U is semi-open in (X, τ).
(h) gp-closed set [11] if pcl(A)⊆U whenever A⊆U and U is open in (X, τ).
(i) g*p-closed set [25] if pcl(A)⊆U whenever A⊆U and U is g-open in (X, τ).
(j) αg^-closed set [1] if αcl(A)⊆U whenever A⊆U and U is g^ -open in (X, τ).
(k) αg *- closed set [14] if cl(A)⊆U whenever A⊆U and U is α-open in (X, τ).
(l) sαg* - closed set [12] if αcl(A)⊆U whenever A⊆ U and U is g *-open in (X, τ).
(m) wgα-closed set [27] if αcl(int(A))⊆U whenever A⊆U and U is α-open in (X, τ).
(n) wαg-closed set [27] if αcl(int(A))⊆U whenever A⊆U and U is open in (X, τ).
(o) ψ-closed set [22] if scl(A)⊆U whenever A⊆U and U is sg-open in (X, τ).
(p) ψg -closed set [18] if ψcl(A)⊆U whenever A⊆U and U is open in (X, τ).
(q) g *ψ -closed set [26] if ψcl(A)⊆U whenever A⊆U and U is g-open in (X, τ).
(r) ψg^ - closed set [18] if ψcl(A)⊆U whenever A⊆U and U is g^ -open in (X, τ).
(s) αψ- closed set [5] if ψcl(A)⊆U whenever A⊆U and U is α-open in (X, τ).
(t) A regular weakly generalized semi closed (rwg closed) [15] if cl (int(A))⊆ U whenever A⊆ U and U is regular open.
(u) A generalized semi-preclosed star closed ((gsp)* closed)[17] if cl(A) ⊆ U whenever A ⊆U and U is gsp open.
(v) A regular^ generalized closed (r^g closed) [19] if gcl(A) ⊆U whenever A⊆ U and U is regular open.
(w) ψ*g * -closed set[3] if ψcl(A)⊆U whenever A⊆U and U is ψg -open in (X, τ).

The complements of the above mentioned sets are called their respective open-sets.

Remark 2.3: r-closed(r-open) → closed (open) → α-closed (α-open) → semi-closed(semi-open) → ψ -closed(ψ -open) → semi pre-closed(semi pre-open).

Remark 2.4: r-closed → closed → ψ-closed →ψ*g*-closed → g*ψ-closed → ψg^-closed → ψg -closed → gsp-closed

III. BASIC PROPERTIES OF (ψ*g*)*- CLOSED SETS

Definition 3.1: A subset A of a topological space (X, τ) is said to be **(ψ*g** *)* **-closed set** if cl(A)⊆U whenever A⊆U and U is **ψ*g** * -open in (X, τ). The class of all (ψ*g*)*-closed sets of (X,τ) is denoted by ψ*g*C(X, τ) .

Preposition 3.2: Every closed set in (X, τ) is (ψ*g*)*-closed but not conversely.

Proof: Let A be a closed set of (X, τ). Let U be any **ψ*g** * -open set containing A. Since A is closed set. Therefore cl(A) = A
Hence cl(A) ⊆U. Therefore A is (ψ*g*)*- closed.

Example 3.3: Let X = {a, b, c} with τ ={φ, X, {a},{b, c}}. Then the subset {b} is (ψ*g*)* -closed but not closed in (X, τ).

Preposition 3.4: Every regular closed set in (X, τ) is (ψ*g*)*-closed but not conversely.

Proof: As every regular closed set is closed and by preposition 3.2 the proof follows.

Example 3.5: Let X = {a, b, c} with τ = {φ, X, {a}, {b}, {a, b}, {b, c}}. Then the subset {c} is (ψ*g*)*-closed but not regular closed in (X, τ).

Preposition 3.6: Every (ψ*g*)*-closed set in (X, τ) is ψg -closed but not conversely.

Proof: Let A be a (ψ*g *)*-closed set and U be any open set containing A in X. Since every open set is ψ*g* -open, Therefore ψcl(A)⊆ cl(A) ⊆U. Hence A is ψg -closed.

Example 3.7: Let X = {a, b, c} with τ ={φ, X, {a},{a, c}}. Then the subset {a, b} is ψg - closed but not (ψ*g*)*-closed in (X, τ).

Preposition 3.8: Every (ψ*g*)*-closed set in (X, τ) is gsp- closed but not conversely.

Proof: Let A be a (ψ*g*)*-closed set and U be any open set containing A in X. Since every open set is ψ*g*-open and spcl(A)⊆cl(A) ⊆U. Hence A is gsp-closed.

Example 3.9: Let X = {a, b, c} with τ ={φ, X, {a, b}}. Then the subset {a} is gsp-closed but not (ψ*g*)*-closed in (X, τ).

Preposition 3.10: Every (gsp)* -closed set in (X, τ) is (ψ*g*)* - closed but not conversely.

Proof: Let A be a (gsp)*-closed set and U be any ψ*g*-open set containing A in X. Since every ψ*g*-open set is gsp-open. Therefore cl(A) ⊆U. Hence A is (ψ*g*)*-closed.

Example 3.11: Let X = {a, b, c} with τ ={φ, X, {a}, {b, c}}. Then the subset {a, b} is (ψ*g*)*-closed but not (gsp)*-closed in (X, τ).

Preposition 3.12: Every (ψ*g*)*-closed set in (X, τ) is gpr- closed but not conversely.

Proof: Let A be a (ψ*g*)*-closed set and U be any regular open set containing A in X. Since every regular open set is ψ*g*-open and pcl(A)⊆cl(A) ⊆U. Hence A is gpr-closed.

Example 3.13: Let X = {a, b, c} with τ ={φ, X, {a}, {b}, {a, b}}. Then the subset {a, b} is gpr-closed but not (ψ*g*)*-closed in (X, τ).

Preposition 3.14: Every (ψ*g*)*-closed set in (X, τ) is rwg- closed but not conversely.

Proof: Let A be a (ψ*g*)*-closed set and U be any regular open set containing A in X. Since every regular open set is ψ*g*-open and cl(int(A))⊆cl(A) ⊆U. Hence A is rwg-closed.

Example 3.15: Let X = {a, b, c} with τ ={φ, X, {a}, {b}, {a, b}}. Then the subset {a,b} is rwg-closed but not (ψ*g*)*-closed in (X, τ).

Preposition 3.16: Every (ψ*g*)*-closed set in (X, τ) is r^g- closed but not conversely.

Proof: Let A be a (ψ*g*)*-closed set and U be any regular open set containing A in X. Since every regular open set is ψ*g*-open and gcl(A) ⊆cl(A) ⊆U. Hence A is r^g- closed

Example 3.17: Let X = {a, b, c} with τ ={φ, X,{a},{b},{a, b}}. Then the subset {a, b} is r^g-closed but not (ψ*g*)*-closed in (X, τ).

Preposition 3.18: Every (ψ*g*)*-closed set in (X, τ) is rg-closed but not conversely.

Proof: Let A be a (ψ*g*)*-closed set and U be any regular open set containing A in X. Since every regular open set is ψ*g* -open and cl(A)⊆U. Hence A is rg-closed.

Example 3.19: Let X = {a, b, c} with τ ={φ, X,{a},{b},{a, b}}. Then the subset {a, b} is rg-closed but not (ψ*g*)* -closed in (X, τ).

Preposition 3.20: (ψ*g*)*-closedness is independent from ψgˆ-closedness, ψ- closedness, semi closedness, g*ψ -closedness, g*-closedness and α -closedness.

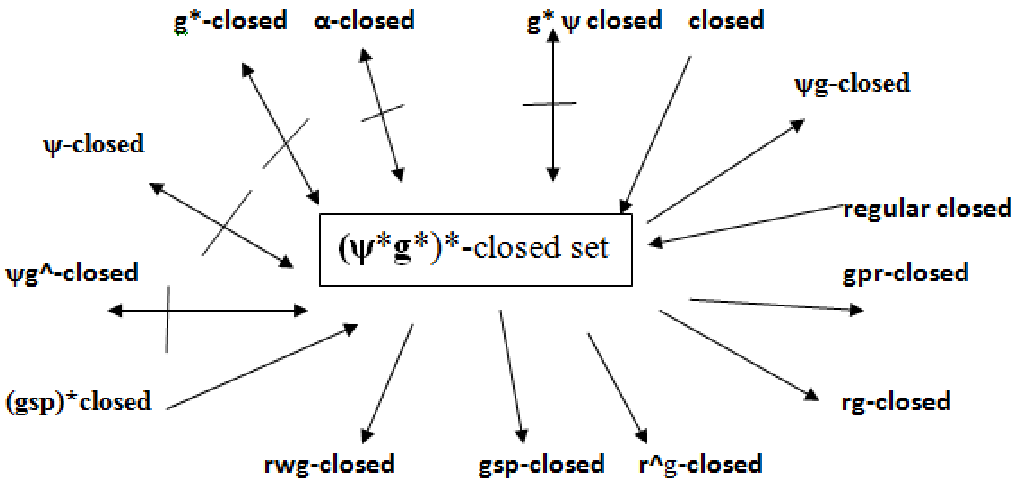
Example 3.21: Let X = {a, b, c} with τ ={φ, X, {a}, {b, c}}.
Then the subset {a, c} is (ψ*g*)* -closed but not ψgˆ-closed, ψ- closed, semi closed, g*ψ- closed, g*-closed, ψ*g*-closed and α –closed.

Example 3.22: Let X = {a, b, c} with τ ={φ, X, {a},{a, c}}. Then the subset {c} is ψgˆ -closed, ψ- closed, semi closed, g*ψ- closed, ψ*g*-closed and α –closed but not (ψ*g*)* -closed in (X, τ). The subset {a, b}is g*-closed but not (ψ*g*)* -closed in (X, τ).

IV. (ψ*g*)*- OPEN SET

Definition 4.1: A subset A of a topological space (X, τ), is called (ψ*g*)*- open set if and only if Ac is (ψ*g*)*-closed in X. We denote the family of all (ψ*g*)*-open sets in X by (ψ*g*)*- O(X).

Theorem 4.2:
(i) Every open set is (ψ*g*)*-open.
(ii) Every regular open set is (ψ*g*)*-open set.
(iii) Every (gsp)* -open set is (ψ*g*)*-open set.
(iv) Every (ψ*g*)*-open set is gsp-open set.
(v) Every (ψ*g*)*-open set is strongly ψg-open set.
(vi) Every (ψ*g*)*-open set is gpr-open set.
(vii) Every (ψ*g*)*-open set is rwg-open set.
(viii) Every (ψ*g*)*-open set is r^g-open set.
(ix) Every (ψ*g*)*-open set is rg-open set.



A $\longrightarrow$ B represents A implies B. But not converse and A $\longleftrightarrow$ B represents A and B are independent of each other.

V. CHARACTERISTICS OF (ψ*g*)*-CLOSED AND (ψ*g*)*-OPEN SETS

Theorem 5.1: If A and B are (ψ*g*)*-closed sets in X then A∪B is (ψ*g*)*-closed set in X.

Proof: Let A and B are (ψ*g*)*-closed sets in X and U be any ψ*g*-open set containing A and B. Therefore cl(A) ⊆ U, cl (B) ⊆U. Since A ⊆U, B ⊆U then A∪B ⊆U. Hence cl (A∪B) = cl(A)∪cl(B)⊆U. Therefore A∪B is (ψ*g*)*-closed set in X.

Theorem 5.2: If a set A is (ψ*g*)*-closed set iff cl(A)-A contains no non empty ψ*g*-closed set.

Proof:
Necessity: Let F be a ψ*g*-closed set in X such that F ⊆ cl (A) –A. Then A ⊆X – F. Since A is (ψ*g*)*- closed set and X - F is ψ*g*-open, then cl (A) ⊆X – F. (i.e.) F ⊆X –cl (A).
So F ⊆(X –cl (A)) ∩(cl (A) –A). Therefore F = ϕ

Sufficiency: Let us assume that cl (A) – A contains no non empty ψ*g*-closed set. Let *A* ⊆*U,* U is ψ*g*- open set. Suppose that cl(A) is not contained in U, cl(A)∩Uc is a nonempty ψ*g*- closed set of cl(A)-A which is contradiction. Therefore cl(A) ⊆U. Hence A is (ψ*g*)*-closed.

Theorem 5.3: The Intersection of any two subsets of (ψ*g*)*-closed sets in X is (ψ*g*)*-closed set in X.

Proof: Let A and B are any two sub sets of (ψ*g*)*-closed sets. A ⊆U, U is any ψ*g*-open and B ⊆U, U is ψ*g*-open. Then cl (A) ⊆ U, cl (B) ⊆ U, therefore cl (A∩B) ⊆ U, U is ψ*g*-open in X. Since A and B are (ψ*g*)*-closed set, Hence A ∩ B is a (ψ*g*)*-closed set.

Theorem 5.4: If A is (ψ*g*)*-closed set in X and A $\subseteq$ B $\subseteq$ cl (A), Then B is (ψ*g*)*-closed set in X.

Proof: Since B$\subseteq$cl(A), we have cl(B) $\subseteq$cl(A) then cl(B) - B $\subseteq$cl(A) – A. By theorem 5.2, cl (A)-A contains no non empty ψ*g*-closed set. Hence cl (B) – B contains no non empty ψ*g*- closed set. Therefore B is (ψ*g*)*-closed set in X.

Theorem 5.5: If A $\subseteq$ Y $\subseteq$ X and suppose that A is (ψ*g*)* closed set in X then A is (ψ*g*)*- closed set relative to Y.

Proof: Given that A $\subseteq$ Y $\subseteq$ X and A is (ψ*g*)*-closed set in X. To prove that A is (ψ*g*)*- closed set relative to Y. Let us assume that A $\subseteq$ Y $\cap$ U, where U is ψ*g*- open in X. Since A is (ψ*g*)*- closed set, A $\subseteq$ U implies cl (A) $\subseteq$ U. It follows that Y $\cap$ cl (A) $\subseteq$ Y $\cap$ U. That is A is (ψ*g*)*- closed set relative to Y.

Theorem 5.6: If A is both ψ*g*-open and(ψ*g*)*-closed set in X, then A is ψ*g*-closed set.

Proof: Since A is ψ*g*-open and (ψ*g*)* closed in X, cl(A) $\subseteq$ U. But Always A $\subseteq$ cl(A). Therefore A = cl (A). Hence A is ψ*g*-closed set.

Theorem 5.7: For $x \in X$, then the set X-{x} is a (ψ*g*)*-closed set or ψ*g*- open.

Proof: Suppose that X – {x} is not ψ*g*-open, then X is the only ψ*g*- open set containing X – {x}.
(i.e.) cl(X-{x}) $\subseteq$ X. Then X-{x} is (ψ*g*)*-closed in X.

Theorem 5.8: If A and B are (ψ*g*)*-open sets in a space X. Then A $\cap$ B is also (ψ*g*)*-open set in X.

Proof: If A and B are (ψ*g*)*-open sets in a space X. Then Ac and Bc are (ψ*g*)*-closed sets in a space X. By theorem 5.1 Ac $\cup$ Bc is also (ψ*g*)*-closed set in X. (i.e.) Ac $\cup$ Bc = (A $\cap$ B) c is a (ψ*g*)*-closed set in X. Therefore A $\cap$ B is (ψ*g*)*-open set in X.

Theorem 5.9: If A and B are (ψ*g*)*-open sets in X then A $\cup$ B is a (ψ*g*)*-open set in X.

Proof: If A and B are (ψ*g*)*-open sets in a space X. Then Ac and Bc are (ψ*g*)*-closed sets in a space X. By theorem 4. Ac $\cap$ Bc is also (ψ*g*)*-closed set in X. (i.e.) Ac $\cap$ Bc = (A $\cup$ B)c is a (ψ*g*)*-closed set in X. Therefore A $\cup$ B is (ψ*g*)*-open set in X.

Theorem 4.15: If Int (B) $\subseteq$ B $\subseteq$ A and if A is (ψ*g*)*-open in X, then B is (ψ*g*)*-open in X.

Proof: Suppose that Int (B) $\subseteq$ B $\subseteq$ A and A is (ψ*g*)*-open in X then Ac $\subseteq$ Bc $\subseteq$ Cl (Ac). Since Ac is (ψ*g*)*-closed in X, by Theorem 5.4 Bc is (ψ*g*)*-closed set in X. Therefore B is (ψ*g*)*-open in X.

VI. CONCLUSION

In this paper we have introduced (ψ *g*)*closed sets and (ψ *g*)*open sets and studied some properties. This class of sets can be used to discuss the notion of Continuity,

Compactness and connectedness and also can be extended to other topological spaces like Fuzzy & Bitopological Spaces.

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