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ON DECOMPOSITIONS OF γ-CONTINUITY WITH RESPECT TO AN OPERATOR IN IDEAL TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce the notions of α^*-$\gamma\iota$-set, t-$\gamma\iota$-set, s-$\gamma\iota$-set, β^*-$\gamma\iota$-set, $C_\gamma I$-continuity, $B_\gamma I$-continuity, $S_\gamma I$-continuity and $\beta_\gamma I$-continuity to obtain decompositions of γ-continuity with respect to an operator in ideal topological spaces.

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1. INTRODUCTION AND PRELIMINARIES

Kasahara [**12**] defined the concept of an operation on topological space and introduced α-closed graphs of an operation. Ogata [8] called the operation α as γ operation and introduced the notion of τ_γ which is the collection of all γ-open sets in a topological space (X, τ). In [10], the authors introduced the notions of α-γ-open sets and $\tau_{\alpha\text{-}\gamma}$ which is the collection of all α-γ-open sets in a topological space (X, τ). In [6,**7**], the authors introduced and studied the notions of semi-γ-open set, pre-γ-open set and β-γ-open set. In [**11**] , the authors introduced the notions of α-$\gamma\iota$- open sets and $\tau_{\alpha\text{-}\gamma\text{-}\iota}$ which is the collection of all α-$\gamma\iota$-open sets and also studied the notions of semi-$\gamma\iota$-open set, pre-$\gamma\iota$-open set, β-$\gamma\iota$-open set in ideal topological space. In this paper, we introduce the notions of α^*-$\gamma\iota$-set, t-$\gamma\iota$-set, s-$\gamma\iota$-set, β^*-$\gamma\iota$-set, $C_\gamma I$-continuity, $B_\gamma I$-continuity, $S_\gamma I$-continuity and $\beta_\gamma I$-continuity to obtain decompositions of γ-continuity in ideal topological spaces.

An operation γ on a topology τ is a mapping from τ on to power set P(X) of X such that $V\subset\gamma(V)$ for each $V \in \tau$, where $\gamma(V)$ denotes the value of γ at V. A subset A of X with an operation γ on τ is called γ-open if for each $x\in A$, there exists an open set U such that $x \in U$ and $\gamma(U)\subset A$. τ_γ denotes the set of all γ-open sets in X. For any topological space (X, τ), $\tau_\gamma\subset\tau$ [8]. Complements of γ-open sets are called γ-closed. The γ-closure of a subset A of X with an operation γ on τ is denoted by $Cl_\gamma(A)$ and is defined to be the intersection of all γ-closed sets containing A. The γ-interior of a subset A of X with an operation γ on τ is denoted by $Int_\gamma(A)$ and is defined to be the union of all γ-open sets containing A. A topological space X with an operation γ on τ is said to be γ-regular if for each $x\in X$ and for each neighborhood V of x, there exists an open neighborhood U of x such that $\gamma(U)$ contained in V. It is also to be note that $\tau = \tau_\gamma$ if and only if X is a γ-regular space [8]. An ideal on a topological space (X, τ) is a nonempty collection of subsets of X which is satisfies *(i)* $A\in$I and $B\subset A$ implies $B\in$I, *(ii)* $A\in$I and $B\in$I implies $A\cup B\in$I [9**]**. An ideal topological space is a topological space (X, τ) with an ideal I on X [9] and if P(X) is the set of all subsets of X, a set operator (.) : P(X) $\rightarrow$ P(X) called a local function of A with respect to τ and I is defined as follows: for $A\subset X$, $A^*(I, \tau)=\{x\in X : U - A\notin I$ for every $U\in \tau(x)\}$ where $\tau(x)=\{U\in \tau : x\in U\}$, simply write A^* instead of $A^*(I, \tau)$ [9]. For every ideal topological space, there exists a topology $\tau^*(I)$ or briefly τ^*, finer than τ, generated by $\beta(I, \tau)=\{U - W : U\in \tau$ and $W\in I\}$, but in general $\beta(I, \tau)$ is not always a topology [4]. Also $Cl^*(A)=A\cup A^*$ defines a Kuratowski closure operator for $\tau^*(I)$ [4]. If $A\in\tau^*$, $Int^*(A)=A$ [4] and $Int^*(A)$ will denote the τ^* interior of A. If I is an ideal on X then (X, τ, I) is called an ideal topological space [4].

Throughout this paper, (X, τ) and (Y, σ) represent topological space on which no separation axioms are assumed unless otherwise mentioned. $Cl(A)$ and $Int(A)$ denote the closure of A and the interior of A, respectively, in topological space (X, τ). Let us recall some of basic definitions used in the sequel.

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Definition 1: Let (X,τ) be a topological space and A be a subset of X and γ be an operation on τ. Then A is said to be

1. α-open set [10] if $A \subset Int_\gamma(Cl_\gamma(Int_\gamma(A)))$,
2. pre-γ-open set [7] if $A \subset Int_\gamma(Cl_\gamma(A))$,
3. semi-γ-open set [6] if $A \subset Cl_\gamma(Int_\gamma(A))$,
4. β-γ-open set [7] if $A \subset Cl_\gamma(Int_\gamma(Cl_\gamma(A)))$,
5. γ-regular open set [1] if $Int_\gamma(Cl_\gamma(A)) = A$.

Definition 2: [11] Let (X,τ,I) be an ideal topological space and A be a subset of X and γ be an operation on τ. Then A is said to be

1. α-$\gamma\iota$-open set if $A \subset Int_\gamma(Cl^{\gamma^*}(Int_\gamma(A)))$,
2. pre-$\gamma\iota$-open set if $A \subset Int_\gamma(Cl^{\gamma^*}(A))$,
3. semi- $\gamma\iota$- open set if $A \subset Cl^{\gamma^*}(Int_\gamma(A))$,
4. β-$\gamma\iota$-open set if $A \subset Cl_\gamma(Int_\gamma(Cl^{\gamma^*}(A)))$.

Definition 3: [5] A subset A of a topological space (X,τ) with an operation γ is called

1. α^*-γ-set if $Int\gamma(Cl_\gamma(Int_\gamma(A))) = Int_\gamma(A)$,
2. t-γ-set if $Int_\gamma(Cl_\gamma(A)) = Int_\gamma(A)$,
3. s-γ-set if $Cl_\gamma(Int_\gamma(A)) = Int_\gamma(A)$,
4. β^*-γ-set if $Cl_\gamma(Int_\gamma(Cl^{\gamma^*}(A))) = Int_\gamma(A)$.

Definition 4:[5] A subset A of a topological space (X,τ) with an operation γ is called

1. Cγ-set if $A = U - V$, where $U \in \tau_\gamma$ and V *is an* α^*-γ-set,
2. Bγ-set if $A = U - V$, where $U \in \tau_\gamma$ and V *is a* t-γ-set,
3. Sγ-set if $A = U - V$, where $U \in \tau_\gamma$ and V *is a* s-γ-set,
4. $\beta\gamma$-set if $A = U - V$, where $U \in \tau_\gamma$ and V *is a* β^*-γ-set.

Definition 5: Let (X,τ) and (Y,σ) be two topological spaces and let γ be an operation on τ. A mapping f $: (X,\tau) \to (Y,\sigma)$ is said to be γ-continuous [3] (resp. α-γ-continuous [10], pre-γ-continuous [7], semi-γ-continuous [6], β-γ-continuous [7]) if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists a γ-open set U containing x (resp. α-γ-open set, pre-γ-open set, semi-γ-open set, β-γ-open set) such that $f(U) \subset V$.

Definition 6: [**11**] Let (X,τ,I) be an ideal topological space and (Y,σ) be a topological space and $\gamma: \tau \to P(X)$ be the operation on τ. A mapping f : $(X,\tau,I) \to (Y,\sigma)$ is said to be α-$\gamma\iota$-continuous (resp. pre-γ-continuous, semi-$\gamma\iota$-continuous, β-$\gamma\iota$-continuous) if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists an α-$\gamma\iota$-open set U containing x (resp. pre-γ-open set, semi-$\gamma\iota$-open set, β-$\gamma\iota$-open set) such that $f(U) \subset V$.

Definition 7: [5] Let (X,τ) and (Y,σ) be two topological spaces and let $\gamma : \tau \to P(X)$ be the operation on τ. Let f : $(X,\tau) \to (Y,\sigma)$ be a function. If for each $V \in \sigma$, $f^{-1}(V)$ is a C_γ-set (resp. B_γ-set, S_γ-set, β_γ-set), then f is said to be C_γ-continuous (resp. B_γ-continuous, S_γ-continuous, β_γ-continuous).

2. *γ-local* FUNCTION

In [11], the authors defined Kuratowski* closure operator as $Cl^{\gamma^*}(A) = A \cup A^*$. According to this definition, we can explain that an ideal topological space with an operation γ is a topological space (X,τ) with an ideal I and an operation γ on X. Therefore, if $P(X)$ is the set of all subsets of X, a set operator $(.)^{\gamma^*} : P(X) \to P(X)$ called a *γ-local* function of A with respect to τ, an operator γ and I is defined as follows: for $A \subset X$, $A^{\gamma^*}(I,\tau) = \{x \in X : U - A \notin I \text{ for every } U \in \tau_\gamma(x)\}$ where $\tau_\gamma(x) = \{U \in \tau_\gamma : x \in U\}$, simply write A^{γ^*} instead of $A^{\gamma^*}(I,\tau)$. Therefore, we can give some properties of *γ-local* function in the following.

Remark 1:

1. The minimal ideal is $\{\phi\}$ and the maximal ideal is $P(X)$ in any ideal topological space (X,τ,I) with an operation γ. Then $A^{\gamma^*}(\{\phi\}) = Cl_\gamma(A) \neq Cl(A)$ and $A^{\gamma^*}(P(X)) = \phi$ for every $A \subset X$.
2. If $A \in I$, then $A^{\gamma^*} = \phi$.
3. Neither $A \subset A^{\gamma^*}$ nor $A^{\gamma^*} \subset A$ in general.

Theorem 1: Let (X,τ,I) be an ideal topological space with an operation γ on τ and A, B subsets of X. The following properties hold:

1. $(\phi)^{\gamma^*} = \phi$,
2. If $A \subset B$, then $A^{\gamma^*} \subset B^{\gamma^*}$,
3. $J \supset I$ on X, $A^{\gamma^*}(J) \subset A^{\gamma^*}(I)$, J another ideal,

4. $A^{\gamma*} \subset Cl\gamma(A)$,
5. $A^{\gamma*} = Cl_\gamma(A^{\gamma*}) \subset Cl_\gamma(A)$ and $A^{\gamma*}$ is γ- closed,
6. $(A^{\gamma*})^{\gamma*} \subset A^{\gamma*}$,
7. $(A \cup B)^{\gamma*} = A^{\gamma*} \cup B^{\gamma*}$,
8. $A^{\gamma*} - B^{\gamma*} = (A - B)^{\gamma*} - B^{\gamma*} \subset (A - B)^{\gamma*}$,
9. If $U \in \tau^\gamma$, then $U \cap A^{\gamma*} = U \cap (U \cap A)^{\gamma*} \subset (U \cap A)^{\gamma*}$,
10. If $U \in$ I, then $(A - U)^{\gamma*} \subset A^{\gamma*} = (A \cup U)^{\gamma*}$.

Proof: *S*traightforward.□

Now we define $\tau^{\gamma*}$ in terms of the closure operator $Cl^{\gamma*}(A) = A \cup A^{\gamma*}$.

Theorem 2: Let (X, τ, I) be an ideal topological space with an operation γ on τ, $Cl^{\gamma*}(A) = A \cup A^{\gamma*}$ and A, B subsets of X. Then

1. $Cl^{\gamma*}(\phi) = \phi$,
2. $A \subset Cl^{\gamma*}(A)$,
3. $Cl^{\gamma*}(A \cup B) = Cl^{\gamma*}(A) \cup Cl^{\gamma*}(B)$,
4. $Cl^{\gamma*}(A) = Cl^{\gamma*}(Cl^{\gamma*}(A))$.

Proof: (1) and (2) are obvious by Theorem 1.

3. $Cl^{\gamma*}(A \cup B) = (A \cup B)^{\gamma*} \cup (A \cup B) = (A^{\gamma*} \cup B^{\gamma*}) \cup (A \cup B)$
$= Cl^{\gamma*}(A) \cup Cl^{\gamma*}(B)$.

4. $Cl^{\gamma*}(Cl^{\gamma*}(A)) = Cl^{\gamma*}(A^{\gamma*} \cup A) = (A^{\gamma*} \cup A)^{\gamma*} \cup (A^{\gamma*} \cup A)$
$= ((A^{\gamma*})^{\gamma*} \cup A^{\gamma*}) \cup (A^{\gamma*} \cup A) = A^{\gamma*} \cup A = Cl^{\gamma*}(A)$.□

A basis for γ-open sets of $\tau^{\gamma*}$ described as follows:
Then, for $A \subset X$, A is $\tau^{\gamma*}$-closed if and only if $A^{\gamma*} \subset A$. Thus we have $U \in \tau^{\gamma*}$ if and only if X- U is $\tau^{\gamma*}$- closed if and only if $U \subset X - (X - U)^{\gamma*}$. Thus if $x \in U$, $x \notin (X - U)^{\gamma*}$, that is, there exists a γ-open set V such that $V \cap (X - U) \in I$. Hence let $I_o = V \cap (X - U)$ and we have $x \in V - I_o \subset U$, where V is γ-open set containing x and $I_o \in$ I. Let us denote $\beta(I, \tau_\gamma) = \{V - I_o : V$ is γ-open, $I_o \in \mathrm{I}\}$, simplicity $\beta(I, \tau_\gamma)$ for β. Therefore, β is a basis for $\tau^{\gamma*}$.

Remark 2: The topology $\tau^{\gamma*}$ finer than τ_γ. If $A \in \tau^{\gamma*}$, $Int^{\gamma*}(A) = A$ and $Int^{\gamma*}(A)$ will denote the $\tau^{\gamma*}$ interior of A. If I is an ideal on X, then (X, τ, I) is called an ideal topological space with an operation γ.
Now, we can give the following definitions to obtain new decompositions of γ-continuity.

3. C_γI-sets, B_γI-sets, S_γI-sets AND β_γI-sets

Definition 8: A subset A of an ideal topological space (X, τ, I) with an operation γ is called

1. α*-γι-set if $Int_\gamma(Cl^{\gamma*}(Int_\gamma(A))) = Int_\gamma(A)$,
2. t-γι-set if $Int_\gamma(Cl^{\gamma*}(A)) = Int_\gamma(A)$,
3. s-γι-set if $Cl^{\gamma*}(Int_\gamma(A)) = Int_\gamma(A)$,
4. β*-γι-set if $Cl_\gamma(Int_\gamma(Cl^{\gamma*}(A))) = Int_\gamma(A)$,
5. weak β-γι-open set if $A \subset Cl_\gamma(Int^{\gamma*}(Cl_\gamma(A)))$ and the complement of weak β-γι-open set is a weak β-γι-closed set if $Int_\gamma(Cl^{\gamma*}(Int_\gamma(A)) \subset A$.
6. γι-regular open set if $Int_\gamma(Cl^{\gamma*}(A)) = A$.

Proposition 1: The following are equivalent for a subset A of an ideal topological space (X, τ, I) with an operator γ,

1. A is α*-γι-set,
2. A is weak β-γι-closed set,
3. $Int_\gamma(A)$ is beta-regular open set change with gammaI-regular open set.

Proof: Straightforward.

Proposition 2: Let A be a subset of an ideal topological space (X, τ, I) with an operator γ,

1. A semi-γι-open set A is a t-γι- set if and only if A is an α*-γι-set.
2. A is an α-γι-open set and A is an α*-γι-set if and only if A is γι-regular open set.

Proof:

1. Let A be a semi-γι-open and an α*-γι-set. Since A is a semi-γι- open, $Cl^{\gamma*}((Int_\gamma(A)) = Cl^{\gamma*}(A)$ and $Int_\gamma(Cl^{\gamma*}(A)) = Int_\gamma(Cl^{\gamma*}(Int_\gamma(A))) = Int_\gamma(A)$. Therefore, A is a t-γι-set.
2. Let A be an α-γι- open set and an α*-γι-set. By Proposition **1** and the definition of α-γι-open set, we have $Int_\gamma(Cl^{\gamma*}(A)) = A$ and hence $Int_\gamma(Cl^{\gamma*}(A)) = Int_\gamma(Cl^{\gamma*}(Int_\gamma(A))) = A$.
The converse is obvious.

Proposition 3: Let (X,τ,I) be an ideal topological space with an operation γ and A a subset of X. Then the following hold:

1. If A is a t-γ_I-set, then A is an α^*-γ_I-set,
2. If A is a s-γ_I-set, then A is an α^*-γ_I-set,
3. If A is a β^*-γ_I-set, then A is both t-γ_I-set and s-γ_I-set. 55
4. t-γ_I-set and s-γ_I-set are independent.

Proof: Straightforward from the definitions of γ-interior and τ^{γ^*}- closure.

Remark 3: The converses of the statements in Proposition 3 are false as seen in the followig examples.

Example 1: Let $X = \{a,b,c,d\}$, $\tau = \{\phi, X, \{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},\{a,b,c\}, \{a,b,d\}\}$ and $I = \{\phi, \{a\}\}$. We define an operator $\gamma : \tau \to P(X)$ by $\gamma(A) = Cl(A)$ if $A \neq \{a\}$ and $\gamma(A) = Int(Cl(A))$ if $A = \{a\}$. Then $\tau_\gamma = \{\phi,\{a\},\{c\},\{a,c\},\{a,b,d\},X\}$.

If we take $A = \{a,b\}$, it is both s-γ_I-set and α^*-γ_I-set, but not a t-γ_I-*set* and not a β^*-γ_I-set.

Example 2: Let $X = \{a,b,c\}$, $\tau = \{\phi, X,\{a\},\{c\},\{a,c\},\{a,b\}\}$ and $I =\{\phi,\{c\}\}$. We define an operator $\gamma : \tau \to P(X)$ by $\gamma(A) = A \cup \{a,c\}$ if $A \neq \{a\}$ and $\gamma(A) = A$ if $A = \{a\}$. Then $\tau_\gamma = \{\phi,X,\{a\},\{c\},\{a,c\}\}$. If we take $A = \{a,b\}$, then A is both α^*-γ_I-set and t-γ_I-set, but it is not a s-γ_I-set and not a β^*-γ_I-set.

Definition 9: A subset A of an ideal topological space (X,τ,I) with an operation γ is called

1. $C_\gamma I$-set if $A = U - V$, where $U \in \tau_\gamma$ and V is an α^*-γ_I-set,
2. $B_\gamma I$-set if $A = U - V$, where $U \in \tau_\gamma$ and V is a t-γ_I-set,
3. $S_\gamma I$- set if $A = U - V$, where $U \in \tau_\gamma$ and V is a s-γ_I-set,
4. $\beta_\gamma I$-set if $A = U - V$, where $U \in \tau_\gamma$ and V is a β^*-γ_I-set.

Proposition 4: Let (X,τ,I) be an ideal topological space with an operation γ and A a subset of X. Then the following hold:

1. If A is an α^*-γ_I-set, then A is $C_\gamma I$-set,
2. If A is a t-γ_I-set, then A is $B_\gamma I$-set,
3. If A is a s-γ_I-set, then A is $S_\gamma I$-set,
4. If A is a β^*-γ_I- set, then A is $\beta_\gamma I$-set.

Proof: 1. Let A be an α^*-γ_I-set. If we take $U = X \in \tau_\gamma$, then $A = U - A$ and hence A is a $C_\gamma I$-set.
The proof of (2), (3) and (4) are same.

Remark 4: The converses of the statements in Proposition 4 are false as seen in the following example.

Example 3: In Example 1, let us take $I = \{\phi\}$. Then if we take $A = \{a,c\}$, since $\{a,c\} \in \tau_\gamma$ and $\{a,c\} = A \cap X$, A is $C_\gamma I$-*set* (resp. $B_\gamma I$-set, $S_\gamma I$-*set* and $\beta_\gamma I$-set), but it is not an α^*-γ_I-set (resp. a t-γ_I-set, a *s*-γ-set and a β^*-γ_I-set).

Proposition 5: Let (X,τ,I) be an ideal topological space with an operation γ and A a subset of X. Then the following hold*:*

1. A $B_\gamma I$-set is a $C_\gamma I$-set,
2. A $S_\gamma I$-set is a $C_\gamma I$-set,
3. A $\beta_\gamma I$-set is both a $B_\gamma I$-set and a $S_\gamma I$-set.

Remark 5: The converses of the statements in Proposition 5 are false and $B_\gamma I$-set and $S_\gamma I$-set are independent notions as seen in the following examples.

Example 4: In Example 2, if we take $A = \{a,b\}$, then A is both $B_\gamma I$-set and $C_\gamma I$-set, but it is not $S_\gamma I$-set and not $\beta_\gamma I$-set.

Example 5: Let $X = \{a,b,c\}$, $\tau = \{\phi,X,\{a\}, \{a,b\}\}$ and $I = \{\phi\}$. We define an operator $\gamma : \tau \to P(X)$ by $\gamma(A) = A$ if $A = \{a,c\}$ or $A = \phi$ and $\gamma(A) = X$ if otherwise. Then $\tau_\gamma = \{\phi, X\}$. If we take $A = \{b\}$, then A is a $S_\gamma I$-set and a $C_\gamma I$-set, but not a $B_\gamma I$-set and not a $\beta_\gamma I$-set.

Proposition 6: Let (X, τ, I) be an ideal topological space with an operation γ and A a subset of X. Then the following hold:

1. A B_γ -set is a $B_\gamma I$-set,
2. A C_γ-set is a $C_\gamma I$-set,
3. A S_γ -set is a $S_\gamma I$-set,
4. A β_γ-set is a $\beta_\gamma I$-set.

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Proof: It follows from $\tau_\gamma \subset \tau^{\gamma^*}$.□

Remark 6: The converses of the statements in Proposition 6 are false as seen in the following examples.

Example 6: In Example 1, if we take $A = \{a,b\}$, it is a $C_\gamma I$-set, but not a C_γ-set.

Example 7: In Example 2, let us take $I = \{\phi, \{a\}\}$. Then if we take $A = \{a,b\}$, then A is a $S_\gamma I$-set, but not a S_γ-set.

Example 8: Let $X = \{a,b,c\}$, $\tau = \{\phi, X, \{a\},\{a,b\}\}$ and $I = \{\phi,\{b\}\}$. We define an operator $\gamma : \tau \rightarrow P(X)$ by $\gamma(A) = A$ if $A = \{a,c\}$ or $A = \phi$ and $\gamma(A) = X$ if otherwise. Then $\tau_\gamma = \{\phi, X\}$. If we take $A = \{b\}$ is a $B_\gamma I$-set and a $\beta_\gamma I$-set, but it is not a B_γ-set and a β_γ -set.

Theorem 3: For a subset A of a space (X, τ, I) with an operation γ, the following properties are equivalent:

1. A is γ-open,
2. A is an α-γI-open set and a $C_\gamma I$-set,
3. A is a pre-γI-open set and a $B_\gamma I$-set,
4. A is a semi-γI-open set and a $S_\gamma I$-set,
5. A is a β-γI-open set and a $\beta_\gamma I$-set.

Proof: The proof of (1)⇒(2), (1) ⇒ (3), (1) ⇒ (4), (1) ⇒ (5) are obvious.

(**5**) ⇒ (**1**) Let A be a β-γI-open set and a $\beta_\gamma I$-set. Since A is a $\beta_\gamma I$-set, we have $A = U \cap V$, where U is a γ-open set and V is a β^*-γI-set. By the hypothesis, A is also β-γI-open and we have

$$A \subset Cl_\gamma(Int_\gamma(Cl^{\gamma^*}(A))) = Cl_\gamma(Int_\gamma(Cl^{\gamma^*}(U \cap V))) \subset Cl_\gamma(Int_\gamma(Cl^{\gamma^*}(U) \cap Cl^{\gamma^*}(V)))$$
$$= Cl_\gamma(Int_\gamma(Cl^{\gamma^*}(Cl^{\gamma^*}((U)) \cap Int_\gamma(Cl^{\gamma^*}(V))) \subset Cl_\gamma(Int_\gamma(Cl^{\gamma^*}(U))) \cap Cl_\gamma(Int_\gamma(Cl^{\gamma^*}(V)))$$
$$\subset Cl_\gamma(Int_\gamma(Cl^{\gamma^*}(U))) \cap Int_\gamma(V). \text{ Hence } A = U \cap V = (U \cap V) \cap U$$
$$\subset (Cl_\gamma(Int_\gamma(Cl^{\gamma^*}(U))) \cap Int_\gamma(V)) \cap U = (Cl_\gamma(Int_\gamma(Cl^{\gamma^*}(U))) \cap U) \cap Int_\gamma(V).$$

Notice $A = U \cap V \supset U \cap Int_\gamma(V)$. Therefore, we obtain $A = U \cap Int_\gamma(V)$.

(2)⇒(1), (3)⇒(1), (4)⇒(1) are shown similarly.

DECOMPOSITIONS OF GAMMA CONTINUITY

Definition 10: Let (X, τ, I) be an ideal topological space and (Y, σ) be a topological space and let $\gamma : \tau \rightarrow P(X)$ be the operation on τ. Let $f : (X, \tau, I) \rightarrow (Y, \sigma)$ be a function. If for each $V \in \sigma$, $f^{-1}(V)$ is a $C_\gamma I$-set (resp. $B_\gamma I$-set, $S_\gamma I$-set, $\beta_\gamma I$-set), then f is said to be $C_\gamma I$-continuous (resp. $B_\gamma I$-continuous, $S_\gamma I$-continuous, $\beta_\gamma I$-continuous). *By Proposition 5, we obtain the following proposition.*

Proposition 6:

1. A $B_\gamma I$-continuous function is $C_\gamma I$-continuous,
2. A $S_\gamma I$-continuous function is $C_\gamma I$-continuous,
3. A $\beta_\gamma I$-continuous is both $B_\gamma I$continuous and $S_\gamma I$-continuous.

By Theorem 3, we have the following main theorem.

Theorem 4: Let (X, τ, I) be an ideal topological space and (Y, σ) be a topological space and let $\gamma : \tau \rightarrow P(X)$ be the operation on τ. For a function $f : (X, \tau, I) \rightarrow (Y, \sigma)$, the following properties are equivalent:

1. A is γ- continuous
2. A is α-γI- continuous and $C_\gamma I$-continuous,
3. A is pre-γI- continuous and $B_\gamma I$-continuous,
4. A is semi-γI- continuous and $S_\gamma I$-continuous,
5. A is β-γI- continuous and $\beta_\gamma I$-continuous.

Proof: This is an immediate consequence of Theorem 3.

Remark 7: α-$\gamma\imath$-continuity and C_γI-continuity, pre-$\gamma\imath$-continuity and B_γI-*continuity*, semi-$\gamma\imath$-continuity and S_γI-continuity, β-$\gamma\imath$-continuity and β_γI-continuity are independent notions of each other as seen in the following examples.

Example 9: Let $X = Y = \{a,b,c\}$, $\tau = \{\phi, X, \{a\}, \{c\}, \{a,c\}, \{a,b\}\}$ and I = $\{\phi, \{c\}\}$ and $\sigma = \{\phi, Y, \{a\}\}$. We define an operator $\gamma : \tau \to$ P(X) by $\gamma(A) = A \cup \{a,c\}$ if $A \neq \{a\}$ and $\gamma(A) = A$ if $A = \{a\}$. Then $\tau_\gamma = \{\phi, X, \{a\}, \{c\}, \{a,c\}\}$. Define a function $f : (X, \tau, I) \to (Y, \sigma)$ as $f(a) = f(b) = a$, $f(c) = c$. Then f is C_γI-continuous (resp. B_γI-continuous, β-$\gamma\imath$-continuous and semi-$\gamma\imath$-continuous), but it is not α-$\gamma\imath$-continuous (resp. pre-$\gamma\imath$-continuous β_γI-continuous and S_γI-continuous)

Example 10: Let $X = Y = \{a,b,c\}$, $\tau = \{\phi, X, \{a\}, \{a,b\}\}$ and I = $\{\phi\}$ and $\sigma = \{\phi, Y, \{b\}\}$. We define an operator
$\gamma : \tau \to$ P(X) by γ(A) = A if $A = \{a,c\}$ or $A = \phi$ and $\gamma(A) = X$ if otherwise. Then $\tau_\gamma = \{\phi, X\}$. Define a function f :(X,τ) → (Y,σ) as $f(a) = f(c) = a$, $f(b) = b$. Then f is both S_γI-continuous and pre-$\gamma\imath$-continuous, but it is neither semi-$\gamma\imath$-continuous nor B_γI-continuous. In this example, take I = $\{\phi, \{b\}\}$. Then $A = \{b\}$ is β_γI-continuous, but it is not β-$\gamma\imath$- continuous.

Example 11: Let $X = Y = \{a,b,c\}$, $\tau = \{\phi, X, \{a\}, \{c\}, \{a,c\}, \{b,c\}\}$ and I = $\{\phi, \{c\}\}$ and $\sigma = \{\phi, Y, \{a\}\}$. We define an operator $\gamma : \tau \to P(X)$ by $\gamma(A)$ = Int(Cl(A)) if A=$\{a\}$ and $\gamma(A)$ = X if A $\neq \{a\}$. Then $\tau_\gamma = \{\phi, \{a\}, X\}$. Define a function f : $(X, \tau, I) \to$ (Y,σ) as f(a) = f(c) = a, f(b) = b. Then f is α-γı-continuous, but it is not $C_\gamma I$-continuous.

Corollary 1: Let (X, τ, I) be an ideal topological space with an operator γ and I = $\{\phi\}$ and (Y, σ) be a topological space. For a function $f : (X, \tau, I) \to (Y, \sigma)$, the following properties and the properties of Theorem 3 are equivalent:

1. f is γ-continuous,
2. f is pre-γ-continuous and B_γ-continuous [5],
3. f is α-γ-continuous and C_γ-*continuous* [5],
4. f is semi-γ-continuous set and S_γ-continuous [5],
5. f *is* β-γ-continuous set and β_γ-*continuous* [5].

Proof: It follows from $A^{\gamma^*}(\{\phi\}) = Cl_\gamma(A)$ for every $A \subset X$.

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