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GENERALIZED PAIRWISE STAR SEPARATION AXIOM IN BITOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce the notions of generalized pairwise star separation axioms on bitopological spaces and study some of their properties. The properties of the space (X,τ_1,τ_2) are studied through the study of the space (X,τ_{12}) which is a supra topology associated to the bitopological space (X,τ_1,τ_2). The importance of this approach is that we study properties of the bitopological space (X,τ_1,τ_2) via one family τ_{12} such that (X,τ_{12}) is a supra topological space associated to (X,τ_1,τ_2). Also, we are dealing with one family instead of two families τ_1 and τ_2. Finally this method enable us to study on X more that two topologies. The relation between these approaches has studied.

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Keywords*: Bitopological space, Supra topology, generalized pairwise star T_0 space, generalized pairwise star T_1 space, generalized pairwise star T_2 space, generalized pairwise star R_0 space generalized pairwise star R_1 space generalized pairwise star regular space and generalized pairwise star normal space, .*

1. INTRODUCTION

In 1963, Kelly was first introduced the concept of bitopological space, where is a non-empty set and τ_1,τ_2 are topologies on X as method of generalizes topological space (X,τ) [1]. Every bitopological space (X,τ_1,τ_2) can be regarded as a topological space (X,τ) if $\tau_1=\tau_2=\tau$. Furthermore, he extended some of the standard results of separation axioms and mappings in a topological space to a bitopological space. In 1983 Mashhor *et al.* [2], [3] introduced supra topological spaces by dropping only the intersection condition. Kandil,[4] generated a supra topological space (X,τ_{12}) from the bitopological space (X,τ_1,τ_2) and they studied some properties of the space (X,τ_1,τ_2) via properties of the associated space (X,τ_{12}). Thereafter, a large number of papers have been written in separation axiom on bitopological space. It has been studied in different versions adopted in the study of the concept of Fukutake by using open and closed collections in the two spaces τ_1,τ_2 or using the new space (X,τ_{12}) resulting from them. [5, 6, 7, 8, 9].

In this paper we introduce the notion of generalized pairwise star T_0 space, generalized pairwise starT_1 space, generalized pairwise star T_2 space, generalized pairwise star R_0 space, generalized pairwise star R_1space generalized pairwise star regular space and generalized pairwise star normal space by using the concept generalized pairwise open sets and generalized pairwise closed sets in (X,τ_{12}) and we study some of relation between this spaces.

2.PRELIMINARIES

This section contains the basic concept and the proprieties of generalized closed sets, generalized open set and pairwise separation axiom in bitopological spaces.

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Definition 2.1: [1] *A bitopological spaces (bts, for short) is a triple* (X,τ_1,τ_2) *where* τ_1 *and* τ_2 *are arbitrary topologies on* X.

Definition 2.2: [10] *Let* (X,τ_1,τ_2) *be a bitopological space. Then,* $A\subseteq X$ *is said to be pairwise open (P-open, for short) if* $A=U_1\cup U_2$, $U_i\in\tau_i$, $(i=1,2)$. *A set* A *is a P-closed if its complement* A^c *is P-open.*

Note that the notion of P-open sets as well as P-closed sets has studied in [11, 4] under the name of P^*-open and P^*-closed.

Definition 2.3: [3] *A family* $\eta\subseteq P(X)$ *is said to be a supra topology on* X, *if* η *contains* X,ϕ *and closed under arbitrary union. The element of* η *are supra open sets and their complements are said to be supra closed sets.*

Proposition 2.1: [11] *Let* (X,τ_1,τ_2) *be a bts. The family of all P-open subsets of* X, *denoted by* $\tau_{12}=\{U_1\cup U_2: U_i\in\tau_i, i=1,2\}$ is a supratopology on X and (X,τ_{12}) is the supra topological space associated to the bts (X,τ_1,τ_2).

Proposition 2.2: [11] *Let* (X,τ_1,τ_2) *be a bts. Then, the operator* $cl_{12}: P(X)\rightarrow P(X)$ *defined by* $cl_{12}(A)=\overline{A}^1\cap\overline{A}^2$, *is a supra closure operator such that,* $\tau_{12}=\{A\subseteq X: cl_{12}(A^c)=A^c\}$ *where* $\overline{A}^i$ *is the closure of* A *with respect to* τ_i *and* $i=1,2$.

Proposition 2.3: [11] *Let* (X,τ_1,τ_2) *be a bts. Then the operator,*$int_{12}: P(X)\rightarrow P(X)$ *defined by,* $int_{12}(A)=A^{o1}\cup A^{o2}$ *is a supra interior operator such that* $\tau_{12}=\{A\subseteq X: int_{12}(A)=A\}$, *where* A^{oi}, $(i=1,2)$ *is the* τ_i*- interior with respect to* τ_i.

Definition 2.4: [12] *Let* (X,τ_1,τ_2) *be a* bts, $A\subseteq X$ *and* $(A,\tau_1|_A,\tau_2|_A)$ *be a bitopological subspace of* (X,τ_1,τ_2). *Then,* $\tau_{12}(A)=\tau_1|_A\cup\tau_2|_A=\{U_1\cup U_2: U_1\in\tau_1|_A, U_2\in\tau_2|_A\}$ *such that* $\tau_i|_A=\{A\cap U_i: U_i\in\tau_i, i=1,2\}$.

Theorem 2.1: [12] *Let* (X,τ_1,τ_2) *be a* bts *and* $A\subseteq X$. *Then,* $\tau_{12}|_A=\tau_{12}(A)$, *where* $\tau_{12}|_A=\{A\cap U: U\in\tau_{12}\}$.

Definition 2.5: [12] *Let* (X,τ_1,τ_2) *be a bts and* (X,τ_{12}) *be its associated supra topological space. Then,* $A\subseteq X$ *is called a generalized pairwise closed set (gp-closed, for short) if,* $cl_{12}(A)\subseteq O$ *whenever* $A\subseteq O$, □ *is a P-open. The set of all generalized pairwise closed sets denoted by* $GPC(X,\tau_{12})$.

Remark 2.1: [12]

1. If A is a P-closed set, then A is a gp-closed set.
2. If $A$and B are gp-closed sets, then $A\cup B$ and $A\cap B$ are not necessary gp-closed sets.

Theorem 2.2: [12] *Let* (X,τ_1,τ_2) *be a bts and* $A\subseteq X$. *If* A *is a* gp *closed set and* $A\subseteq B\subseteq cl_{12}(A)$. *Then,* B*is a gp-closed set.*

Theorem 2.3: [12] *Let* (X,τ_1,τ_2) *be a bts,* τ_{12} *be supra topology on* X *induced by* τ_1,τ_2. *Then,* $\tau_{12}=\tau_{12}^c$ *if and only if every subset of* X *is a gp- closed set.*

Theorem 2.4: [12] *Let* (X,τ_1,τ_2) *be a* bts. *Suppose that* $B\subseteq A\subseteq X$, B *is a gp-closed set relative to* A *and* A *is a gp-closed set of* X. *Then,* B *is a gp-closed set relative to* X.

Corollary 2.1: [12] *If* A *is a gp-closed set and* F *is P-closed set, then* $A\cap F$ *is a gp-closed set.*

Theorem 2.5: [12] *Let* (X,τ_1,τ_2) *be a bts and* $A\subseteq Y\subseteq X$. *Suppose that* A *is gp-closed in* X. *Then,* A *is gp-closed relative to* Y.

Definition 2.6: [10] *Let* (X,τ_1,τ_2) *be a bts and* $A,B\subseteq X$. *Then* A *and* B *are* P^*- *separated in* X *if* $A\cap cl_{12}(B)=\phi$ *and* $cl_{12}(A)\cap B=\phi$.

Not that if A and B are P^*-separated and $C\subseteq A, D\subseteq B$, then $CandD$ are P^*- separated.

Theorem 2.6: [12] *Let* (X,τ_1,τ_2) *be a bts. If* A *and* B *are* P^**-separated and gp-open sets, then* $A\cup B$ *is a gp-open set.*

Definition 2.7: [12] *A set* A *is called a generalized pairwise open (for short, gp-open) set if and only if* A^c *is a gp-closed set. The set of all generalized closed set is denoted by* $GPO(X,\tau_{12})$.

Theorem 2.7: [12] *A space (X,τ_1,τ_2) is a P^*-symmetric if and only if $\{x\}$ is a gp- closed $\forall x\in X$.*

Theorem 2.8: [12] *If A is a gp-closed set in X and $f:X\to Y$ is P^*-cts and is P^*-closed function, then $f(A)$ is a gp-closed set.*

Theorem 2.9: [12] *If $f:(X,\tau_1,\tau_2)\to(Y,\upsilon_1,\upsilon_2)$ is a P^*-cts and P^*-closed function, B is a gp-closed(or gp-open) subset of Y, then $f^{-1}(B)$ is gp-closed (or gp-open) set in X.*

Definition 2.8: [10] *Let (X,τ_1,τ_2) be a bitopological space. X is called a P^*T_0-space (resp. P^*T_1-space, P^*T_2-space, P^*T_3-space, P^*T_4-space,P^*R_0-space, P^*R_1-space) if X satisfies the following P^*T_0-separation (resp. P^*T_1-separation, P^*T_2-separation, P^*T_3-separation, P^*T_4-separation, P^*R_0- separation, P^*R_1-separation) condition.*

1. P^*T_0 -separation: If $x,y\in X$ and $x\neq y$, then $\exists U\in\tau_{12}$ such that either $U\cap\{x,y\}=\{x\}$or$U\cap\{x,y\}=\{y\}$.
2. P^*T_1 -separation: If $x,y\in X$ and $x\neq y$, then there are $U_x,U_y\in\tau_{12}$ such that $U_x\cap\{x,y\}=\{x\}$ and $U_y\cap\{x,y\}=\{y\}$.
3. P^*T_2-separation: If $x,y\in X$and $x\neq y$, then $\exists U_x\in\tau_{12}$ and $U_y\in\tau_{12}$ such that $U_x\cap U_y=\phi$.
4. P^*regular: If $x\notin F$ and F is P-closed, then $\exists U_x\in\tau_{12}$ and $U_F\in\tau_{12}$ such that $F\subseteq U_F$ and $U_x\cap U_F=\phi$. (X,τ_1,τ_2) is P^*T_3-separation if it is P^*T_1 and P^* regular.
5. P^* normal: If F_1 and F_2 are P-closed and $F_1\cap F_2=\phi$, then $\exists U_1,U_2\in\tau_{12}$ such that $F_1\subseteq U_1$ and $F_2\subseteq U_2$ such that $U_1\cap U_2=\phi$. (X,τ_1,τ_2) is P^*T_4-separation if it is P^*T_1 and P^*normal.
6. P^*R_0-separation: If $x\in U\in\tau_{12}$, then $cl_{12}(\{x\})\subseteq U$.
7. P^*R_1 -separation: If $x\neq y$, and $cl_{12}\{x\}\neq cl_{12}\{y\}$ then $\exists U_1,U_2\in\tau_{12}$ such that, $cl_{12}(\{x\})\subseteq U_1$ and $cl_{12}(\{y\})\subseteq U_2$.

3. GENERALIZED PAIRWISE STAR SEPARATION AXIOMS

In this section we defined the concepts of generalized pairwise star T_0 space, generalized pairwise star T_1 space, generalized pairwise star T_2 space, generalized pairwise star T_3, generalized pairwise star T_4, generalized pairwise star R_0 space and generalized pairwise star R_1 space, we also study some of their properties.

Definition 3.1: *Let (X,τ_1,τ_2) be a bts and (X,τ_{12}) be its associated supra topological space. Then X is called a generalized pairwise star T_0 (gp^*T_0, for short) space if $\forall x,y\in X$, $x\neq y\exists$ gp -open set A such that $x\in A,y\notin A$. In another word for every two different elements in X, there exist gp- open set containing one of them but not the other.*

Theorem 3.1: *Let (X,τ_1,τ_2) be a bitopological space, (X,τ_{12}) its associated supra topological spaces. If (X,τ_{12}) is a gp^*T_0 space and A is P- closed set in (X,τ_{12}), then $(A,\tau_{12}|_A)$ is a gp^*T_0 space.*

Proof: Let (X,τ_{12}) be a gp^*T_0 space and A be a P- closed set in (X,τ_{12}). Suppose that $x,y\in A,x\neq y$. Thus, $x,y\in X$ but, (X,τ_{12}) is a gp^*T_0 this implies that, $\exists G\in GPO(X,\tau_{12})$ such that $x\in G,y\notin G$. Therefore, $G^c\in GPC(X,\tau_{12})$. By using corollary2.1 we get, $G^c\cap A\in GPC(X,\tau_{12})$. Also we have, $G^c\cap A\subseteq A\subseteq X$. Then, $A\cap G^c\in GPC(A,\tau_{12}|_A)$ by Theorem2.5. This implies that, $A\cap G\in GPO(A,\tau_{12}|_A)$ and $x\in A\cap G,y\notin A\cap G$. Hence,$(A,\tau_{12}|_A)$ is a gp^*T_0 space.

Theorem 3.2: *Let (X,τ_1,τ_2) and (Y,γ_1,γ_2) be a bitopological spaces, (X,τ_{12}) and (Y,γ_{12}) are associated supra topological spaces. Suppose that $f:X\to Y$ is an injective, P^* continuous function and P^*-closed, if (Y,γ_1,γ_2) is a gp^*T_0 space, then (X,τ_1,τ_2) is gp^*T_0 space.*

Proof: Let (X,τ_1,τ_2) and (Y,γ_1,γ_2) be a bitopological spaces, (X,τ_{12}) and (Y,γ_{12}) are associated supra topological spaces. Suppose that $f:X\to Y$ is a injective, P^* continuous function and P^*-closed, (Y,γ_{12}) is gp^*T_0 space.Let $x_1,x_2\in X$ and $x_1\neq x_2$ but we have f is injective, this implies that $f(x_1)\neq f(x_2)$. But, (Y,γ_{12}) is gp^*T_0. This implies that, $\exists$ $gp-$open set V such that, $f(x_1)\in V,f(x_2)\notin V$. Therefore, $x_1=f^{-1}f(x_1)\in f^{-1}(V),x_2\notin f^{-1}(V)$. By using Theorem2.9 we get, $f^{-1}(V)$ is $gp-$open sets. Hence, (X,τ_1,τ_2) is gp^*T_0 space.

Remark 3.1: *Every P^*T_0 space is gp^*T_0 space but the converse is not true. Example3.1 explain that.*

Example 3.1: *Let $X=\{a,b,c,d\}$, $\tau_1=\{X,\phi,\{a,b\}\}$, $\tau_2=\{X,\phi,\{c,d\}\}$. Then, $\tau_{12}=\{X,\phi,\{a,b\}\},\{c,d\}\}$. Then (X,τ_{12}) is gp^*T_0 space but not P^*T_0. Since, the set of all generalized open set $(GPO(X,\tau_{12}))=P(X)$.*

Corollary 3.1: *If (X,τ_1) or (X,τ_2) is T_0 space, then (X,τ_{12}) is gp^*T_0.*

Proof: The prove is trivially, since $\tau_1\subseteq\tau_{12}$ and $\tau_2\subseteq\tau_{12}$.

Definition 3.2: *Let (X, τ_1, τ_2) be a bts and (X, τ_{12}) be its associated supra topological space. Then X is called a generalized pairwise star T_1 (gp^*T_1, for short) space if $\forall x, y \in X,\ x \neq y$, then $\exists\ gp$ -open sets A and B such that $x \in A, y \notin A,\ y \in B$ and $x \notin B$. In other word, for every two distant elements in X, there exist two gp- open sets containing one of them and not either.*

Theorem 3.3: *Let (X, τ_1, τ_2) be a bitopological space, (X, τ_{12}) its associated supra topological space. If (X, τ_{12}) is a gp^*T_1 space and A is a P- closed set in (X, τ_{12}), then $(A, \tau_{12}|_A)$ is a gp^*T_1 space.*

Proof: Let (X, τ_{12}) be a gp^*T_1 space and A be a P- closed set in (X, τ_{12}). Suppose that $x, y \in A, x \neq y$. Thus, $x, y \in X$ but, (X, τ_{12}) is a gp^*T_1, this implies that, $\exists G, H \in GPO(X, \tau_{12})$ such that $x \in G, y \notin G$ and $y \in H, x \notin H$. Therefore, $G^c, H^c \in GPC(X, \tau_{12})$. By using corollary2.1 we get, $G^c \cap A, H^c \cap A \in GPC(X, \tau_{12})$. Also we have, $G^c \cap A \subseteq A \subseteq X$ and $H^c \cap A \subseteq A \subseteq X$. Then, $A \cap G^c,\ H^c \cap A \in GPC(A, \tau_{12}|_A)$ by Theorem2.5. This implies that, $A \cap G, A \cap H \in GPO(A, \tau_{12}|_A)$ so, $x \in A \cap G,\ y \notin A \cap G$ and $y \in A \cap H,\ x \notin A \cap H$. Hence,$(A, \tau_{12}|_A)$ is a gp^*T_1 space.

Theorem 3.4: *Let (X, τ_1, τ_2) and (Y, γ_1, γ_2) be bitopological spaces, (X, τ_{12}) and (Y, γ_{12}) are associated supra topological spaces. Suppose that $f: X \to Y$ is an injective, P^* continuous function and P^* −closed. If (Y, γ_1, γ_2) is gp^*T_1 space, then (X, τ_1, τ_2) is gp^*T_1 space.*

Proof: Let (X, τ_1, τ_2) and (Y, γ_1, γ_2) be bitopological spaces, (X, τ_{12}) and (Y, γ_{12}) are associated supra topological spaces. Suppose that $f: X \to Y$ is an injective, P^* continuous function and P^* −closed, (Y, γ_{12}) is gp^*T_1 space. Let $x_1, x_2 \in X$ and $x_1 \neq x_2$ but we have f is injective, this implies that $f(x_1) \neq f(x_2)$. But, (Y, γ_{12}) is gp^*T_1. This implies that, $\exists\ gp$ − open sets V_1, V_2 such that, $f(x_1) \in V_1, f(x_2) \notin V_1$ and $f(x_2) \in V_2, f(x_1) \notin V_2$. Therefore, $x_1 = f^{-1}f(x_1) \in f^{-1}(V_1), x_2 \notin V_1$ and $x_2 = f^{-1}f(x_2) \in f^{-1}(V_2), x_1 \notin V_2$. By using Theorem2.9 we get, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are gp −open sets. Hence, (X, τ_1, τ_2) is gp^*T_1 space.

Remark 3.2:
1. Every P^*T_1 space is gp^*T_1 space but the converse is not true. See Example3.1.
2. Every gp^*T_1 space is gp^*T_0 space but the converse is not true. Example3.2 explain that.

Example 3.2: Let $X = \{1,2\}$, $\tau_1 = \{X, \phi\}$, $\tau_2 = \{X, \phi, \{1\}\}$. Then, $\tau_{12} = \{X, \phi, \{1\}\}$. Then (X, τ_{12}) is gp^*T_0 space but not gp^*T_1. Since, the set of all generalized pairwise open set $(GPO(X, \tau_{12})) = \{X, \phi, \{1\}\}$.

Corollary 3.2: *If (X, τ_1) or (X, τ_2) is T_1 space, then (X, τ_{12}) is gp^*T_1.*

Proof: The prove is trivially, since $\tau_1, \tau_2 \subseteq \tau_{12}$.

Theorem 3.5: *Every singleton set is gp −closed in gp^*T_1 space.*

Proof: Let (X, τ_1, τ_2) be a gp^*T_1 space. To prove that $\{x\}$ is gp −closed set. Let $\{x\} \subseteq O$,O is P-open set and we must prove that $cl_{12}\{x\} \subseteq O$. Let $y \notin O$, then $y \neq x$ and therefore $\exists\ gp$ −open set U such that, $y \in U$ and $x \notin U$. Now, $x \in U^c$ and U^c is a gp −closed, then $cl_{12}(U^c) \subseteq G$ whenever, $U^c \subseteq G,\ G$ is P-open. it follows that $cl_{12}\{x\} \subseteq G$, whenever $\{x\} \subseteq G,\ G$ is P-open. Hence, $\{x\}$ is gp −closed.

Definition 3.3: *Let (X, τ_1, τ_2) be a bts and (X, τ_{12}) be its associated supra topological space. Then X is called a generalized pairwise star T_2 (gp^*T_2, for short)space if $\forall x, y \in X$ and $x \neq y$, $\exists$ disjoint gp -open setsA and B such that $x \in A$ and $y \in B$. In other word for every two distinct elements in X, there exist two disjoint $gp-$ open sets containing one of them and not either.*

Theorem 3.6: *Let (X, τ_1, τ_2) be a bitopological space, (X, τ_{12}) its associated supra topological spaces. If (X, τ_{12}) is a gp^*T_2 space and A is a P- closed set in (X, τ_{12}), then $(A, \tau_{12}|_A)$ is a gp^*T_2 space.*

Proof: Let (X, τ_{12}) be a gp^*T_2 space and A be a P- closed set in (X, τ_{12}). Suppose that $x, y \in A, x \neq y$. Thus, $x, y \in X$ but, (X, τ_{12}) is a gp^*T_2, this implies that $\exists G, H \in GPO(X, \tau_{12})$ such that $x \in G,\ y \in H$ and $G \cap H = \phi$. Therefore, $G^c, H^c \in GPC(X, \tau_{12})$. By using corollary2.1 we get, $G^c \cap A, H^c \cap A \in GPC(X, \tau_{12})$. Also we have, $G^c \cap A \subseteq A \subseteq X$ and $H^c \cap A \subseteq A \subseteq X$. Then, $A \cap G^c, H^c \cap A \in GPC(A, \tau_{12}|_A)$ by Theorem2.5. This implies that, $A \cap G, A \cap H \in GPO(A, \tau_{12}|_A)$, $x \in A \cap G$ and $y \in A \cap H$. Also, $(A \cap G) \cap (A \cap H = \phi)$. Hence, $(A, \tau_{12}|_A)$ is a gp^*T_2 space.

Theorem 3.7: *Let* (X,τ_1,τ_2) *and* (Y,γ_1,γ_2) *be a bitopological spaces,* (X,τ_{12}) *and* (Y,γ_{12}) *are associated supra topological spaces. Suppose that* $f: X\to Y$ *is an injective,* P^* *continuous function and* P^*-*closed. If* (Y,γ_1,γ_2) *is* gp^*T_2 *space, then* (X,τ_1,τ_2) *is* gp^*T_2 *space.*

Proof: Let (X,τ_1,τ_2) and (Y,γ_1,γ_2) be a bitopological spaces, (X,τ_{12}) and (Y,γ_{12}) are associated supra topological spaces. Suppose that $f: X\to Y$ is an injective, P^* continuous function and P^*-closed, (Y,γ_{12}) is $gp-T_2$ space. Let $x_1,x_2\in X$ and $x_1\neq x_2$, but we have f is injective, this implies that $f(x_1)\neq f(x_2)$. But, (Y,γ_{12}) is $gp-T_2$. This implies that, $\exists$ disjoint $gp-$open sets V_1,V_2 such that, $f(x_1)\in V_1$ and $f(x_2)\in V_2$. Therefore, $x_1=f^{-1}f(x_1)\in f^{-1}(V_1)$ and $x_2=f^{-1}f(x_2)\in f^{-1}(V_2)$. By using Theorem2.9 we get, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are $gp-$open sets, also $f^{-1}(V_1),f^{-1}(V_2)$ are disjoint sets. Hence, (X,τ_1,τ_2) is gp^*T_2 space.

Remark 3.3:
1. Every P^*T_2 space is gp^*T_2 space but the converse is not true. See Example3.1.
2. Every gp^*T_2 space is gp^*T_1 space but the converse is not true. Example3.3 explain that.

Example 3.3: *Let* $X=\{1,2,3,4\}$, $\tau_1=\{X,\phi,\{1,2,3\},\{2,3,4\},\{2,3\}\}$, $\tau_2=\{X,\phi,\{1,3,4\},\{1,2,4\},\{1,4\}\}$
Then, $\tau_{12}=\{X,\phi,\{1,2,3\}\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,4\},\{2,3\}$. Then (X,τ_1,τ_2) is gp^*T_1 space. Since,
$GPO(X,\tau_{12})=\tau_{12}$ and

$1\neq 2$ we have $\{1,3,4\}$and $\{2,3,4\}$ are $gp-$ open sets.
$1\neq 3$ we have $\{1,4\}$and $\{3,4\}$ are $gp-$ open sets.
$1\neq 4$ we have $\{1,2,3\}$and $\{2,3,4\}$ are $gp-$ open sets.
$2\neq 3$ we have $\{1,2,4\}$and $\{1,3,4\}$ are $gp-$ open sets.
$2\neq 4$ we have $\{2,3\}$and $\{1,4\}$ are $gp-$ open sets.
$3\neq 4$ we have $\{2,3\}$and $\{1,4\}$ are $gp-$ open sets.

But (X,τ_1,τ_2) is not gp^*T_2 since, $1\neq 4$ and all $gp-$open containing 1 intersect all $gp-$open sets containing 4.

Remark 3.4: *For a bts* (X,τ_1,τ_2) *we have the following chart.*

$$\begin{array}{ccccc} P^*T_2 & \to & P^*T_1 & \to & P^*T_0 \\ \downarrow & & \downarrow & & \downarrow \\ gp^*T_2 & \to & gp^*T_1 & \to & gp^*T_0 \end{array}$$

Definition 3.4: *Let* (X,τ_1,τ_2) *be a bts and* (X,τ_{12}) *its associated supra topological space. Then* X *is called a generalized pairwise star regular (*gp^* *regular, for short) space if for each* $gp-$*closed set* F *and* $x\notin F$*, there exist disjoint gp-open sets* G *and* H *such that,* $x\in H$ *and* $F\subseteq G$*.*

Theorem 3.8: *Let* (X,τ_1,τ_2) *be a bitopological space,* (X,τ_{12}) *its associated supra topological spaces. If* (X,τ_{12}) *is a* gp^**regular space and* A *is P- closed set in* (X,τ_{12})*, then* $(A,\tau_{12}|_A)$ *is a* gp^* *regular space.*

Proof: Let (X,τ_{12}) be a gp^* regular space and A be a P- closed set in (X,τ_{12}). Suppose that $x\in A, x\notin F$ and $F\in GPC(A,\tau_{12}|_A)$. Thus, $x\in X$ and $F\in GPC(X,\tau_{12})$. But, (X,τ_{12}) is a gp^* regular space this implies that, $\exists G,H\in GPO(X,\tau_{12})$ such that $x\in G$, $F\subseteq H$ and $G\cap H=\phi$. Therefore, $G^c,H^c\in GPC(X,\tau_{12})$. By using corollary2.1 we get, $G^c\cap A, H^c\cap A\in GPC(X,\tau_{12})$. Also we have, $G^c\cap A\subseteq A\subseteq X$ and $H^c\cap A\subseteq A\subseteq X$. Then, $A\cap G^c, H^c\cap A\in GPC(A,\tau_{12}|_A)$ by Theorem2.5. This implies that, $A\cap G, A\cap H\in GPO(A,\tau_{12}|_A)$, $x\in A\cap G$ and $F\subseteq A\cap H$ and therefore $(A\cap G)\cap(A\cap H)=\phi$. Hence,$(A,\tau_{12}|_A)$ is a gp^* regular space.

Definition 3.5: *Let* (X,τ_1,τ_2) *be a bts and* (X,τ_{12}) *be its associated supra topological space. Then a*$gp^*-regular$ gp^*T_1 *space is called a generalized pairwise star* T_3 *(*gp^*T_3*, for short) space.*

Theorem 3.9: *Every* gp^*T_3 *space is* gp^*T_2 *space.*

Proof: Let (X,τ_{12}) be a gp^*T_3 and $a,b\in X$ such that $a\neq b$. Then, $\{b\}$ is $gp-$closed set since, (X,τ_{12}) is gp^*T_1. This implies that, $a\notin\{b\}$. But, (X,τ_{12}) is gp^*regular space. Therefore, their exists disjoint $gp-$open sets G,H such that, $a\in G,\{b\}\subseteq H$. Hence, (X,τ_{12}) is gp^*T_2 space.

Definition 3.6: *Let* (X,τ_1,τ_2) *be a bitopological space and* (X,τ_{12}) *be its associated supra topological space. Then* X *is called a generalized pairwise star normal (*gp^* *normal, for short) space if for all disjoint* $gp-$*closed sets* F *and* U*, their exists disjoint gp-open sets* G *and* H *such that,* $U\subseteq H$ *and* $F\subseteq G$*.*

Theorem 3.10: *Let (X,τ_1,τ_2) be a bitopological space, (X,τ_{12}) is an associated supra topological spaces. If (X,τ_{12}) is a gp^*normal space and A is P- closed set in (X,τ_{12}), then $(A,\tau_{12}|_A)$ is a gp^*normal space.*

Proof: Let (X,τ_{12}) be a gp^* normal space and A be a P- closed set in (X,τ_{12}). Suppose that $F,V \in GPC(A,\tau_{12}|_A)$ and $F\cap V=\phi$. Thus, $F,V\in GPC(X,\tau_{12})$. But, (X,τ_{12}) is a $gp-$ normal space, this implies that $\exists G,H\in GPO(X,\tau_{12})$ such that $F\subseteq G$, $V\subseteq H$ and $G\cap H=\phi$. Therefore, $G^c,H^c\in GPC(X,\tau_{12})$. By using corollary 2.1 we get, $G^c\cap A, H^c\cap A\in GPC(X,\tau_{12})$. Also we have, $G^c\cap A\subseteq A\subseteq X$ and $H^c\cap A\subseteq A\subseteq X$. Then, $A\cap G^c, H^c\cap A\in GPC(A,\tau_{12}|_A)$ by Theorem2.5. This implies that, $A\cap G, A\cap H\in GPO(A,\tau_{12}|_A)$, $F\subseteq A\cap G$ and $V\subseteq A\cap H$ also $(A\cap G)\cap(A\cap H)=\phi$. Hence,$(A,\tau_{12}|_A)$ is a gp^* normal space.

Theorem 3.11: *If (X,τ_1,τ_2) is gp^* normal, then (X,τ_1,τ_2) is P^*normal.*

Proof: Let (X,τ_1,τ_2) be a gp^* normal, F,H be disjoint P-closed set in (X,τ_1,τ_2). Then F,H are disjoint $gp-$closed sets. But, (X,τ_1,τ_2) is gp^* normal. This implies that, there exist disjoint $gp-$open sets U,V such that $F\subseteq U$ and $H\subseteq V$. But F,H are P-closed set and U,V $gp-$open sets. Then, $F\subseteq int_{12}(U)$ and $H\subseteq int_{12}(V)$. Also, $int_{12}(U)\cap int_{12}(V)=\phi$ and $int_{12}(U), int_{12}(V)\in\tau_{12}$. Hence,$(X,\tau_1,\tau_2)$ is P^*normal.

Remark 3.5: *A gp^* normal space is not gp^* regular space in general, Examples 3.4 explain that.*

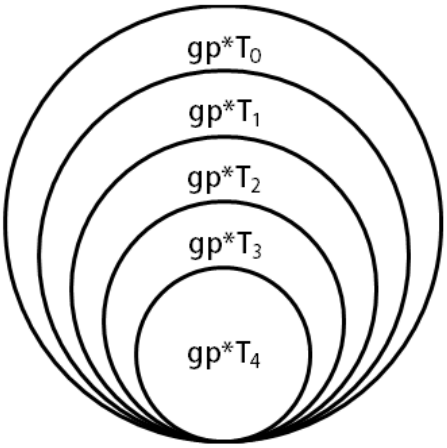
Example 3.4: *Let $X=\{a,b,c,d\}$, $\tau_1=\{X,\phi,\{a,b\},\{a,b,c\}\}$, $\tau_2=\{X,\phi,\{c\}\}$. Then $\tau_{12}=\{X,\phi,\{a,b\},\{c\},\{a,b,c\}\}$ and $\tau_{12}^c=\{X,\phi,\{c,d\},\{d\},\{a,b,d\}\}$. $GPC(X,\tau_{12})=\{X,\phi,\{c,d\},\{d\},\{a,d\},\{b,d\},\{a,b,d\},\{b,c,d$*and
$GPO(X,\tau_{12})=\{X,\phi,\{a,b\},\{a,b,c\},\{b,c\},\{a,c\},\{c\},\{a\}\}$ (X,τ_{12}) is gp^*normal space but it is not gp^* regular space. Since, $a\notin\{d\}$ and there is no disjoint $gp-$open set cantoning a and $\{d\}$.

Definition 3.8: *Let (X,τ_1,τ_2) be a bts and (X,τ_{12}) be its associated supra topological space. Then a $gp^*-normal$ gp^*T_1 space is called a generalized pairwise star T_4 (gp^*T_4, for short)*

Theorem 3.12: *Every gp^*T_4 space is gp^*T_3 space.*

Proof: Let (X,τ_{12}) be a gp^*T_4 and $a\notin F$ such that F is $gp-$closed. Then, $\{a\}$ is $gp-$closed set since, (X,τ_{12}) is gp^*T_1. This implies that, $\{a\}\cap F=\phi$. But, (X,τ_{12}) is gp^*normal space. Therefore, their exists disjoint $gp-$open sets G,H such that, $\{a\}\in G, F\subseteq H$. Hence, (X,τ_{12}) is gp^*T_3 space.

Remark 3.5: *For a bitopological space (X,τ_1,τ_2) we have the following chart.*



Definition 3.8: *Let (X,τ_1,τ_2) be a bts and (X,τ_{12}) be its associated supra topological space. Then X is called a generalized pairwise star R_0 space (gp^*R_0, for short) if $\forall x\in G$, G is gp-open set, then $cl_{12}\{x\}\subseteq G$.*

Theorem 3.13: *Let (X,τ_1,τ_2) be a bitopological space, (X,τ_{12}) its associated supra topological spaces. If (X,τ_{12}) is a gp^*R_0 space and A is P- closed set in (X,τ_{12}), then $(A,\tau_{12}|_A)$ is a gp^*R_0 space.*

Proof: Let (X,τ_{12}) be a gp^*T_2 space and A be a P- closed set in (X,τ_{12}). Suppose that $x\in G$, $G\in GPO(A,\tau_{12}|_A)$. Then $A\backslash G\in GPC(A,\tau_{12}|_A)$. So, $A\backslash G=A\cap(X\backslash G)$ and $X\backslash G\in GPC(X,\tau_{12})$. This implies that, $X\backslash A\cup G\in GPO(X,\tau_{12})$ and $x\in X\backslash A\cup G$ but, X is gp^*R_0. Then, $cl_{12}\{x\}\subseteq X\backslash A\cup G$. Therefore, $A\cap cl_{12}\{x\}\subseteq A\cap G=G$ consequently, $cl_{12}|_A\{x\}\subseteq G$. Hence, $(A,\tau_{12}|_A)$ is gp^*R_0 space.

Theorem 3.14: *Let (X,τ_1,τ_2) and (Y,γ_1,γ_2) be bitopological spaces, (X,τ_{12}) and (Y,γ_{12}) are associated supra topological spaces. Suppose that $f: X \to Y$ is an injective, P^* continuous function and P^* −closed, if (Y,γ_{12}) is gp^*R_0 space, then (X,τ_{12}) is gp^*R_0 space.*

Proof: Let (X,τ_1,τ_2) and (Y,γ_1,γ_2) be bitopological spaces, (X,τ_{12}) and (Y,γ_{12}) are associated supra topological spaces. Suppose that $f: X \to Y$ is an injective, P^* continuous and P^* −closed, (Y,γ_{12}) is gp^*R_0 space. Let $x \in G$ and G be gp −open set in τ_{12}. Then,G^c be gp −closed,Theorem 2.8, $f(G^c) = (f(G))^c$ is gp −closed and hence, $f(G)$ is $gp-$ open and $f(x) \in f(G)$. We have (Y,γ_{12}) is $gp - R_0$. Thus, $cl_{12}|_Y(\{f(x)\}) \subseteq f(G)$. Then, $f^{-1}cl_{12}|_Y(\{f(x)\}) \subseteq f^{-1}f(G) = G$. Then, $f^{-1}cl_{12}|_Y(\{f(x))\}$ is P-closed and containing x since, f is cts^*. Therefore, $f^{-1}cl_{12}|_Y(f(x)) = cl_{12}|_X f^{-1}f(x) = cl_{12}|_X(x)$. Hence, $cl_{12}|_X(\{x\}) \subseteq G$ which means that, (X,τ_1,τ_2) is gp^*R_0.

Theorem 3.15: *Let (X,τ_1,τ_2) is gp^*R_0 space. For any $F \in GPC(X,\tau_{12})$, $x \notin F$, then $F \subseteq U$ and $x \notin U$ for some $U \in GPO(X,\tau_{12})$.*

Proof: Let (X,τ_1,τ_2) be gp^*R_0 space. Suppose that $F \in GPC(X,\tau_{12})$, $x \notin F$, then $cl_{12}\{x\} \subseteq F^c$. This implies that, $F \subseteq [cl_{12}\{x\}]^c$. Take $U = [cl_{12}\{x\}]^c$ which is $gp-$ open set and containing x.

Theorem 3.16: *If $F \cap cl_{12}[\{x\}] = \phi$ $\forall x \notin F$ and $F \in GPC(X,\tau_{12})$, then $(X,\tau_1,\tau_2) gp^*R_0$ space.*

Proof: Let $x \in G$ and $G \in GPO(X,\tau_{12})$. Then, $x \notin G^c$ and $G^c \in GPC(X,\tau_{12})$. This implies that, $cl_{12}\{x\} \cap G^c = \phi$. Thus, $cl_{12}\{x\} \subseteq G$. Hence, (x,τ_1,τ_2) is gp^*R_0 space.

Theorem 3.17: *If (X,τ_1,τ_2) is gp^*R_0 space and gp^*T_0 space then, it is a gp^*T_1 space.*

Proof: Let (X,τ_1,τ_2) be gp^*R_0 space and gp^*T_0 space. suppose that $x,y \in X$ and $x \neq y$. Then $\exists G \in GPO(X,\tau_{12})$ such that $x \in G, y \notin G$. But we have, (X,τ_1,τ_2) is gp^*R_0, this implies that, $cl_{12}(\{x\}) \subseteq G$. Therefore, $G^c \subseteq [cl_{12}(\{x\})]^c$ and $y \in [cl_{12}(\{x\})]^c$. But $[cl_{12}(\{x\})]^c$ is gp −open set containing y and not containing x. Hence, (X,τ_1,τ_2) is gp^*T_1.

Definition 3.9: *Let (X,τ_1,τ_2) be a bts and (X,τ_{12}) be its associated supra topological space. Then X is called a generalized pairwise star R_1 space (gp^*R_1, for short) if $\forall x,y \in X$, and $cl_{12}\{x\} \neq cl_{12}\{y\}$, $\exists$ gp-open sets G and H such that $cl_{12}\{x\} \subseteq G$, $cl_{12}\{y\} \subseteq H$ and $G \cap H = \phi$.*

Remark 3.6: *If (X,τ_1,τ_2) is gp^*R_1 space, then it is gp^*R_0 space by definition. But, gp^*R_0 is not necessary gp^*R_1. Examples 3.5 explain that.*

Example 3.5: *Take R is real line, $\tau_1 = \{R,\phi\}$, τ_2 = cofinite topology. Then τ_{12} = cofinite topology, The set of all generalized closed sets is equal to the set of all pairwise closed sets. So, This space is $gp * R_0$ but not gp^*R_1. Since $cl_{12}\{1\} \neq cl_{12}\{2\}$ and there is no disjoint gp-open sets containing 1 and 2.*

Theorem 3.18: *Let (X,τ_1,τ_2) be a bitopological space, (X,τ_{12}) its associated supra topological spaces. If (X,τ_{12}) is a gp^*R_1 space and A is P- closed set in (X,τ_{12}), then $(A,\tau_{12}|_A)$ is a gp^*R_1 space.*

Proof: Let (X,τ_{12}) be a gp^*R_1 space and A be a P - closed set in (X,τ_{12}). Suppose that $x,y \in A$ and $cl_{12}|_A\{x\} \neq cl_{12}|_A\{y\}$. Then, $cl_{12}\{x\} \neq cl_{12}\{y\}$. But, (X,τ_{12}) is a gp^*R_1 space. This implies that, $\exists G,H \in GPO(X,\tau_{12})$ such that $cl_{12}\{x\} \subseteq G$, $cl_{12}\{y\} \subseteq H$ and $G \cap H = \phi$. By using corollary2.1and Theorem2.5 we get, $G \cap A, F \cap A \in GPO(A,\tau_{12}|_A)$ and $(G \cap A) \cap (F \cap A) = \phi$. Also, $cl_{12}|_A\{x\} \subseteq A \cap G$, $cl_{12}|_A\{y\} \subseteq A \cap H$. Hence,$(A,\tau_{12}|_A)$ is a gp^*R_1 space.

Theorem 3.19: *Let (X,τ_1,τ_2) and (Y,γ_1,γ_2) be a bitopological spaces, (X,τ_{12}) and (Y,γ_{12}) are associated supra topological spaces. Suppose that $f: X \to Y$ is an injective, P^* continuous function and P^* −closed. If (Y,γ_1,γ_2) is gp^*R_1 space, then (X,τ_1,τ_2) is gp^*R_1 space.*

Proof: Let (X,τ_1,τ_2) and (Y,γ_1,γ_2) be bitopological spaces, (X,τ_{12}) and (Y,γ_{12}) are associated supra topological spaces. Suppose that $f: X \to Y$ is an injective, P^* continuous function and P^* −closed, (Y,γ_{12}) is gp^*R_1 space. Let $x_1,x_2 \in X$ and $x_1 \neq x_2$ such that $cl_{12}|_X(\{x_1\}) \neq cl_{12}|_X(\{x_2\})$. Then, we have f is injective, this implies that, $f(x_1) \neq f(x_2)$. But,(Y,γ_{12}) is gp^*R_1 $f(cl_{12}|_X(\{x_1\})) \neq f(cl_{12}|_X(\{x_2\}))$. Thus, $cl_{12}|_Y(f(\{x_1\})\}) \neq cl_{12}|_Y(f(\{x_2\}))$. Then, $\exists gp$ −open sets H_1,H_2 in (Y,γ_{12}) such that $cl_{12}|_Y(f(\{\{x_1\})) \subseteq H_1$, $cl_{12}|_Y(f(\{x_2\})) \subseteq H_2$ and $H_1 \cap H_2 = \phi$. But, f is an injective $P^* -$ continuous function. Then, $f^{-1}(H_1), f^{-1}(H_2)$ are $gp-$ open sets in (X,τ_{12}) say $f^{-1}(H_1) = G_1, f^{-1}(H_2) = G_2$. Also, $f^{-1}cl_{12}|_Y(f(\{x_1\})) = cl_{12}|_X(x_1)$ and $f^{-1}cl_{12}|_Y(f(\{x_2\})) = cl_{12}|_X(x_2)$. This implies that, $cl_{12}|_X(\{x_1\}) \subseteq G_1$ and $cl_{12}|_X(\{x_2\}) \subseteq G_2$. Hence, (X,τ_{12}) is gp^*R_1.

Theorem 3.20: *If* (X,τ_1,τ_2) *is* gp^*R_0 *and* gp^*T_1 *space then, it is* gp^*T_2 (X,τ_1,τ_2).

Proof: Let (X,τ_1,τ_2) be gp^*R_0 space and gp^*T_1 space. suppose that $x,y\in X$ and $x\neq y$. Then $\exists G,H\in GPO(X,\tau_{12})$ such that $x\in G, y\notin G$ and $y\in H, x\notin H$. But we have, (X,τ_1,τ_2) is gp^*R_0, this implies that, $cl_{12}(\{x\})\subseteq G$. Therefore, $G^c\subseteq[cl_{12}(\{x\})]^c\ni y$. But $[cl_{12}(\{x\})]^c$ is gp −open set containing y and not containing x. Hence, (X,τ_1,τ_2) is gp^*T_2.

Theorem 3.21: *For a bitopological space* (X,τ_1,τ_2), *the following statements are equivalent:*

1. (X,τ_1,τ_2) is gp^*R_1 space.
2. For all $x,y\in X$ one of the following holds:
   (i) If $x\in U\in GPO(X,\tau_{12})$, then $y\in U$.
   (ii) There exist disjoint sets $G,H\in GPO(X,\tau_{12})$ such that $x\in G$ and $y\in H$.
3. If $x,y\in X$ such that $cl_{12}\{x\}\neq cl_{12}\{y\}$, then $\exists F_1,F_2 GPC(X,\tau_{12})$ such that, $x\notin F_1$, $y\notin F_2$ and $F_1\cup F_2=X$.

Proof: (1) ⇒ (2).Let (X,τ_1,τ_2) be gp^*R_1 space and $cl_{12}\{x\}=cl_{12}\{y\}$. Suppose that, $x\in U\in GPO(X,\tau_{12})$ such that $cl_{12}\{x\}\subseteq U$. Since, every gp^*R_1 space is gp^*R_0 space. But, $cl_{12}\{x\}=cl_{12}\{y\}$. Then, $y\in cl_{12}\{x\}\subseteq U$. If $cl_{12}\{x\}\neq cl_{12}\{y\}$ then, by definition of gp^*R_1 space we get ∃ gp-open sets G and H such that $cl_{12}\{x\}\subseteq G$, $cl_{12}\{y\}\subseteq H$ and $G\cap H=\phi$.

$(\mathbf{2})\Rightarrow(\mathbf{3})$: Suppose that, $x,y\in X$ such that $cl_{12}\{x\}\neq cl_{12}\{y\}$. Then, by (2) there exist disjoint sets $G,H\in GPO(X,\tau_{12})$ such that $x\in G$ and $y\in H$. Thus, $F_1=X\backslash U$ and $F_2=X\backslash V$ which are $gp-closed$ sets and There exist disjoint sets $F_1,F_2\in GPC(X,\tau_{12})$ such that $x\notin F_1$ and $y\notin F_2$ and $F_1\cup F_2=X$.

$(\mathbf{3})\Rightarrow(\mathbf{1})$: Let $x,y\in X$ such that $cl_{12}\{x\}\neq cl_{12}\{y\}$. Then, $\exists F_1,F_2 GPC(X,\tau_{12})$ such that, $x\notin F_1$, $y\notin F_2$ and $F_1\cup F_2=X$. So, $F_1^c,F_2^c GPO(X,\tau_{12})$ such that, $x\in F_1^c$, $y\in F_2^c$ and $F_1^c\cap F_2^c=\phi$. Hence, (X,τ_1,τ_2) is gp^*R_1 space.

4. CONCLUSION

1. The notation of P^* −regular and gp^* regular are not equal, when we tried to given examples for one not the other, we have two cases: (i) $\tau_{12}=GPC(X,\tau_{12})$ and (ii) $GPC(X,\tau_{12})=P(X)$.
2. Also, the notation of P^* −normal and gp^* normal are not equal,when we tried to given examples for one not the other, we have the above two cases.
3. Also, we tried to find an example to show the relation between gp^* −regular and gp^* normal, we see that $GPO(X,\tau_{12})=GPC(X,\tau_{12})=P(X)$. These examples remain missing and this is an open area of research.

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