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PAIRED EQUITABLE DOMINATION IN FUZZY GRAPHS

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ABSTRACT

In this paper, introduce the concept of paired equitable dominating set (ped-set), paired equitable domination number in a fuzzy graph. The relation between equitable domination number and paired equitable domination number are established. Bound for paired equitable domination numbers are obtained.

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Keywords: *Fuzzy graph, equitable dominating set, equitable domination number, paired equitable dominating set, paired equitable domination number.*

1. INTRODUCTION

Zadeh[8] introduced the concept of fuzzy sets in the year 1965. In 1975, fuzzy graph was introduced by Rosenfeld [5]. The notation of domination in fuzzy graphs was developed by A. Somasundaram and S. Somasundaram [6]. Nagoorgani and Chandrasekaran [4] discussed about domination in a fuzzy graph using strong arcs. The concept of degree equitable domination in graphs was introduced by Venkatasubramanian Swaminathan and Kuppusamy Markandan Dharmalingam [7]. The concept of equitable domination in fuzzy graphs was introduced by Dharmalingam and Rani [2]. S. Yahya Mohamad and S.Suganthi [8] introduced the concept of matching in fuzzy labeling graph

In this paper, paired equitable dominating set and paired equitable domination number in fuzzy graphs are defined. The relation between paired equitable domination numbers and other well known parameters in fuzzy graph are obtained.

2. PRELIMINARIES

Definition 2.1: Let $G^* = (V, E)$ be a graph with vertex V and edge set $E \subseteq V \times V$. Let σ and μ be a fuzzy set of V and E respectively. Then $G = (\sigma, \mu)$ be a *fuzzy graph* if $\mu(u,v) \le \sigma(u) \wedge \sigma(v)$ for all $(u,v) \in E$ and is denoted by $G = (\sigma, \mu)$.

Definition 2.2: The complement of a fuzzy $G = (\sigma, \mu)$ is a fuzzy graph $G = (\sigma', \mu')$ where $\sigma = \sigma'$and $\mu'(u,v) = \sigma(u) \wedge \sigma(u) - \mu(u,v)$ for all u, v in V.

Definition 2.3: Let $G = (\sigma, \mu)$ be a fuzzy graph then the *order* and *size* are defined as $p = \sum_{u \in V} \sigma(u)$ and $q = \sum_{(u,v) \in E} \mu(u,v)$

Definition 2.4: The *neighbourhood degree* of a vertex u is defined to be the sum of the weights of the vertices adjacent to u and is denoted by $d_N(u)$, the *minimum neighbourhood degree* of G is $\delta_N(G) = min\{d_N(u): u \in V\}$ and the *maximum neighbourhood degree* of G is $\Delta_N(G) = max\{d_N(u): u \in V\}$

Definition 2.5: The *effective degree* of a vertex u is defined to be the sum of the weights of the effective edges incident at u and is denoted by $d_E(u)$, the *minimum effective degree* of G is $\delta_E(G) = min\{d_E(u): u \in V\}$ and the *maximum effective degree* of G is $\Delta_E(G) = max\{d_E(u): u \in V\}$

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Definition 2.6: A fuzzy graph $G = (\sigma, \mu)$ is said to be *bipartite* if the vertex set V can be partitioned into two sets V_1 defined on σ_1 and V_2 defined on σ_1 such that $\mu(v_1, v_2) = 0$ if $(v_1, v_2) \in V_1 \times V_1$ or $(v_1, v_2) \in V_2 \times V_2$

Definition 2.7: A fuzzy graph is said to be *connected* if there exists atleast one path between every pair of vertices.

Definition 2.8: The *degree of vertex u* is defined as the sum of the weights of the edges incident at u and it is denoted by $d(u)$.

Definition 2.9: The maximum and minimum fuzzy equitable degree of a vertex in G are denoted respectively by $\Delta^e(G)$ and $\delta^e(G)$ .That is $\Delta^e(G) = \max_{u \in V(G)} |N^e(u)|$and $\delta^e(G) = \min_{u \in V(G)} |N^e(u)|$

Definition 2.10: A *path* in a fuzzy graph G is a sequence of distinct vertices $u_0, u_1, u_2, \ldots u_n$ such that (u_{i-1}, u_i) $1 \leq i \leq n$ be a strong arc and is denoted by P_σ and n is called the length of the path. The path in a fuzzy graph is called a *cycle* if $u_0 = u_n, n \geq 2$ and is denoted by c_σ

Definition 2.11: A connected acyclic fuzzy graph is said to be a *tree.*

Definition 2.12: A vertex in a fuzzy graph having only one neighbour is called a *pendent vertex*. Otherwise it is called *non – pendent vertex*.

Definition 2.13: An arc (u, v) in a fuzzy graph $G = (\sigma, \mu)$ is said to be strong if $\mu^\infty(u, v) \leq \mu(u, v)$then u, v are called *strong neighbours*.

Definition 2.14: A fuzzy graph $G = (\sigma, \mu)$ be a *perfect matching* in a fuzzy graph a subset of strong arc in which no two strong arcs are adjacent which means no two strong arc are incident on a common vertex.

Definition 2.15: The *strong neighbourhood* of the vertex u is defined as $N_S(u) = \{v \in V \mid (u, v) is\ a\ strong\ arc\}$.

Definition 2.16: A vertex $u \in V$ dominates $v \in V$ if (u, v) is a strong arc. A subset D of V is called a *dominating set* of a fuzzy graph G if for every $v \in V - D$, there exists $u \in D$ such that u dominates v. The smallest number of vertices in a dominating set of G is called domination number and is denoted by $\gamma(G)$.

Definition 2.17 [2]: Let u and v be two vertices in a fuzzy graph G. A subset D of V is called an *equitable dominating set* if for every $v \in V - D$ there exists a vertex $u \in D$ such that $(u, v) \in E(G)$ and $|d(u) - d(v)| \leq 1$ and $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$. The minimum cardinality of an *equitable dominating set* in a fuzzy graph is denoted by γ_e.

3. PAIRED EQUITABLE DOMINATION IN FUZZY GRAPHS

Definition 3.1: Let $G = (\sigma, \mu)$ be a connected fuzzy graph. An equitable dominating set $D \subseteq V$ is called *paired equitable dominating set (ped-set)* then $< D >$ has a perfect matching. The minimum cardinality taken over all of paired equitable dominating set is called *paired equitable domination number* of G and is denoted by $\gamma_{ped}(G)$.

Remark 3.2: The paired equitable dominating set exists in a fuzzy graph G, it is non-trivial connected fuzzy graph G.

Example 3.3: From the fuzzy graph G given in figure (1), Order of G = 3.0, Size of G = 2.5
Paired Equitable dominating set is $\{c, d\}$, $\gamma_{ped}(G) = 1.2$

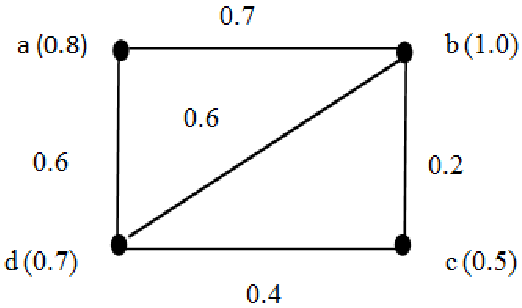

Figure-1

Observations 3.4:
1) A paired equitable dominating set exists only if a G contains no isolated vertices. Hereafter we consider G is a connected fuzzy graph without isolated vertices.
2) A paired equitable dominating set of fuzzy graph G is also an equitable dominating set of G.
3) If D is a paired equitable dominating set then any super set of D need not be paired equitable dominating set.

Example 3.5: Consider the fuzzy graph given in figure (2)

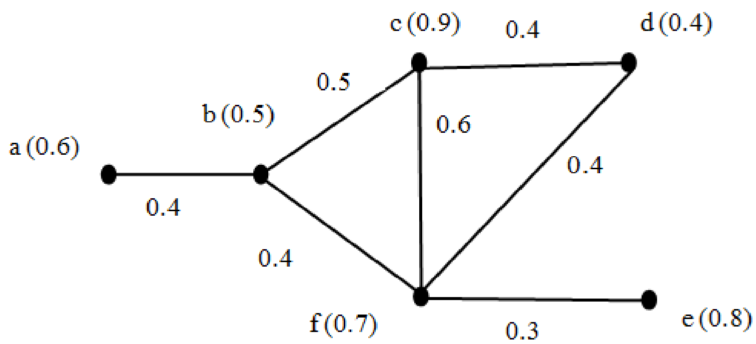

Figure-2

ped-set, $D = \{a, b, f, e\}$ but the super set $\{a, b, f, e, d\}$ of D is not a ped-set.

Definition 3.6: A paired equitable dominating set D is said to be minimal paired equitable dominating set if no proper subset of D is a paired equitable dominating set in a fuzzy graph. The maximum cardinality of a minimal paired equitable dominating set is denoted by $\Gamma_{ped}(G)$.

Example 3.7: Consider the fuzzy graph G in figure (1), $\Gamma_{ped}(G) = 1.5$

Theorem 3.8: A paired equitable dominating set D of a fuzzy graph G is minimal ped – set if and only if any two vertices $v_1, v_2 \in D$ one of the following conditions holds,

(i) $G\left\{D - \{v_1, v_2\}\right\}$ does not contain a perfect matching

(ii) there exist a vertex $u \in V - D$ such that $N(u) \cap D \subseteq \{v_1, v_2\}$ and $\left|d(v_1) - d(u)\right| \leq 1$; $\left|d(v_2) - d(u)\right| \leq 1$

Proof: Suppose D is a minimal paired equitable dominating set of a fuzzy graph G. then for any two vertices $v_1, v_2 \in D$, $D - \{v_1, v_2\}$ is not a ped-set of G. Therefore $G\left\{D - \{v_1, v_2\}\right\}$ does not contains a perfect matching. If $u \in V - D$ is not dominated by $D - \{v_1, v_2\}$, but it is equitable dominated by D then u adjacent to either v_1 or v_2 or both. This implies that $\left|d(v_1) - d(u)\right| \leq 1$, $\left|d(v_2) - d(u)\right| \leq 1$

Conversely, suppose D is minimal ped-set of a fuzzy graph of G. Then for any two vertices $v_1, v_2 \in D$ one of the two stated conditions holds. Suppose D is not a minimal paired equitable dominating set. Then there exist two vertices say $v_1, v_2 \in D$ such that $D - \{v_1, v_2\}$ is a ped-set. Therefore condition (1) does not hold. If $D - \{v_1, v_2\}$ is a ped-set then every vertex in $V - D$ is adjacent to atleast one vertex in $D - \{v_1, v_2\}$. Therefore for any two vertices $v_1, v_2 \in D$ condition (ii) does not holds. Hence neither condition (i) nor (ii) holds. Which is a contradiction.

Theorem 3.9: For any connected fuzzy graph, $G = (\sigma, \mu)$ then $\gamma(G) \leq \gamma_e(G) \leq \gamma_{ped}(G)$

Proof: Every equitable dominating set is a dominating set. Therefore $\gamma(G) \leq \gamma_e(G)$. By observation (2), $\gamma_e(G) \leq \gamma_{ped}(G)$. Hence, $\gamma(G) \leq \gamma_e(G) \leq \gamma_{ped}(G)$.

Observation 3.10: In a complete fuzzy graph K_n, $\gamma_{ped}(G) = \sigma_0 + \sigma_1$ Where $\sigma_0 = \min_{u \in V} \sigma(u)$ and $\sigma_1 = \min_{v \in V - \{u\}} \sigma(v)$.

Theorem 3.11: For any fuzzy graph $G = (\sigma, \mu)$ of order p, $2.min_{u \in V}\, \sigma(u) \leq \gamma_{ped}(G) \leq p$

Proof: Let D is a minimum paired equitable dominating set of a fuzzy graph G. This D has atleast two vertices u_1, u_2 which is an equitable dominating set and a dominating set also it has a perfect matching. Hence (u_1, u_2) is a strong arc. Suppose $\sigma(u) = \min_{v \in V}\left(\sigma(v)\right)$ then $2 \min \sigma(u) \leq \gamma_{ped}(G)$. By the definition of ped-set, $\gamma_{ped}(G) \leq p$. Hence, $2\, min\, \sigma(u) \leq \gamma_{ped}(G) \leq p$.

Theorem 3.12: Let G and $\bar{G}$ be connected fuzzy graph without isolated vertices, $G = (\sigma, \mu)$ then $\gamma_{ped}(G) + \gamma_{ped}(\bar{G}) \leq 2p$.

Proof: By the definition, $\gamma_{ped}(G) \leq p$. For complementary fuzzy graph the order will be same (i.e. p). The complementary fuzzy graph without isolated vertex and connected $\bar{G} = (\sigma, \mu')$ and $< \bar{D} >$ has a paired equitable dominating set, which implies that, $\gamma_{ped}(\bar{G}) \leq p$. Hence, $\gamma_{ped}(G) + \gamma_{ped}(\bar{G}) \leq 2p$.

Theorem 3.13: Every k – connected graph G^* is an underlying fuzzy graph G has a ped-set if $< D^* >$ has even order does not contain induced subgraph isomorphic to $K^*{}_{1,n+1}$, $n \geq 2$

Proof: Let us consider D be a ped-set of a connected fuzzy graph G and its induced subgraph $< D^* >$ has 4 vertices isomorphic to $K^*{}_{1,3}$. V_1 has only one vertex and other vertices are in V_2. This implies that the induced fuzzy subgraph of a dominating set $< D^* >$ does not contians a perfect matching.

Theorem 3.14: For any fuzzy graph $G = (\sigma, \mu)$, D is a minimal equitable dominating set and $< D >$ has cycle with even number of vertices then G has atleast two distinct minimal ped-sets.

Proof: Let D be the minimal equitable dominating set and $< D >$ has a cycle with even number of vertices satisfies the condition $|d(u) - d(v)| \leq 1, u \in D \; and \; v \in V - D$ of G then $< D >$ has a perfect matching in a fuzzy graph. Here every vertex of D incident with two strong arcs. If there is two adjacent edges, one edge in matching in M_1 and another edge in matching in M_2. Thus D has two distinct perfect matching. Hence G has atleast two distinct minimal ped-sets.

Theorem 3.15: If v is a support vertex in a fuzzy graph G then every ped-set D contains v.

Proof: Let v be support vertex of a connected fuzzy graph $G = (\sigma, \mu)$ and D be a ped-set. Suppose that $v \notin D$ in ped-set, then the end vertex u is not dominated by any other vertices of a fuzzy graph G, this implies that D is not a ped-set. Hence v must be in the every ped-set.

Theorem 3.16: Let H be a spanning fuzzy subgraph of connected fuzzy graph G then $\gamma_{ped}(G) \leq \gamma_{ped}(H)$

Proof: Let $H = (\sigma', \mu')$ be a spanning fuzzy subgraph of fuzzy graph G. Suppose D is a minimum ped-set then D is paired equitable dominate all vertices in $V(H) - D$, this implies that D is a ped-set of G. Hence $\gamma_{ped}(G) \leq \gamma_{ped}(H)$

Theorem 3.17: If $G = (\sigma, \mu)$ be a fuzzy graph of order p, size q then $\gamma_{ped}(G) \leq 2q - p + 2$

Proof: From the definition of the ped-set we have $\gamma_{ped}(G) \leq p$ then $\gamma_{ped}(G) \leq p = 2(p-1) - p + 2 \leq 2q - p + 2$.

Theorem 3.18: For any fuzzy graph G, $p - q \leq \gamma_{ped}(G) \leq p - \Delta_E$

Proof: Let D be a ped-set and $\gamma_{ped} - set$ be the minimum ped-set number of G, then the scalar cardinality of $V - D$ is less than or equal to the scalar cardinality of $V \times V$. Hence $p - q \leq \gamma_{ped}(G)$ (1)
Now let v_i be the vertex with maximum strong arc incident degree Δ_E clearly $V - \{v_i\}$ is ped-set and hence

$$\gamma_{ped}(G) \leq p - \Delta_E \tag{2}$$

From (1) & (2), $p - q \leq \gamma_{ped}(G) \leq p - \Delta_E$ is true.

Theorem 3.19: For any fuzzy graph G, $p - q \leq \gamma_{ped}(G) \leq q - \delta_E$

Proof: Let D be a ped-set and $\gamma_{ped} - set$ be the minimum ped-set number of G, then the scalar cardinality of $V - S$ is less than or equal to the scalar cardinality of $V \times V$. Hence, $p - q \leq \gamma_{ped}(G)$ (1)

Now let v_i be the vertex with minimum strong arc incident degree δ_E clearly $V - \{v_i\}$ is ped-set and hence

$$\gamma_{ped}(G) \leq p - \delta_E \tag{2}$$

From above (1) & (2), $p - q \leq \gamma_{ped}(G) \leq p - \delta_E$ is true.

Corollary 3.20:
 i) For any fuzzy graph G, $|p - q| \leq \gamma_{ped}(G) \leq p - \Delta_N$
 ii) For any fuzzy graph G, $|p - q| \leq \gamma_{ped}(G) \leq q - \delta_N$

Theorem 3.21: Let G be a fuzzy graph without any equitable isolated point with $\Delta^e \leq p - 1$ then $\gamma_{ped}(G) \leq p - \Delta^e$

Proof: Let $v_i \in V(G)$ and $d^e(v_i) = \Delta^e$. Since G is connected fuzzy graph and $\Delta^e \leq p - 1$ there exist two adjacent vertices v_j and v_k such that $v_j \in N(v_i)$ and $v_k \notin N(v_i)$. Now let $S = \{v_k\} \cup \left(N(v_i) - \{v_j\}\right)$. Clearly $V - D$ be the ped-set of G, hence $\gamma_{ped}(G) \leq p - \Delta^e$

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