International Journal of Mathematical Archive-9(6), 2018, 82-85
Available online through www.ijma.info ISSN 2229 – 5046

G-DOMATIC NUMBER OF A GRAPH

S. KALAVATHI*[1] AND K. PALANI[2]

[1]Research Scholar,
Manonmaniam Sundaranar University, Tirunelveli, Tamil Nadu, India.

[2]Department of Mathematics,
A.P.C Mahalaxmi College for Women, Thoothukudi.
Manonmaniam Sundaranar University, Tamil Nadu, India.

(Received On: 03-05-18; Revised & Accepted On: 05-06-18)

ABSTRACT

Let $G = (V, E)$ be a graph. The maximum order of a partition of V into (G, D)-sets of G is called the G-domatic number of G and is denoted by $d_G(G)$. In this paper we initiate the study of this parameter.

Keywords: *Domination, Geodomination, (G, D)-sets, G-domatic Number.*

AMS Subject Classification: *05C69.*

1. INTRODUCTION

Throughout this paper, we consider the graph G as a finite undirected simple graph with no loops and multiple edges. The study of domination in graphs was begun by Ore and Berge[6]. Let G = (V, E) be any graph. A dominating set of a graph G is a set D of vertices of G such that every vertex in V–D is adjacent to atleast one vertex in D and the minimum cardinality among all dominating sets is called the domination number γ(G). The concept of geodominating (or geodetic) set was introduced by Buckley and Harary in [1] and Chartrand, Zhang and Harary in [2, 3, 4]. Let u, v ∈ V(G). A u-v geodesic is a u-v path of length d(u, v). A vertex x ∈ V(G) is said to lie on a u-v geodesic P if x is a vertex of P including the vertices u and v. A set S of vertices of G is a geodominating (or geodetic) set if every vertex of G lie on an x-y geodesic for some x, y in S. The minimum cardinality of a geodominating set is the geodomination (or geodetic) number of G and is denoted as g(G)[1, 2, 3, 4]. A (G, D)-set of G is a subset S of V(G) which is both a dominating and geodetic set of G. A (G, D)-set S of G is said to be a minimal (G, D)-set of G if no proper subset of S is a (G, D)-set of G. The minimum cardinality of all (G, D)-sets of G is called the (G, D)-number of G and it is denoted by γ_G(G). Any (G, D)-set of G of cardinality γ_G is called a γ_G-set of G [8, 9, 10].

An excellent treatment of fundamentals of domination is given in [6] by Haynes et al. and survey papers on several advanced topics are given in [7] edited by Haynes *et al.*. A domatic partition of G is a partition of V(G) into classes that are pairwise disjoint dominating sets. The domatic number of G is the maximum cardinality of a domatic partition of G and it is denoted by d(G). The domatic number was introduced by Cockayne and Hedetniemi [5] and we extend the definition of domatic number as follows: Let $G = (V, E)$ be a graph. The maximum order of a partition of V into (G, D)-sets of G is called the G-domatic number of G and is denoted by $d_G(G)$. A vertex v in G is an *extreme* (or simplicial or link complete) *vertex* of G if the subgraph induced by its neighbours is complete. A *dominating vertex* is a vertex which forms a dominating set, i.e. a vertex adjacent to all other vertices. The *complement* $\bar{G}$ of a graph G is the graph with vertex set $V(G)$ such that two vertices are adjacent in $\bar{G}$ if and only if they are not adjacent in G. A *perfect matching* of a graph is a matching (ie, an independent edge set) in which every vertex of the graph is incident to exactly one edge of the matching. A graph G is called *acyclic* if it has no cycles. A connected acyclic graph is called a *tree.*

Theorem 1.1: [8] Let $G = (V, E)$ be any graph. Then, every (G, D)-set of G contains all the extreme vertices of G.

Corresponding Author: S. Kalavathi*[1],
[1]Research Scholar, Manonmaniam Sundaranar University, Tirunelveli, Tamil Nadu, India.

2. G-DOMATIC NUMBER OF GRAPHS

Definition 2.1: Let $G = (V, E)$ be a graph. The maximum order of a partition of V into (G, D)-sets of G is called the G-domatic number of G and is denoted by $d_G(G)$.

Example 2.2: (i) If $G \cong K_n$, then $d_G(G) = 1$. (ii) Consider the graph G as in figure (2.1). In G, $X = \{v_1, v_4, v_7\}$ and $Y = \{v_2, v_3, v_5, v_6\}$ are disjoint (G, D)-sets and $X \cup Y = V(G)$. Also, {X, Y} is the unique G-domatic partition of G and hence $d_G(G) = 2$.

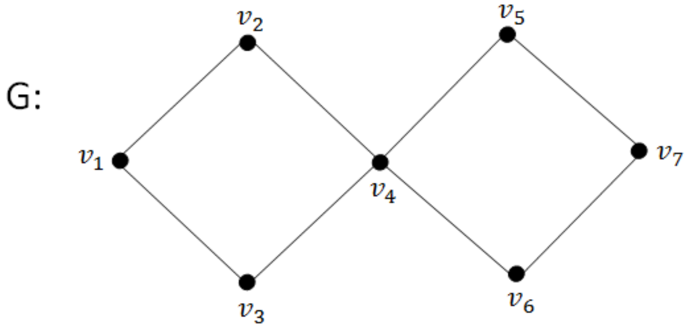

Figure-2.1

Proposition 2.3: If $G \cong K_{2n} - X, n \geq 2$ where X is a perfect matching, then $d_G(G) = n$.

Proof: Let $V(K_{2n}) = \{v_1, v_2, \dots, v_{2n}\}$ and $X = \{e_1, e_2, \dots, e_n\}$. Suppose $e_k = v_i v_j \in X$. Let $S_k = \{v_{i\,,}\, v_j\}$. Then, v_i is not adjacent to v_j in $K_{2n} - X$. Clearly, both v_i and v_j are adjacent to all other vertices and each vertex in the set $\{V(K_{2n} - X) - \{v_i, v_j\}\}$ lie in a geodesic joining v_i and v_j. So, for every $k = 1, 2, \dots, n$, S_k is a (G,D)-set of $K_{2n} - X$. Further, since X is a perfect matching, $G(X)$ is a spanning subgraph of G. Therefore, $\{S_k: 1 \leq k \leq n\}$ forms a partition of $V(G)$ into (G,D)-sets. Since $|S_k|=2$ for every k=1 to n, $\{S_k$: k=1 to n} is a G-domatic partition of G with maximum cardinality. Hence, $d_G(G) = n$.

Lemma 2.4: $\gamma_G(\overline{C_n}) = 3$.

Proof: Let $V(\overline{C_n}) = \{v_1, v_2, \dots, v_n\}$. Then, $E(\overline{C_n}) = E(K_n) - E(C_n)$. For any graph G, $2 \leq \gamma_G(G) \leq n$. Suppose $\gamma_G(\overline{C_n}) = 2$. For any two consecutive vertices v_i and v_j, $d(v_i, v_j) = 2$ and any two non-consecutive vertices v_i and v_j, $d(v_i, v_j) = 1$. For $1 \leq i \leq n$, take $S_i = \{v_i, v_{i+1}\}$ where $v_{n+1} = v_1$. Then, S_i dominate all the vertices of $V(\overline{C_n}) - S_i$. But, S_i geodominate only the vertices of $(V(\overline{C_n}) - S_i) - \{v_{i-1}, v_{i+2}\}$. That is, exactly two vertices in $V(\overline{C_n}) - S_i$ does not lie on any geodesic joining the vertices of S_i. Clearly, $X_i = S_i \cup \{v_{i-1}\}$ or $Y_i = S_i \cup \{v_{i+2}\}$ dominate and geodominate all the vertices of $V(\overline{C_n}) - S_i$. Therefore, X_i or Y_i are minimum (G, D)-sets of $\overline{C_n}$ for every i=1 to n. Hence, $\gamma_G(\overline{C_n}) = |X_i| = |Y_i| = |S_i| + 1 = 3$.

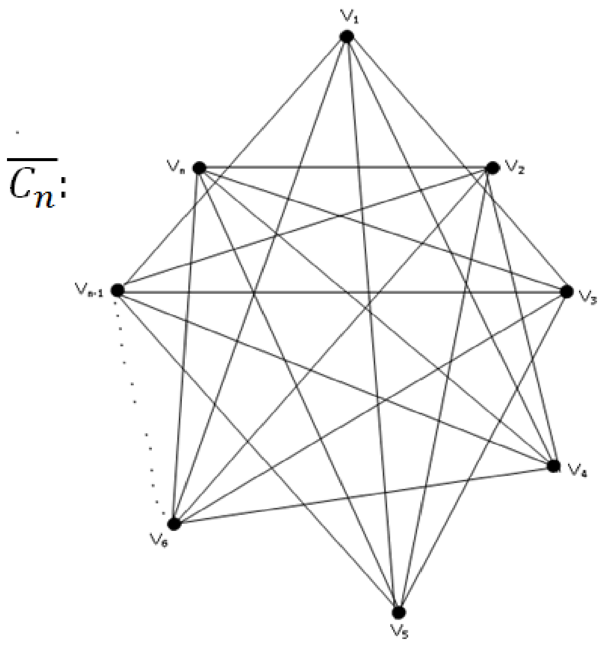

Figure-2.2

Proposition 2.5: For n$\equiv 0(mod\ 3)$ and $n \neq 3$, $d_G(\overline{C_n}) = \frac{n}{3}$, where $\overline{C_n}$ denote the complement of C_n.

Proof: Let $V(\overline{C_n}) = \{v_1, v_2, \ldots, v_{3k}\}$. By lemma (2.4), $\gamma_G(\overline{C_n}) = 3$. Therefore, $X = \{\{v_1, v_2, v_3\}, \{v_4, v_5, v_6\}, \ldots, \{v_{3k-2}, v_{3k-1}, v_{3k}\}\}$ is a maximum partition of $V(\overline{C_n})$ into (G, D)-sets.. Therefore, $d_G(\overline{C_n}) = |X| = \frac{3k}{3} = \frac{n}{3}$.

Proposition 2.6: If G is any connected graph which contains atleast one pendant vertex, then $d_G(G) = 1$.

Proof: Let $A \subset V(G)$ be a set of pendant vertices of G. Then, the set A must be contained in every (G, D)-set of G. Therefore, $V - S$ cannot contain any (G, D)-set of G. But, V is always a (G, D)-set of G. Therefore, $\{V\}$ forms a partition of $V(G)$. Hence, $d_G(G) = 1$.

Corollary 2.7: For any tree T, $d_G(T) = 1$.

Proof: Since any tree contains atleast two end vertices, the proof follows by proposition 2.6.

Theorem 2.8: If G is a graph which contains atleast one extreme vertex, then $d_G(G)$=1.

Proof: By theorem (1.1), every extreme vertex lie in every (G, D)-set. Therefore, the proof follows the lines of proposition 2.6.

Remark 2.9: (i) If G is a graph with an isolated vertex, then $d_G(G) = 1$.(ii) $d_G(P_n) = 1$ and (iii) $d_G(K_n) = 1 = d_G(\overline{K_n})$.

Theorem 2.10: For any graph $G, 1 \leq d_G(G) \leq \left\lfloor\frac{n}{2}\right\rfloor$.

Proof: Since the vertex set itself is a (G, D)-set, $\{V\}$ forms a partition of $V(G)$. Therefore, $1 \leq d_G(G)$. Since minimum value of $\gamma_G(G)$ is 2, the maximum partition of $V(G)$ contains $\left\lfloor\frac{n}{2}\right\rfloor$ elements. Therefore, $d_G(G) \leq \left\lfloor\frac{n}{2}\right\rfloor$.
Hence, $1 \leq d_G(G) \leq \left\lfloor\frac{n}{2}\right\rfloor$.

Remark 2.11: In the above inequality, the bounds are sharp for $d_G(K_n) = 1$ and $d_G(K_n - X) = \left\lfloor\frac{n}{2}\right\rfloor = \frac{n}{2}$, where $n \geq 4$ is even and X is a perfect matching of K_n.

Proposition 2.12: If G contains a dominating vertex, then $d_G(\bar{G}) = 1$.

Proof: Let v be a dominating vertex of G. Then, v is an isolated vertex in $\bar{G}$. So, v belongs to every (G, D)-set of $\bar{G}$. Thus, $\bar{G}$ has $\{V\}$ as its only G-domatic partition. Therefore, $d_G(\bar{G}) = 1$.

Proposition 2.13: $d_G(G_1 \cup G_2) = \min\{d_G(G_1), d_G(G_2)\}$ for any two graphs G_1 and G_2.

Proof: Let G_1, G_2 be two graphs with $d_G(G_1) = m$ and $d_G(G_2) = n$ with $m < n$. Let $D_1 = \{S_1, S_2, \ldots, S_m\}$ and $D_2 = \{S_1', S_2', \ldots, S_n'\}$ be maximum G-domatic partitions of G_1 and G_2 respectively. Then, $S_1 \cup S_1'$, $S_2 \cup S_2', \ldots, S_m \cup (S_m' \cup S_{m+1}' \cup \ldots \cup S_n')$ is obviously a partition of $V(G_1 \cup G_2)$.

Hence, $d_G(G_1 \cup G_2) \geq m ------(1)$

Further, it is obvious that corresponding to any partition of $V(G_1 \cup G_2)$ into (G, D)-sets of $G_1 \cup G_2$, there exist partitions of $V(G_1)$ and $V(G_2)$ into (G, D)-sets of G_1 and G_2 respectively and vice versa. Therefore,
$d_G(G_1 \cup G_2) \leq \min\{m, n\} \ = m ------(2)$

Hence, by (1) and (2), $d_G(G_1 \cup G_2) = m = min\{m, n\} = min\{d_G(G_1), d_G(G_2)\}$.

Remark 2.14: Let $G \cong G_1$, where G_1 is given in figure (2.3). Then, (v_1, v_6), (v_2, v_5), (v_3, v_8) and (v_4, v_7) are (G, D)-sets of G. Therefore, $d_G\ (G) = 4 = \delta(G) + 1$.

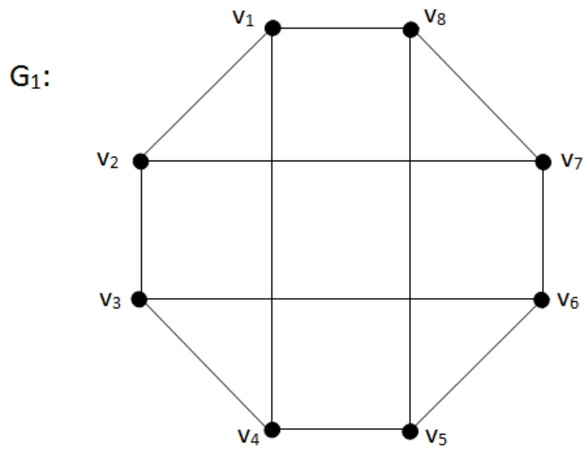

Figure-2.3

Proposition 2.15: For $n \equiv 0(mod\ 3)$ and $n \neq 3,\ d_G(C_n) = \delta(C_n) + 1$.

Proof: Let $V(C_n) = \{v_1, v_2, \dots, v_{3k}\}$. Then, the sets $A = \{v_1, v_4, v_7, \dots, v_{3k-2}\}, B = \{v_2, v_5, v_8, \dots, v_{3k-1}\}$ and $C = \{v_{3,} v_6, v_9, \dots, v_{3k}\}$ form a maximum partition of $V(G)$ into (G, D)-sets and so, $d_G(C_n) = 3 = \delta(C_n) + 1$.

REFERENCES

1. Buckley F, Harary F and Quintas V L, Extremal results on the geodetic number of a graph, Scientia, volume A2 (1988), 17-26.
2. Chartrand G, Harary F and Zhang P, Geodetic sets in graphs, Discussiones Mathematicae Graph theory, 20 (2000), 129-138e.
3. Chartrand G, Harary F and Zhang P, On the Geodetic number of a graph, Networks, Volume 39(1) (2002), 1-6.
4. Chartrand G, Zhang P and Harary F, Extremal problems in Geodetic graph Theory, Congressus Numerantium 131 (1998), 55-66.
5. Cockayne E J and Hedetniemi S T, Towards a theory of domination in graphs, Networks 7 (1977), 247-261.
6. Haynes T W, Hedetniemi S T and Slater P J, Fundamentals of Domination in Graphs, Marcel Dekker Inc., 1998.
7. Haynes T W, Hedetniemi S T and Slater P J, Domination in Graphs – Advanced Topics, Marcel Dekker Inc., 1998.
8. Palani K and Nagarajan A, (G, D)-Number of a graph, International Journal of Mathematics Research, Volume 3, Number 3 (2011), 285-299.
9. Palani K and Kalavathi S, (G,D) – Number of some special graphs, International Journal of Engineering and Mathematical Sciences, January-June 2014, Volume 5, Issue-1, pp.7-15, ISSN(Print) – 2319 – 4537, (Online) – 2319 – 4545.
10. Palani K , Nagarajan A and Mahadevan G, Results connecting domination, geodetic and (G, D) – number of graph, International Journal Of Combinatorial graph theory and applications, Volume 3, No.1, January – June (2010) (pp.51 -59).

Source of support: Nil, Conflict of interest: None Declared.