

## INTUITIONISTIC ANTI - FUZZY HX BI-IDEAL OF A HX RING

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### **ABSTRACT**

***In this paper, we define the notion of intuitionistic anti-fuzzy HX bi-ideal of a HX ring and some of their related properties are investigated. We define the necessity and possibility operators of an intuitionistic fuzzy subset of an intuitionistic anti-fuzzy HX bi-ideal of a HX ring and discuss some of its properties. We discuss the concept of an image, pre-image of an intuitionistic anti-fuzzy HX bi-ideal of a HX ring and homomorphic, anti homomorphic properties of an intuitionistic anti-fuzzy HX bi-ideal of a HX ring are discussed.***

***Keywords:*** intuitionistic fuzzy set, fuzzy HX ring, intuitionistic anti-fuzzy HX bi-ideal, homomorphism and anti homomorphism of an intuitionistic anti-fuzzy HX bi-ideal, anti-image and pre-image of an intuitionistic anti-fuzzy HX bi-ideal.

### **INTRODUCTION**

In 1965, Zadeh [13] introduced the concept of fuzzy subset  $\mu$  of a set  $X$  as a function from  $X$  into the closed unit interval  $[0, 1]$  and studied their properties. Fuzzy set theory is a useful tool to describe situations in which the data or imprecise or vague and it is applied to logic, set theory, group theory, ring theory, real analysis, measure theory etc. In 1967, Rosenfeld [11] defined the idea of fuzzy subgroups and gave some of its properties. Li Hong Xing [5] introduced the concept of HX group. In 1982 Wang-jin Liu [7] introduced the concept of fuzzy ring and fuzzy ideal. With the successful upgrade of algebraic structure of group many researchers considered the algebraic structure of some other algebraic systems in which ring was considered as first. In 1988, Professor Li Hong Xing [6] proposed the concept of HX ring and derived some of its properties, then Professor Zhong [2, 3] gave the structures of HX ring on a class of ring. R.Muthuraj *et.al* [10]. Introduced the concept of fuzzy HX ring. In this paper we define a new algebraic structure of an intuitionistic anti fuzzy HX bi-ideal of a HX ring and investigate some related properties. We define the necessity and possibility operators of an intuitionistic fuzzy subset of an intuitionistic anti-fuzzy HX bi-ideal and discuss some of its properties. Also we introduce the anti image and pre-image of an intuitionistic anti-fuzzy HX bi-ideal and discuss some of its properties.

### **PRELIMINARY**

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper,  $R = (R, +, \cdot)$  is a Ring,  $e$  is the additive identity element of  $R$  and  $xy$ , we mean  $x.y$

**2.1 Definition [6]:** Let  $R$  be a ring. In  $2^R - \{\phi\}$ , a non-empty set  $\mathcal{R} \subset 2^R - \{\phi\}$  with two binary operation '+' and '.' is said to be a HX ring on  $R$  if  $\mathcal{R}$  is a ring with respect to the algebraic operation defined by

- i.  $A + B = \{a + b / a \in A \text{ and } b \in B\}$ , which its null element is denoted by  $Q$ , and the negative element of  $A$  is denoted by  $-A$ .
- ii.  $AB = \{ab / a \in A \text{ and } b \in B\}$ ,
- iii.  $A(B + C) = AB + AC$  and  $(B + C)A = BA + CA$ .

**2.2 Definition:** Let  $R$  be a ring. Let  $H = \{\langle x, \mu(x), \eta(x) \rangle / x \in R\}$  be an intuitionistic fuzzy set defined on a ring  $R$ , where  $\mu : R \rightarrow [0,1]$ ,  $\eta : R \rightarrow [0,1]$  such that  $0 \leq \mu(x) + \eta(x) \leq 1$ . Let  $\mathfrak{R} \subset 2^R - \{\phi\}$  be a HX ring. An intuitionistic fuzzy subset  $\lambda^H = \{\langle A, \lambda^\mu(A), \lambda^\eta(A) \rangle / A \in \mathfrak{R}$  and  $0 \leq \lambda^\mu(A) + \lambda^\eta(A) \leq 1\}$  of  $\mathfrak{R}$  is said to be an intuitionistic fuzzy HX bi-ideal or intuitionistic fuzzy bi-ideal induced by  $H$  of a HX ring  $\mathfrak{R}$  if the following conditions are satisfied. For all  $A, B, C \in \mathfrak{R}$ ,

- i.  $\lambda^\mu(A-B) \geq \min\{\lambda^\mu(A), \lambda^\mu(B)\}$
- ii.  $\lambda^\mu(AB) \geq \min\{\lambda^\mu(A), \lambda^\mu(B)\}$
- iii.  $\lambda^\mu(ABC) \geq \min\{\lambda^\mu(A), \lambda^\mu(C)\}$
- iv.  $\lambda^\eta(A-B) \leq \max\{\lambda^\eta(A), \lambda^\eta(B)\}$
- v.  $\lambda^\eta(AB) \leq \max\{\lambda^\eta(A), \lambda^\eta(B)\}$
- vi.  $\lambda^\eta(ABC) \leq \max\{\lambda^\eta(A), \lambda^\eta(C)\}$

where  $\lambda^\mu(A) = \max\{\mu(x) / \text{for all } x \in A \subseteq R\}$  and  $\lambda^\eta(A) = \min\{\eta(x) / \text{for all } x \in A \subseteq R\}$ .

### 3.1 Intuitionistic anti-fuzzy HX bi-ideal of a HX ring

In this section we define the concept of an intuitionistic anti-fuzzy HX bi-ideal of a HX ring and discuss some related results.

**3.1.1 Definition:** Let  $R$  be a ring. Let  $H = \{\langle x, \mu(x), \eta(x) \rangle / x \in R\}$  be an intuitionistic fuzzy set defined on a ring  $R$ , where  $\mu : R \rightarrow [0,1]$ ,  $\eta : R \rightarrow [0,1]$  such that  $0 \leq \mu(x) + \eta(x) \leq 1$ . Let  $\mathfrak{R} \subset 2^R - \{\phi\}$  be a HX ring. An intuitionistic fuzzy subset  $\lambda_H = \{\langle A, \lambda_\mu(A), \lambda_\eta(A) \rangle / A \in \mathfrak{R}$  and  $0 \leq \lambda_\mu(A) + \lambda_\eta(A) \leq 1\}$  of  $\mathfrak{R}$  is called an intuitionistic anti-fuzzy HX bi-ideal of  $\mathfrak{R}$  or an intuitionistic anti-fuzzy bi-ideal induced by  $H$  if the following conditions are satisfied. For all  $A, B \in \mathfrak{R}$ ,

- i.  $\lambda_\mu(A-B) \leq \max\{\lambda_\mu(A), \lambda_\mu(B)\},$
- ii.  $\lambda_\mu(AB) \leq \max\{\lambda_\mu(A), \lambda_\mu(B)\},$
- iii.  $\lambda_\mu(ABC) \leq \max\{\lambda_\mu(A), \lambda_\mu(C)\},$
- iv.  $\lambda_\eta(A-B) \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\},$
- v.  $\lambda_\eta(AB) \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\},$
- vi.  $\lambda_\eta(ABC) \geq \min\{\lambda_\eta(A), \lambda_\eta(C)\}.$

where  $\lambda_\mu(A) = \min\{\mu(x) / \text{for all } x \in A \subseteq R\}$  and  $\lambda_\eta(A) = \max\{\eta(x) / \text{for all } x \in A \subseteq R\}$ .

**3.1.2 Theorem:** If  $H$  is an intuitionistic anti-fuzzy bi-ideal of a ring  $R$  then the intuitionistic fuzzy subset  $\lambda_H$  is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}$ .

**Proof:** Let  $H$  be an intuitionistic anti-fuzzy bi-ideal of  $R$ .

|   |  |
|---|--|
| i. $\max\{\lambda_\mu(A), \lambda_\mu(B)\}$               | $= \max\{\min\{\mu(x) / \text{for all } x \in A \subseteq R\}, \min\{\mu(y) / \text{for all } y \in B \subseteq R\}\}$ |
|   | $= \max\{\mu(x_0), \mu(y_0)\}$   |
|   | $\geq \mu(x_0 - y_0)$  |
|   | $\geq \min\{\mu(x-y) / \text{for all } x-y \in A-B \subseteq R\}$  |
|   | $\geq \lambda_\mu(A-B)$  |
| $\lambda_\mu(A-B)$  | $\leq \max\{\lambda_\mu(A), \lambda_\mu(B)\}.$   |
| ii. $\max\{\lambda_\mu(A), \lambda_\mu(B)\}$              | $= \max\{\min\{\mu(x) / \text{for all } x \in A \subseteq R\},$  |
| $\min\{\mu(y) / \text{for all } y \in B \subseteq R\}\}$  | $= \max\{\mu(x_0), \mu(y_0)\}$   |
|   | $\geq \mu(x_0 y_0)$  |
|   | $\geq \min\{\mu(xy) / \text{for all } xy \in AB \subseteq R\}$   |
|   | $\geq \lambda_\mu(AB)$   |
| $\lambda_\mu(AB)$   | $\leq \max\{\lambda_\mu(A), \lambda_\mu(B)\}.$   |
| iii. $\max\{\lambda_\mu(A), \lambda_\mu(C)\}$             | $= \max\{\min\{\mu(x) / \text{for all } x \in A \subseteq R\},$  |
| $\min\{\mu(z) / \text{for all } z \in C \subseteq R\}\}$  | $= \max\{\mu(x_0), \mu(z_0)\}$   |
|   | $\geq \mu(x_0 z_0)$  |
|   | $\geq \min\{\mu(xyz) / \text{for all } xyz \in ABC \subseteq R\}$  |
|   | $\geq \lambda_\mu(ABC)$  |
| $\lambda_\mu(ABC)$  | $\leq \max\{\lambda_\mu(A), \lambda_\mu(C)\}.$   |
| iv. $\min\{\lambda_\eta(A), \lambda_\eta(B)\}$            | $= \min\{\max\{\eta(x) / \text{for all } x \in A \subseteq R\},$   |
| $\max\{\eta(y) / \text{for all } y \in B \subseteq R\}\}$ |  |

$$\begin{aligned}
 &= \min\{\eta(x_0), \eta(y_0)\} \\
 &\leq \eta(x_0 - y_0) \\
 &\leq \max\{\eta(x-y) / \text{for all } x-y \in A-B \subseteq R\} \\
 &\leq \lambda_\eta(A-B) \\
 &\geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}. \\
 \lambda_\eta(A-B) \\
 \text{v. } \lambda_\eta(AB) &= \min\{\eta(xy) / \text{for all } x \in A \subseteq R \text{ and } y \in B \subseteq R\} \\
 &= \eta(x_0 y_0), \text{ for } x_0 \in A \text{ and } y_0 \in B. \\
 &\geq \min\{\eta(x_0), \eta(y_0)\} \\
 &= \min\{\{\eta(x) / \text{for all } x \in A \subseteq R}, \{\eta(y) / \text{for all } x \in B \subseteq R\}\} \\
 &\geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}. \\
 &\geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}. \\
 \text{vi. } \lambda_\eta(ABC) &= \min\{\eta(xyz) / \text{for all } x \in A \subseteq R, y \in B \subseteq R \text{ and} \\
 z \in C \subseteq R\} \\
 &= \eta(x_0 y_0 z_0), \text{ for } x_0 \in A, y_0 \in B \text{ and } z_0 \in C. \\
 &\geq \min\{\eta(x_0), \eta(z_0)\} \\
 &= \min\{\{\eta(x) / \text{for all } x \in A \subseteq R}, \{\eta(z) / \text{for all } z \in C \subseteq R\}\} \\
 &\geq \min\{\lambda_\eta(A), \lambda_\eta(C)\}. \\
 &\geq \min\{\lambda_\eta(A), \lambda_\eta(C)\}.
 \end{aligned}$$

Hence,  $\lambda_H$  is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}$ .

### 3.1.3 Remark

- i. If  $H$  is not an intuitionistic anti-fuzzy bi-ideal of  $R$  then the intuitionistic fuzzy subset  $\lambda_H$  of  $\mathfrak{R}$  is an intuitionistic anti-fuzzy HX bi-ideal of  $\mathfrak{R}$ , provided  $|X| \geq 2$  for all  $X \in \mathfrak{R}$ .
- ii. If  $H$  is an intuitionistic fuzzy subset of a ring  $R$  and  $\lambda_H$  be an intuitionistic anti-fuzzy HX bi-ideal on  $\mathfrak{R}$ , such that  $\lambda_\mu(A) = \max\{\mu(x) / \text{for all } x \in A \subseteq R\}$  and  $\lambda_\eta(A) = \min\{\eta(x) / \text{for all } x \in A \subseteq R\}$ , then  $H$  may or may not be an intuitionistic anti-fuzzy bi-ideal of  $R$ .

**3.1.4 Theorem:** Let  $H$  be an intuitionistic fuzzy subset on  $R$ . Let  $\mathfrak{R} \subset 2^R - \{\phi\}$  be a HX ring. If  $\lambda_H$  is an intuitionistic anti-fuzzy HX right ideal of a HX ring  $\mathfrak{R}$  then  $\lambda_H$  is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}$ .

**Proof:** Let  $\lambda_H$  be an intuitionistic anti-fuzzy HX right ideal of a HX ring  $\mathfrak{R}$ .

Then for all  $A, B \in \mathfrak{R}$ ,

$$\begin{aligned}
 \text{i. } \lambda_\mu(A-B) &\leq \max\{\lambda_\mu(A), \lambda_\mu(B)\}, \\
 \text{ii. } \lambda_\mu(AB) &\leq \lambda_\mu(A) \\
 \text{iii. } \lambda_\eta(A-B) &\geq \min\{\lambda_\eta(A), \lambda_\eta(B)\} \\
 \text{iv. } \lambda_\eta(AB) &\geq \lambda_\eta(A).
 \end{aligned}$$

Therefore ,

$$\begin{aligned}
 \lambda_\mu(AB) &\leq \max\{\lambda_\mu(A), \lambda_\mu(B)\} \\
 \lambda_\eta(AB) &\geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}
 \end{aligned}$$

Let  $A, B, C \in \mathfrak{R}$ ,

$$\begin{aligned}
 \lambda_\mu(ABC) &= \lambda_\mu(A(BC)) \\
 &\leq \lambda_\mu(A), \\
 &\leq \max\{\lambda_\mu(A), \lambda_\mu(C)\}. \\
 \lambda_\mu(ABC) &\leq \max\{\lambda_\mu(A), \lambda_\mu(C)\}. \\
 \lambda_\eta(ABC) &= \lambda_\eta(A(BC)) \\
 &\geq \lambda_\eta(A) \\
 &\geq \min\{\lambda_\eta(A), \lambda_\eta(C)\} \\
 \lambda_\eta(ABC) &\geq \min\{\lambda_\eta(A), \lambda_\eta(C)\}.
 \end{aligned}$$

Hence,  $\lambda_H$  is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}$ .

**3.1.5 Theorem:** Let  $H$  be an intuitionistic fuzzy subset on  $R$ . Let  $\mathfrak{R} \subset 2^R - \{\phi\}$  be a HX ring. If  $\lambda_H$  is an intuitionistic anti-fuzzy HX left ideal of a HX ring  $\mathfrak{R}$  then  $\lambda_H$  is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}$ .

**Proof:** It is clear.

**3.1.6 Remark:** Every intuitionistic anti-fuzzy HX (right or left) ideal of a HX ring  $\mathfrak{R}$  is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}$ .

**3.1.7 Theorem:** Let G and H be any two intuitionistic fuzzy sets on R. Let  $\gamma_G$  and  $\lambda_H$  be any two intuitionistic anti-fuzzy HX bi-ideals of a HX ring  $\mathfrak{R}$  then their intersection,  $\gamma_G \cap \lambda_H$  is also an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}$ .

**Proof:** Let  $G = \{\langle x, \alpha(x), \beta(x) \rangle / x \in R\}$  and  $H = \{\langle x, \mu(x), \eta(x) \rangle / x \in R\}$  be any two intuitionistic fuzzy sets defined on a ring R.

Then,  $\gamma_G = \{\langle A, \gamma_\alpha(A), \gamma_\beta(A) \rangle / A \in \mathfrak{R}\}$  and  $\lambda_H = \{\langle A, \lambda_\mu(A), \lambda_\eta(A) \rangle / A \in \mathfrak{R}\}$  be any two intuitionistic anti-fuzzy HX bi-ideals of a HX ring  $\mathfrak{R}$ .

$$\gamma_G \cap \lambda_H = \{\langle A, (\gamma_\alpha \cup \lambda_\mu)(A), (\gamma_\beta \cap \lambda_\eta)(A) \rangle / A \in \mathfrak{R}\}$$

Let  $A, B, C \in \mathfrak{R}$ .

|      |   |        |   |
|------|---|--------|---|
| i.   | $(\gamma_\alpha \cap \lambda_\mu)(A-B)$ | =      | $\min \{\gamma_\alpha(A-B), \lambda_\mu(A-B)\}$   |
|      |   | $\leq$ | $\min \{\max \{\gamma_\alpha(A), \gamma_\alpha(B)\}, \max \{\lambda_\mu(A), \lambda_\mu(B)\}\}$   |
|      |   | =      | $\max \{\min \{\gamma_\alpha(A), \lambda_\mu(A)\}, \min \{\gamma_\alpha(B), \lambda_\mu(B)\}\}$   |
|      |   | =      | $\max \{(\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\alpha \cap \lambda_\mu)(B)\}$               |
| ii.  | $(\gamma_\alpha \cap \lambda_\mu)(AB)$  | $\leq$ | $\max \{(\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\alpha \cap \lambda_\mu)(B)\}.$              |
|      |   | =      | $\min \{\gamma_\alpha(AB), \lambda_\mu(AB)\}$   |
|      |   | $\leq$ | $\min \{\max \{\gamma_\alpha(A), \gamma_\alpha(B)\}, \max \{\lambda_\mu(A), \lambda_\mu(B)\}\}$   |
|      |   | =      | $\max \{\min \{\gamma_\alpha(A), \lambda_\mu(A)\}, \min \{\gamma_\alpha(B), \lambda_\mu(B)\}\}$   |
|      |   | =      | $\max \{(\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\alpha \cap \lambda_\mu)(B)\}$               |
| iii. | $(\gamma_\alpha \cap \lambda_\mu)(ABC)$ | $\leq$ | $\max \{(\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\alpha \cap \lambda_\mu)(B)\}.$              |
|      |   | =      | $\min \{\gamma_\alpha(ABC), \lambda_\mu(ABC)\}$   |
|      |   | $\leq$ | $\min \{\max \{\gamma_\alpha(A), \gamma_\alpha(C)\}, \max \{\lambda_\mu(A), \lambda_\mu(C)\}\}$   |
|      |   | =      | $\max \{\min \{\gamma_\alpha(A), \lambda_\mu(A)\}, \min \{\gamma_\alpha(C), \lambda_\mu(C)\}\}$   |
|      |   | =      | $\max \{(\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\alpha \cap \lambda_\mu)(C)\}$               |
|      |   | $\leq$ | $\max \{(\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\alpha \cap \lambda_\mu)(C)\}.$              |
| iv.  | $(\gamma_\alpha \cap \lambda_\mu)(ABC)$ | =      | $\max \{\gamma_\beta(A-B), \lambda_\eta(A-B)\}$   |
|      |   | $\geq$ | $\max \{(\min \{\gamma_\beta(A), \gamma_\beta(B)\}, \min \{\lambda_\eta(A), \lambda_\eta(B)\})\}$ |
|      |   | =      | $\min \{\max \{\gamma_\beta(A), \lambda_\eta(A)\}, \max \{\gamma_\beta(B), \lambda_\eta(B)\}\}$   |
|      |   | =      | $\min \{(\gamma_\beta \cup \lambda_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(B)\}$               |
| v.   | $(\gamma_\beta \cup \lambda_\mu)(A-B)$  | $\geq$ | $\min \{(\gamma_\beta \cup \lambda_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(B)\}.$              |
|      | $(\gamma_\beta \cup \lambda_\mu)(AB)$   | =      | $\max \{\gamma_\beta(AB), \lambda_\eta(AB)\}$   |
|      |   | $\geq$ | $\max \{(\min \{\gamma_\beta(A), \gamma_\beta(B)\}, \min \{\lambda_\eta(A), \lambda_\eta(B)\})\}$ |
|      |   | =      | $\min \{\max \{\gamma_\beta(A), \lambda_\eta(A)\}, \max \{\gamma_\beta(B), \lambda_\eta(B)\}\}$   |
|      |   | =      | $\min \{(\gamma_\beta \cup \lambda_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(B)\}$               |
| vi.  | $(\gamma_\beta \cup \lambda_\mu)(AB)$   | $\geq$ | $\min \{(\gamma_\beta \cup \lambda_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(B)\}.$              |
|      | $(\gamma_\beta \cup \lambda_\mu)(ABC)$  | =      | $\max \{\gamma_\beta(ABC), \lambda_\eta(ABC)\}$   |
|      |   | $\geq$ | $\max \{(\min \{\gamma_\beta(A), \gamma_\beta(C)\}, \min \{\lambda_\eta(A), \lambda_\eta(C)\})\}$ |
|      |   | =      | $\min \{\max \{\gamma_\beta(A), \lambda_\eta(A)\}, \max \{\gamma_\beta(C), \lambda_\eta(C)\}\}$   |
|      |   | =      | $\min \{(\gamma_\beta \cup \lambda_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(C)\}$               |
|      | $(\gamma_\beta \cup \lambda_\mu)(ABC)$  | $\geq$ | $\min \{(\gamma_\beta \cup \lambda_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(C)\}.$              |

Hence,  $\gamma_G \cap \lambda_H$  is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}$ .

**3.1.8 Theorem:** Let G and H be any two intuitionistic fuzzy sets on R. Let  $\gamma_G$  be an intuitionistic anti-fuzzy HX (right or left) ideal and  $\lambda_H$  be an intuitionistic anti-fuzzy HX (right or left) ideal of a HX ring  $\mathfrak{R}$  then their intersection,  $\gamma_G \cap \lambda_H$  is also an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}$ .

**Proof:** It is clear.

**3.1.9 Theorem:** Let G and H be any two intuitionistic fuzzy sets on R. Let  $\gamma_G$  and  $\lambda_H$  be any two intuitionistic anti-fuzzy HX bi-ideals of a HX ring  $\mathfrak{R}$  then their union,  $\gamma_G \cup \lambda_H$  is also an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}$ .

**Proof:** Let  $G = \{ \langle x, \alpha(x), \beta(x) \rangle / x \in R \}$  and  $H = \{ \langle x, \mu(x), \eta(x) \rangle / x \in R \}$  be any two intuitionistic fuzzy sets defined on a ring  $R$ .

Then,  $\gamma_G = \{ \langle A, \gamma_\alpha(A), \gamma_\beta(A) \rangle / A \in \mathfrak{R} \}$  and  $\lambda_H = \{ \langle A, \lambda_\mu(A), \lambda_\eta(A) \rangle / A \in \mathfrak{R} \}$  be any two intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}$ . Then,

$$\gamma_G \cup \lambda_H = \{ \langle A, (\gamma_\alpha \cup \lambda_\mu)(A), (\gamma_\beta \cap \lambda_\eta)(A) \rangle / A \in \mathfrak{R} \}.$$

Let  $A, B \in \mathfrak{R}$ ,

|      |   |        |   |
|------|---|--------|---|
| i.   | $(\gamma_\alpha \cup \lambda_\mu)(A-B)$ | =      | $\min \{ \gamma_\alpha(A-B), \lambda_\mu(A-B) \}$   |
|      |   | $\geq$ | $\min \{ \max \{ \gamma_\alpha(A), \gamma_\alpha(B) \}, \max \{ \lambda_\mu(A), \lambda_\mu(B) \} \}$ |
|      |   | =      | $\max \{ \min \{ \gamma_\alpha(A), \lambda_\mu(A) \}, \min \{ \gamma_\alpha(B), \lambda_\mu(B) \} \}$ |
|      |   | =      | $\max \{ (\gamma_\alpha \cup \lambda_\mu)(A), (\gamma_\alpha \cup \lambda_\mu)(B) \}$                 |
|      |   | $\geq$ | $\max \{ (\gamma_\alpha \cup \lambda_\mu)(A), (\gamma_\alpha \cup \lambda_\mu)(B) \}$                 |
| ii.  | $(\gamma_\alpha \cup \lambda_\mu)(AB)$  | =      | $\min \{ \gamma_\alpha(AB), \lambda_\mu(AB) \}$   |
|      |   | $\geq$ | $\min \{ \max \{ \gamma_\alpha(A), \gamma_\alpha(B) \}, \max \{ \lambda_\mu(A), \lambda_\mu(B) \} \}$ |
|      |   | =      | $\max \{ \min \{ \gamma_\alpha(A), \lambda_\mu(A) \}, \min \{ \gamma_\alpha(B), \lambda_\mu(B) \} \}$ |
|      |   | =      | $\max \{ (\gamma_\alpha \cup \lambda_\mu)(A), (\gamma_\alpha \cup \lambda_\mu)(B) \}$                 |
|      |   | $\geq$ | $\max \{ (\gamma_\alpha \cup \lambda_\mu)(A), (\gamma_\alpha \cup \lambda_\mu)(B) \}$                 |
| iii. | $(\gamma_\alpha \cup \lambda_\mu)(ABC)$ | =      | $\min \{ \gamma_\alpha(ABC), \lambda_\mu(ABC) \}$   |
|      |   | $\geq$ | $\min \{ \max \{ \gamma_\alpha(A), \gamma_\alpha(C) \}, \max \{ \lambda_\mu(A), \lambda_\mu(C) \} \}$ |
|      |   | =      | $\max \{ \min \{ \gamma_\alpha(A), \lambda_\mu(A) \}, \min \{ \gamma_\alpha(C), \lambda_\mu(C) \} \}$ |
|      |   | =      | $\max \{ (\gamma_\alpha \cup \lambda_\mu)(A), (\gamma_\alpha \cup \lambda_\mu)(C) \}$                 |
|      |   | $\geq$ | $\max \{ (\gamma_\alpha \cup \lambda_\mu)(A), (\gamma_\alpha \cup \lambda_\mu)(C) \}$                 |
| iv.  | $(\gamma_\beta \cap \lambda_\eta)(A-B)$ | =      | $\max \{ \gamma_\beta(A-B), \lambda_\eta(A-B) \}$   |
|      |   | $\leq$ | $\max \{ \min \{ \gamma_\beta(A), \gamma_\beta(B) \}, \min \{ \lambda_\eta(A), \lambda_\eta(B) \} \}$ |
|      |   | =      | $\min \{ \max \{ \gamma_\beta(A), \lambda_\eta(A) \}, \max \{ \gamma_\beta(B), \lambda_\eta(B) \} \}$ |
|      |   | =      | $\min \{ (\gamma_\beta \cap \lambda_\eta)(A), (\gamma_\beta \cap \lambda_\eta)(B) \}$                 |
|      |   | $\leq$ | $\min \{ (\gamma_\beta \cap \lambda_\eta)(A), (\gamma_\beta \cap \lambda_\eta)(B) \}$                 |
| v.   | $(\gamma_\beta \cap \lambda_\eta)(AB)$  | =      | $\max \{ \gamma_\beta(AB), \lambda_\eta(AB) \}$   |
|      |   | $\leq$ | $\max \{ \min \{ \gamma_\beta(A), \gamma_\beta(B) \}, \min \{ \lambda_\eta(A), \lambda_\eta(B) \} \}$ |
|      |   | =      | $\min \{ \max \{ \gamma_\beta(A), \lambda_\eta(A) \}, \max \{ \gamma_\beta(B), \lambda_\eta(B) \} \}$ |
|      |   | =      | $\min \{ (\gamma_\beta \cap \lambda_\eta)(A), (\gamma_\beta \cap \lambda_\eta)(B) \}$                 |
|      |   | $\leq$ | $\min \{ (\gamma_\beta \cap \lambda_\eta)(A), (\gamma_\beta \cap \lambda_\eta)(B) \}$                 |
| vi.  | $(\gamma_\beta \cap \lambda_\eta)(ABC)$ | =      | $\max \{ \gamma_\beta(ABC), \lambda_\eta(ABC) \}$   |
|      |   | $\leq$ | $\max \{ \min \{ \gamma_\beta(A), \gamma_\beta(C) \}, \min \{ \lambda_\eta(A), \lambda_\eta(C) \} \}$ |
|      |   | =      | $\min \{ \max \{ \gamma_\beta(A), \lambda_\eta(A) \}, \max \{ \gamma_\beta(C), \lambda_\eta(C) \} \}$ |
|      |   | =      | $\min \{ (\gamma_\beta \cap \lambda_\eta)(A), (\gamma_\beta \cap \lambda_\eta)(C) \}$                 |
|      |   | $\leq$ | $\min \{ (\gamma_\beta \cap \lambda_\eta)(A), (\gamma_\beta \cap \lambda_\eta)(C) \}$                 |

Hence,  $\gamma_G \cup \lambda_H$  is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}$ .

**3.1.10 Theorem:** Let  $\mu$  be an intuitionistic fuzzy set defined on  $R$ . Let  $\lambda^\mu$  is an intuitionistic fuzzy HX bi-ideal of  $\mathfrak{R}$  if and only if  $(\lambda^\mu)^c$  is an intuitionistic anti-fuzzy HX bi-ideal of  $\mathfrak{R}$ .

**Proof:** Let  $\lambda^\mu$  be an intuitionistic fuzzy HX bi-ideal of  $\mathfrak{R}$ .

Let  $A, B, C \in \mathfrak{R}$

|      |  |        |   |
|------|--|--------|---|
| i.   | $\lambda^\mu(A-B)$                     | $\geq$ | $\min \{ \lambda^\mu(A), \lambda^\mu(B) \}$             |
|      | $\Leftrightarrow 1 - \lambda^\mu(A-B)$ | $\leq$ | $1 - \min \{ \lambda^\mu(A), \lambda^\mu(B) \}$         |
|      | $(\lambda^\mu)^c(A-B)$                 | $\leq$ | $\max \{ (1 - \lambda^\mu(A)), (1 - \lambda^\mu(B)) \}$ |
|      | $(\lambda^\mu)^c(A-B)$                 | $\leq$ | $\max \{ (\lambda^\mu)^c(A), (\lambda^\mu)^c(B) \}$     |
| ii.  | $\lambda^\mu(AB)$                      | $\geq$ | $\min \{ \lambda^\mu(A), \lambda^\mu(B) \}$             |
|      | $\Leftrightarrow 1 - \lambda^\mu(AB)$  | $\leq$ | $1 - \min \{ \lambda^\mu(A), \lambda^\mu(B) \}$         |
|      | $(\lambda^\mu)^c(AB)$                  | $\leq$ | $\max \{ (1 - \lambda^\mu(A)), (1 - \lambda^\mu(B)) \}$ |
|      | $(\lambda^\mu)^c(AB)$                  | $\leq$ | $\max \{ (\lambda^\mu)^c(A), (\lambda^\mu)^c(B) \}$     |
| iii. | $\lambda^\mu(ABC)$                     | $\geq$ | $\min \{ \lambda^\mu(A), \lambda^\mu(C) \}$             |
|      | $\Leftrightarrow 1 - \lambda^\mu(ABC)$ | $\leq$ | $1 - \min \{ \lambda^\mu(A), \lambda^\mu(C) \}$         |

$$\begin{array}{llll}
 \Leftrightarrow & (\lambda^{\mu})^c(ABC) & \leq & \max \{(1-\lambda^{\mu}(A)),(1-\lambda^{\mu}(C))\} \\
 & (\lambda^{\mu})^c(ABC) & \leq & \max \{(\lambda^{\mu})^c(A),(\lambda^{\mu})^c(C)\}. \\
 & \text{iv. } \lambda^{\eta}(A-B) & \leq & \max\{\lambda^{\eta}(A),\lambda^{\eta}(B)\} \\
 & \Leftrightarrow & 1-\lambda^{\eta}(A-B) & \geq 1-\max\{\lambda^{\eta}(A),\lambda^{\eta}(B)\} \\
 & \Leftrightarrow & (\lambda^{\eta})^c(A-B) & \geq \min\{(1-\lambda^{\eta}(A)),(1-\lambda^{\eta}(B))\} \\
 & \Leftrightarrow & (\lambda^{\eta})^c(A-B) & \geq \min\{(\lambda^{\eta})^c(A),(\lambda^{\eta})^c(B)\}. \\
 & \text{v. } \lambda^{\eta}(AB) & \leq & \max\{\lambda^{\eta}(A),\lambda^{\eta}(B)\} \\
 & \Leftrightarrow & 1-\lambda^{\eta}(AB) & \geq 1-\max\{\lambda^{\eta}(A),\lambda^{\eta}(B)\} \\
 & \Leftrightarrow & (\lambda^{\eta})^c(AB) & \geq \min\{(1-\lambda^{\eta}(A)),(1-\lambda^{\eta}(B))\} \\
 & & (\lambda^{\eta})^c(AB) & \geq \min\{(\lambda^{\eta})^c(A),(\lambda^{\eta})^c(B)\}. \\
 & \text{vi. } \lambda^{\eta}(ABC) & \leq & \max\{\lambda^{\eta}(A),\lambda^{\eta}(C)\} \\
 & \Leftrightarrow & 1-\lambda^{\eta}(ABC) & \geq 1-\max\{\lambda^{\eta}(A),\lambda^{\eta}(C)\} \\
 & \Leftrightarrow & (\lambda^{\eta})^c(ABC) & \geq \min\{(1-\lambda^{\eta}(A)),(1-\lambda^{\eta}(C))\} \\
 & & (\lambda^{\eta})^c(ABC) & \geq \min\{(\lambda^{\eta})^c(A),(\lambda^{\eta})^c(C)\}.
 \end{array}$$

Hence,  $(\lambda^{\mu})^c$  is an intuitionistic anti-fuzzy HX bi-ideal on  $\mathfrak{R}$ .

**3.1.11 Definition:** Let  $G = \{\langle x, \alpha(x), \beta(x) \rangle / x \in R\}$  and  $H = \{\langle x, \mu(x), \eta(x) \rangle / x \in R\}$  be any two intuitionistic fuzzy sets defined on a ring  $R$ . Let  $\mathfrak{R}_1 \subset 2^R_1 - \{\phi\}$  and  $\mathfrak{R}_2 \subset 2^R_2 - \{\phi\}$  be any two HX rings. Let  $\gamma_G = \{\langle A, \gamma_{\alpha}(A), \gamma_{\beta}(A) \rangle / A \in \mathfrak{R}\}$  and  $\lambda_H = \{\langle A, \lambda_{\mu}(A), \lambda_{\eta}(A) \rangle / A \in \mathfrak{R}\}$  be any two intuitionistic fuzzy subsets of a HX ring  $\mathfrak{R}$ , then the anti product of  $\gamma_G$  and  $\lambda_H$  is defined as

where,  $(\gamma_G \times \lambda_H) = \{\langle (A, B), (\gamma_{\alpha} \cup \lambda_{\mu})(A, B), (\gamma_{\beta} \cap \lambda_{\eta})(A, B) \rangle / (A, B) \in \mathfrak{R}_1 \times \mathfrak{R}_2\}$ ,  
 $(\gamma_{\alpha} \cup \lambda_{\mu})(A, B) = \min \{\gamma_{\alpha}(A), \lambda_{\mu}(B)\}$ , for all  $(A, B) \in \mathfrak{R}_1 \times \mathfrak{R}_2$ ,  
 $(\gamma_{\beta} \cap \lambda_{\eta})(A, B) = \max \{\gamma_{\beta}(A), \lambda_{\eta}(B)\}$ , for all  $(A, B) \in \mathfrak{R}_1 \times \mathfrak{R}_2$ .

**3.1.12 Theorem:** Let  $G$  and  $H$  be any two intuitionistic fuzzy sets of  $R_1$  and  $R_2$  respectively. Let  $\mathfrak{R}_1 \subset 2^R_1 - \{\phi\}$  and  $\mathfrak{R}_2 \subset 2^R_2 - \{\phi\}$  be any two HX rings. If  $\gamma_G$  and  $\lambda_H$  are any two intuitionistic anti-fuzzy HX bi-ideals of  $\mathfrak{R}_1$  and  $\mathfrak{R}_2$  respectively then,  $\gamma_G \times \lambda_H$  is also an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}_1 \times \mathfrak{R}_2$ .

**Proof:** Let  $G = \{\langle x, \alpha(x), \beta(x) \rangle / x \in R_1\}$  and  $H = \{\langle x, \mu(x), \eta(x) \rangle / x \in R_2\}$  be any two intuitionistic fuzzy sets defined on a ring  $R_1$  and  $R_2$  respectively.

Then,  $\gamma_G = \{\langle A, \gamma_{\alpha}(A), \gamma_{\beta}(A) \rangle / A \in \mathfrak{R}_1\}$  and  $\lambda_H = \{\langle A, \lambda_{\mu}(A), \lambda_{\eta}(A) \rangle / A \in \mathfrak{R}_2\}$  be any two intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}_1$  and  $\mathfrak{R}_2$  respectively. Then,

$(\gamma_G \times \lambda_H) = \{\langle (A, B), (\gamma_{\alpha} \cap \lambda_{\mu})(A, B), (\gamma_{\beta} \cup \lambda_{\eta})(A, B) \rangle / (A, B) \in \mathfrak{R}_1 \times \mathfrak{R}_2\}$ ,  
where  $(\gamma_{\alpha} \cap \lambda_{\mu})(A, B) = \max \{\gamma_{\alpha}(A), \lambda_{\mu}(B)\}$ , for all  $(A, B) \in \mathfrak{R}_1 \times \mathfrak{R}_2$ ,  
 $(\gamma_{\beta} \cup \lambda_{\eta})(A, B) = \min \{\gamma_{\beta}(A), \lambda_{\eta}(B)\}$ , for all  $(A, B) \in \mathfrak{R}_1 \times \mathfrak{R}_2$ .

Let  $A, B, I \in \mathfrak{R}_1 \times \mathfrak{R}_2$ , where  $A = (C, D)$ ,  $B = (E, F)$  and  $I = (J, K)$ ,

$$\begin{array}{llll}
 \text{i. } & (\gamma_{\alpha} \cap \lambda_{\mu})(A-B) & = & \max \{\gamma_{\alpha}(C-E), \lambda_{\mu}(D-F)\} \\
 & & \geq & \max \{\max \{\gamma_{\alpha}(C), \gamma_{\alpha}(E)\}, \max \{\lambda_{\mu}(D), \lambda_{\mu}(F)\}\} \\
 & & = & \max \{\max \{\gamma_{\alpha}(C), \lambda_{\mu}(D)\}, \max \{\gamma_{\alpha}(D), \lambda_{\mu}(E)\}\} \\
 & & = & \max \{(\gamma_{\alpha} \cap \lambda_{\mu})(C, D), (\gamma_{\alpha} \cap \lambda_{\mu})(E, F)\} \\
 & & \geq & \max \{(\gamma_{\alpha} \cap \lambda_{\mu})(A), (\gamma_{\alpha} \cap \lambda_{\mu})(B)\}. \\
 \text{ii. } & (\gamma_{\alpha} \cap \lambda_{\mu})(AB) & = & (\gamma_{\alpha} \cap \lambda_{\mu})((C, D) \cdot (E, F)) \\
 & & = & (\gamma_{\alpha} \cap \lambda_{\mu})(CE, DF) \\
 & & = & \max \{\gamma_{\alpha}(CE), \lambda_{\mu}(DF)\} \\
 & & \geq & \max \{\max \{\gamma_{\alpha}(C), \gamma_{\alpha}(E)\}, \max \{\lambda_{\mu}(D), \lambda_{\mu}(F)\}\} \\
 & & = & \max \{\max \{\gamma_{\alpha}(C), \lambda_{\mu}(D)\}, \max \{\gamma_{\alpha}(E), \lambda_{\mu}(F)\}\} \\
 & & = & \max \{(\gamma_{\alpha} \cap \lambda_{\mu})(C, D), (\gamma_{\alpha} \cap \lambda_{\mu})(E, F)\} \\
 & & \geq & \max \{(\gamma_{\alpha} \cap \lambda_{\mu})(A), (\gamma_{\alpha} \cap \lambda_{\mu})(B)\}. \\
 \text{iii. } & (\gamma_{\alpha} \cap \lambda_{\mu})(ABI) & = & (\gamma_{\alpha} \cap \lambda_{\mu})((C, D) \cdot (E, F) \cdot (J, K)) \\
 & & = & (\gamma_{\alpha} \cap \lambda_{\mu})(CEJ, DFK) \\
 & & = & \max \{\gamma_{\alpha}(CEJ), \lambda_{\mu}(DFK)\} \\
 & & \geq & \max \{\max \{\gamma_{\alpha}(C), \gamma_{\alpha}(J)\}, \max \{\lambda_{\mu}(D), \lambda_{\mu}(K)\}\} \\
 & & = & \max \{\max \{\gamma_{\alpha}(C), \lambda_{\mu}(D)\}, \max \{\gamma_{\alpha}(J), \lambda_{\mu}(K)\}\} \\
 & & = & \max \{(\gamma_{\alpha} \cap \lambda_{\mu})(C, D), (\gamma_{\alpha} \cap \lambda_{\mu})(J, K)\}
 \end{array}$$

|     |  |
|-----|--|
| iv. | $(\gamma_\alpha \cap \lambda_\mu)(AB) \geq \max\{(\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\alpha \cap \lambda_\mu)(B)\}.$<br>$= \min\{\gamma_\beta(C-E), \lambda_\eta(D-F)\}$<br>$\leq \min\{\min\{\gamma_\beta(C), \gamma_\beta(E)\}, \min\{\lambda_\eta(D), \lambda_\eta(F)\}\}$<br>$= \min\{\min\{\gamma_\beta(C), \lambda_\eta(D)\}, \min\{\gamma_\beta(E), \lambda_\eta(F)\}\}$<br>$= \min\{(\gamma_\beta \cup \lambda_\eta)(C, D), (\gamma_\beta \cup \lambda_\eta)(E, F)\}$<br>$\leq \min\{(\gamma_\beta \cup \lambda_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(B)\}.$                                 |
| v.  | $(\gamma_\beta \cup \lambda_\eta)(A-B) = (\gamma_\beta \cup \lambda_\eta)((C, D) \cdot (E, F))$<br>$= (\gamma_\beta \cup \lambda_\eta)(CE, DF)$<br>$= \min\{\gamma_\beta(CE), \lambda_\eta(DF)\}$<br>$\leq \min\{\min\{\gamma_\beta(C), \gamma_\beta(E)\}, \min\{\lambda_\eta(D), \lambda_\eta(F)\}\}$<br>$= \min\{\min\{\gamma_\beta(C), \lambda_\eta(D)\}, \min\{\gamma_\beta(E), \lambda_\eta(F)\}\}$<br>$= \min\{(\gamma_\beta \cup \lambda_\eta)(C, D), (\gamma_\beta \cup \lambda_\eta)(E, F)\}$<br>$\leq \min\{(\gamma_\beta \cup \lambda_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(B)\}.$                 |
| vi. | $(\gamma_\beta \cup \lambda_\eta)(AB) = (\gamma_\beta \cup \lambda_\eta)((C, D) \cdot (E, F) \cdot (J, K))$<br>$= (\gamma_\beta \cup \lambda_\eta)(CEJ, DFK)$<br>$= \min\{\gamma_\beta(CEJ), \lambda_\eta(DFK)\}$<br>$\leq \min\{\min\{\gamma_\beta(C), \gamma_\beta(J)\}, \min\{\lambda_\eta(D), \lambda_\eta(K)\}\}$<br>$= \min\{\min\{\gamma_\beta(C), \lambda_\eta(D)\}, \min\{\gamma_\beta(J), \lambda_\eta(K)\}\}$<br>$= \min\{(\gamma_\beta \cup \lambda_\eta)(C, D), (\gamma_\beta \cup \lambda_\eta)(J, K)\}$<br>$\leq \min\{(\gamma_\beta \cup \lambda_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(I)\}.$ |

Hence,  $\gamma^G \times \lambda^H$  is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}$ .

**3.1.13 Definition:** Let  $H$  be an intuitionistic fuzzy set of  $R$ . Let  $\mathfrak{R} \subset 2^R - \{\phi\}$  be a HX ring. Let  $\lambda_H$  be an intuitionistic fuzzy set of  $\mathfrak{R}$ . We define the following “necessity” and possibility” operations:

$$\begin{aligned}\square \lambda_H &= \{\langle A, \lambda_\mu(A), 1-\lambda_\mu(A) \rangle / A \in \mathfrak{R}\} \\ \Diamond \lambda_H &= \{\langle A, 1-\lambda_\eta(A), \lambda_\eta(A) \rangle / A \in \mathfrak{R}\}.\end{aligned}$$

**3.1.14 Theorem:** Let  $H$  be an intuitionistic fuzzy set on  $R$ . Let  $\lambda_H$  be an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}$  then  $\square \lambda_H$  is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}$ .

**Proof:** Let  $\lambda_H$  be an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}$ . Then,

$$\begin{array}{lll} i. & \lambda_\mu(A-B) & \leq \max\{\lambda_\mu(A), \lambda_\mu(B)\}, \\ ii. & \lambda_\mu(AB) & \leq \max\{\lambda_\mu(A), \lambda_\mu(B)\}, \\ iii. & \lambda_\mu(AB) & \leq \max\{\lambda_\mu(A), \lambda_\mu(C)\}, \\ iv. & \lambda_\eta(A-B) & \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}, \\ v. & \lambda_\eta(AB) & \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}, \\ vi. & \lambda_\eta(ABC) & \geq \min\{\lambda_\eta(A), \lambda_\eta(C)\}. \end{array}$$

Now,

$$\begin{aligned}\lambda_\mu(A-B) &\leq \max\{\lambda_\mu(A), \lambda_\mu(B)\} \\ 1-\lambda_\mu(A-B) &\geq 1 - \max\{\lambda_\mu(A), \lambda_\mu(B)\} \\ &\geq \min\{1-\lambda_\mu(A), 1-\lambda_\mu(B)\}.\end{aligned}$$

That is,

$$1-\lambda_\mu(A-B) \geq \min\{1-\lambda_\mu(A), 1-\lambda_\mu(B)\}.$$

We have,

$$\begin{aligned}\lambda_\mu(AB) &\leq \max\{\lambda_\mu(A), \lambda_\mu(B)\} \\ 1-\lambda_\mu(AB) &\geq 1 - \max\{\lambda_\mu(A), \lambda_\mu(B)\} \\ &\geq \min\{1-\lambda_\mu(A), 1-\lambda_\mu(B)\}.\end{aligned}$$

That is,

$$1-\lambda_\mu(AB) \geq \min\{1-\lambda_\mu(A), 1-\lambda_\mu(B)\}.$$

We have,

$$\begin{aligned}\lambda_\mu(ABC) &\leq \max\{\lambda_\mu(A), \lambda_\mu(C)\} \\ 1-\lambda_\mu(ABC) &\geq 1 - \max\{\lambda_\mu(A), \lambda_\mu(C)\} \\ &\geq \min\{1-\lambda_\mu(A), 1-\lambda_\mu(C)\}.\end{aligned}$$

That is,

$$1-\lambda_\mu(ABC) \geq \min\{1-\lambda_\mu(A), 1-\lambda_\mu(C)\}.$$

Hence,  $\square \lambda_H$  is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}$ .

**3.1.15 Theorem:** Let  $H$  be an intuitionistic fuzzy set on  $R$ . Let  $\lambda_H$  be an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}$  then  $\Diamond\lambda_H$  is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}$ .

**Proof:** Let  $\lambda_H$  be an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}$ . Then,

|      |   |
|------|---|
| i.   | $\lambda_\mu(A - B) \leq \max \{\lambda_\mu(A), \lambda_\mu(B)\},$    |
| ii.  | $\lambda_\mu(AB) \leq \max \{\lambda_\mu(A), \lambda_\mu(B)\},$       |
| iii. | $\lambda_\mu(AB) \leq \max \{\lambda_\mu(A), \lambda_\mu(C)\},$       |
| iv.  | $\lambda_\eta(A - B) \geq \min \{\lambda_\eta(A), \lambda_\eta(B)\},$ |
| v.   | $\lambda_\eta(AB) \geq \min \{\lambda_\eta(A), \lambda_\eta(B)\},$    |
| vi.  | $\lambda_\eta(ABC) \geq \min \{\lambda_\eta(A), \lambda_\eta(C)\}.$   |

Now,

|  |
|--|
| $\lambda_\eta(A - B) \geq \min \{\lambda_\eta(A), \lambda_\eta(B)\}$         |
| $1 - \lambda_\eta(A - B) \leq 1 - \min \{\lambda_\eta(A), \lambda_\eta(B)\}$ |
| $\leq \max \{1 - \lambda_\eta(A), 1 - \lambda_\eta(B)\}.$                    |

That is,

$$1 - \lambda_\eta(A - B) \leq \max \{1 - \lambda_\eta(A), 1 - \lambda_\eta(B)\}.$$

We have,

|   |
|---|
| $\lambda_\eta(AB) \geq \min \{\lambda_\eta(A), \lambda_\eta(B)\}$         |
| $1 - \lambda_\eta(AB) \leq 1 - \min \{\lambda_\eta(A), \lambda_\eta(B)\}$ |
| $\leq \max \{1 - \lambda_\eta(A), 1 - \lambda_\eta(B)\}.$                 |

That is,

$$1 - \lambda_\eta(AB) \leq \max \{1 - \lambda_\eta(A), 1 - \lambda_\eta(B)\}.$$

We have,

|  |
|--|
| $\lambda_\eta(ABC) \geq \min \{\lambda_\eta(A), \lambda_\eta(C)\}$         |
| $1 - \lambda_\eta(ABC) \leq 1 - \min \{\lambda_\eta(A), \lambda_\eta(C)\}$ |
| $\leq \max \{1 - \lambda_\eta(A), 1 - \lambda_\eta(C)\}.$                  |

That is,

$$1 - \lambda_\eta(ABC) \leq \max \{1 - \lambda_\eta(A), 1 - \lambda_\eta(C)\}.$$

Hence,  $\Diamond\lambda_H$  is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}$ .

#### 4.2 Homomorphism and anti homomorphism

In this section, we introduce the concept of an image, pre-image of an intuitionistic fuzzy HX bi-ideal of a HX ring and discuss its properties under homomorphism and anti homomorphism.

**4.2.1 Definition:** Let  $R_1$  and  $R_2$  be any two rings. Let  $\mathfrak{R}_1 \subset 2^{R_1} - \{\phi\}$  and  $\mathfrak{R}_2 \subset 2^{R_2} - \{\phi\}$  be any two HX rings defined on  $R_1$  and  $R_2$  respectively. Let  $H$  and  $G$  be any two intuitionistic fuzzy subsets in  $R_1$  and  $R_2$  respectively. Let  $\lambda_H$  and  $\gamma_G$  be any two intuitionistic anti-fuzzy HX bi-ideals of HX rings  $\mathfrak{R}_1$  and  $\mathfrak{R}_2$  respectively induced by  $H$  and  $G$ . Let  $f: \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$  be a mapping then the anti image of  $\lambda_H$  denoted as  $f(\lambda_H)$  is an intuitionistic fuzzy subset of  $\mathfrak{R}_2$  and is defined as for each  $U \in \mathfrak{R}_2$ ,

$$(f(\lambda_\mu))(U) = \begin{cases} \inf \{\lambda_\mu(X): X \in f^{-1}(U)\}, & \text{if } f^{-1}(U) \neq \phi \\ 0 & \text{, otherwise} \end{cases}$$

$$(f(\lambda_\eta))(U) = \begin{cases} \sup \{\lambda_\eta(X): X \in f^{-1}(U)\}, & \text{if } f^{-1}(U) \neq \phi \\ 1 & \text{, otherwise} \end{cases}$$

Also the pre-image of  $\gamma_G$  denoted as  $f^{-1}(\gamma_G)$  under  $f$  is an intuitionistic fuzzy subset of  $\mathfrak{R}_1$  defined as for each  $X \in \mathfrak{R}_2$ ,  $(f^{-1}(\gamma_\alpha))(X) = \gamma_\alpha(f(X))$  and  $(f^{-1}(\gamma_\beta))(X) = \gamma_\beta(f(X))$ .

**4.2.2 Theorem:** Let  $\mathfrak{R}_1$  and  $\mathfrak{R}_2$  be any two HX rings on the rings  $R_1$  and  $R_2$  respectively. Let  $f: \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$  be a homomorphism onto HX rings. Let  $H$  be an intuitionistic fuzzy subset of  $R_1$ . Let  $\lambda_H$  be an intuitionistic anti-fuzzy HX bi-ideal of  $\mathfrak{R}_1$  then  $f(\lambda_H)$  is an intuitionistic anti-fuzzy HX bi-ideal of  $\mathfrak{R}_2$ , if  $\lambda_H$  has an infimum property and  $\lambda_H$  is  $f$ -invariant.

**Proof:** Let  $H = \{(x, \mu(x), \eta(x)) / x \in R_1\}$  be an intuitionistic fuzzy sets defined on a ring  $R_1$ .

Then,  $\lambda_H = \{ (X, \lambda_\mu(X), \lambda_\eta(X)) / X \in \mathfrak{R}_1 \}$  be an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}_1$ .

Then,  $f(\lambda_H) = \{ (f(X), f(\lambda_\mu)(f(X)), f(\lambda_\eta)(f(X))) / X \in \mathfrak{R}_1 \}$ .

There exist  $X, Y \in \mathfrak{R}_1$  such that  $f(X), f(Y) \in \mathfrak{R}_2$ ,

$$\begin{aligned}
 \text{i. } & (f(\lambda_\mu))(f(X) - f(Y)) & = & (f(\lambda_\mu))(f(X-Y)), \\
 & & = & \lambda_\mu(X-Y) \\
 & & \leq & \max\{\lambda_\mu(X), \lambda_\mu(Y)\} \\
 & & = & \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\} \\
 & & \leq & \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\} \\
 \text{ii. } & (f(\lambda_\mu))(f(X)f(Y)) & = & (f(\lambda_\mu))(f(XY)) \\
 & & = & \lambda_\mu(XY) \\
 & & \leq & \max\{\lambda_\mu(X), \lambda_\mu(Y)\} \\
 & & = & \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\} \\
 & & \leq & \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\}. \\
 \text{iii. } & (f(\lambda_\mu))(f(X)f(Y)f(Z)) & = & (f(\lambda_\mu))(f(XYZ)) \\
 & & = & \lambda_\mu(XYZ) \\
 & & \leq & \max\{\lambda_\mu(X), \lambda_\mu(Z)\} \\
 & & = & \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Z))\} \\
 & & \leq & \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Z))\}. \\
 \text{iv. } & (f(\lambda_\eta))(f(X) - f(Y)) & = & (f(\lambda_\eta))(f(X-Y)), \\
 & & = & \lambda_\eta(X-Y) \\
 & & \geq & \min\{\lambda_\eta(X), \lambda_\eta(Y)\} \\
 & & = & \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\} \\
 & & \geq & \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\} \\
 & & = & (f(\lambda_\eta))(f(XY)) \\
 & & = & \lambda_\eta(XY) \\
 & & \geq & \min\{\lambda_\eta(X), \lambda_\eta(Y)\} \\
 & & = & \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\} \\
 & & \geq & \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\}. \\
 \text{v. } & (f(\lambda_\eta))(f(X)f(Y)) & = & (f(\lambda_\eta))(f(XYZ)) \\
 & & = & \lambda_\eta(XYZ) \\
 & & \geq & \min\{\lambda_\eta(X), \lambda_\eta(Z)\} \\
 & & = & \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Z))\} \\
 & & \geq & \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Z))\}. \\
 \text{vi. } & (f(\lambda_\eta))(f(X)f(Y)f(Z)) & = & (f(\lambda_\eta))(f(XYZ)) \\
 & & = & \lambda_\eta(XYZ) \\
 & & \geq & \min\{\lambda_\eta(X), \lambda_\eta(Z)\} \\
 & & = & \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Z))\} \\
 & & \geq & \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Z))\}.
 \end{aligned}$$

Hence,  $f(\lambda_H)$  is an intuitionistic anti-fuzzy HX bi-ideal of  $\mathfrak{R}_2$ .

**4.6.3 Theorem:** Let  $\mathfrak{R}_1$  and  $\mathfrak{R}_2$  be any two HX rings on  $R_1$  and  $R_2$  respectively. Let  $f: \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$  be a homomorphism on HX rings. Let  $G$  be an intuitionistic fuzzy subset of  $R_2$ . Let  $\gamma_G$  be an intuitionistic anti-fuzzy HX bi-ideal of  $\mathfrak{R}_2$ , then  $f^{-1}(\gamma_G)$  is an intuitionistic anti-fuzzy HX bi-ideal of  $\mathfrak{R}_1$ .

**Proof:** Let  $G = \{\langle y, \alpha(y), \beta(y) \rangle / y \in R_2\}$  be an intuitionistic fuzzy sets defined on a ring  $R_2$ .

Then,  $\gamma_G = \{\langle Y, \gamma_\alpha(Y), \gamma_\beta(Y) \rangle / Y \in \mathfrak{R}_2\}$  be an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}_2$ .

Then,  $f^{-1}(\gamma_G) = \{\langle X, f^{-1}(\gamma_\alpha)(X), f^{-1}(\gamma_\beta)(X) \rangle / X \in \mathfrak{R}_1\}$ .

For any  $X, Y \in \mathfrak{R}_1$ ,  $f(X), f(Y) \in \mathfrak{R}_2$ ,

$$\begin{aligned}
 \text{i. } & (f^{-1}(\gamma_\alpha))(X-Y) & = & \gamma_\alpha(f(X-Y)) \\
 & & = & \gamma_\alpha(f(X) - f(Y)) \\
 & & \leq & \max\{\gamma_\alpha(f(X)), \gamma_\alpha(f(Y))\} \\
 & & = & \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\} \\
 \text{ii. } & (f^{-1}(\gamma_\alpha))(XY) & \leq & \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\}. \\
 & & = & \gamma_\alpha(f(XY)) \\
 & & = & \gamma_\alpha(f(X)f(Y)) \\
 & & \leq & \max\{\gamma_\alpha(f(X)), \gamma_\alpha(f(Y))\} \\
 & & = & \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\} \\
 \text{iii. } & (f^{-1}(\gamma_\alpha))(XYZ) & \leq & \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\} \\
 & & = & \gamma_\alpha(f(XYZ)) \\
 & & = & \gamma_\alpha(f(Z)f(Y)f(X)) \\
 & & \leq & \max\{\gamma_\alpha(f(Z)), \gamma_\alpha(f(X))\}
 \end{aligned}$$

|     |  |
|-----|--|
|     | $\begin{aligned} &= \max \{\gamma_\alpha(f(X)), \gamma_\alpha(f(Z))\} \\ &= \max \{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Z)\} \\ &\leq \max \{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Z)\} \\ &= \gamma_\beta(f(X-Y)) \\ &= \gamma_\beta(f(X) - f(Y)) \\ &\geq \min \{\gamma_\beta(f(X)), \gamma_\beta(f(Y))\} \\ &= \min \{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\} \\ &\geq \min \{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\}. \\ &= \gamma_\beta(f(XY)) \\ &= \gamma_\beta(f(X)f(Y)) \\ &\geq \min \{\gamma_\beta(f(X)), \gamma_\beta(f(Y))\} \\ &= \min \{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\} \\ &\geq \min \{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\}. \\ &= \gamma_\beta(f(XYZ)) \\ &= \gamma_\beta(f(Z)f(Y)f(X)) \\ &\geq \min \{\gamma_\beta(f(Z)), \gamma_\beta(f(X))\} \\ &= \min \{\gamma_\beta(f(X)), \gamma_\beta(f(Z))\} \\ &= \min \{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Z)\} \\ &\geq \min \{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Z)\}. \end{aligned}$ |
| iv. | $(f^{-1}(\gamma_\alpha))(XYZ)$   |
| v.  | $(f^{-1}(\gamma_\beta))(XY)$   |
| vi. | $\begin{aligned} &(f^{-1}(\gamma_\beta))(XY) \\ &(f^{-1}(\gamma_\beta))(XYZ) \\ &(f^{-1}(\gamma_\beta))(XYZ) \end{aligned}$  |

Hence,  $f^{-1}(\gamma_G)$  is an intuitionistic anti-fuzzy HX bi-ideal of  $\mathfrak{R}_1$ .

**4.6.4 Theorem:** Let  $\mathfrak{R}_1$  and  $\mathfrak{R}_2$  be any two HX rings on the rings  $R_1$  and  $R_2$  respectively. Let  $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$  be an anti homomorphism onto HX rings. Let  $H$  be an intuitionistic fuzzy subset of  $R_1$ . Let  $\lambda_H$  be an intuitionistic anti-fuzzy HX bi-ideal of  $\mathfrak{R}_1$  then  $f(\lambda_H)$  is an intuitionistic anti-fuzzy HX bi-ideal of  $\mathfrak{R}_2$ , if  $\lambda_H$  has an infimum property and  $\lambda_H$  is  $f$ -invariant.

**Proof:** Let  $H = \{\langle x, \mu(x), \eta(x) \rangle / x \in R_1\}$  be an intuitionistic fuzzy sets defined on a ring  $R_1$ .

Then,  $\lambda_H = \{\langle X, \lambda_\mu(X), \lambda_\eta(X) \rangle / X \in R_1\}$  be an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}_1$ .

Then,  $f(\lambda_H) = \{\langle f(X), f(\lambda_\mu)(f(X)), f(\lambda_\eta)(f(X)) \rangle / X \in R_1\}$ .

There exist  $X, Y \in \mathfrak{R}_1$  such that  $f(X), f(Y) \in \mathfrak{R}_2$ ,

|      |   |
|------|---|
|      | $\begin{aligned} i. \quad (f(\lambda_\mu))(f(X) - f(Y)) &= (f(\lambda_\mu))(f(X-Y)), \\ &= \lambda_\mu(X-Y) \\ &\leq \max \{\lambda_\mu(X), \lambda_\mu(Y)\} \\ &= \max \{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\} \\ &\leq \max \{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\} \\ ii. \quad (f(\lambda_\mu))(f(X)f(Y)) &= (f(\lambda_\mu))(f(YX)) \\ &= \lambda_\mu(YX) \\ &\leq \max \{\lambda_\mu(Y), \lambda_\mu(X)\} \\ &= \max \{\lambda_\mu(X), \lambda_\mu(Y)\} \\ &= \max \{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\} \\ &\leq \max \{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\}. \\ iii. \quad (f(\lambda_\mu))(f(X)f(Y)f(Z)) &= (f(\lambda_\mu))(f(ZYX)) \\ &= \lambda_\mu(ZYX) \\ &\leq \max \{\lambda_\mu(Z), \lambda_\mu(X)\} \\ &= \max \{\lambda_\mu(X), \lambda_\mu(Z)\} \\ &= \max \{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Z))\} \\ &\leq \max \{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Z))\}. \\ iv. \quad (f(\lambda_\eta))(f(X) - f(Y)) &= (f(\lambda_\eta))(f(X-Y)), \\ &= \lambda_\eta(X-Y) \\ &\geq \min \{\lambda_\eta(X), \lambda_\eta(Y)\} \\ &= \min \{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\} \\ &\geq \min \{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\} \\ v. \quad (f(\lambda_\eta))(f(X)f(Y)) &= (f(\lambda_\eta))(f(YX)) \\ &= \lambda_\eta(YX) \\ &\geq \min \{\lambda_\eta(Y), \lambda_\eta(X)\} \end{aligned}$ |
| i.   | $(f(\lambda_\mu))(f(X) - f(Y))$   |
| ii.  | $(f(\lambda_\mu))(f(X)f(Y))$  |
| iii. | $(f(\lambda_\mu))(f(X)f(Y)f(Z))$  |
| iv.  | $(f(\lambda_\eta))(f(X) - f(Y))$  |
| v.   | $(f(\lambda_\eta))(f(X)f(Y))$   |

$$\begin{aligned}
 &= \min \{\lambda_\eta(X), \lambda_\eta(Y)\} \\
 &= \min \{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\} \\
 &\geq \min \{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\}. \\
 \text{vi. } (f(\lambda_\eta))(f(X)f(Y) f(Z)) &= (f(\lambda_\eta))(f(ZYX)) \\
 &= \lambda_\eta(ZYX) \\
 &\geq \min \{\lambda_\eta(Z), \lambda_\eta(X)\} \\
 &= \min \{\lambda_\eta(X), \lambda_\eta(Z)\} \\
 &= \min \{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Z))\} \\
 &\geq \min \{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Z))\}.
 \end{aligned}$$

Hence,  $f(\lambda_H)$  is an intuitionistic anti-fuzzy HX bi-ideal of  $\mathfrak{R}_2$ .

**4.6.5 Theorem:** Let  $\mathfrak{R}_1$  and  $\mathfrak{R}_2$  be any two HX rings on  $R_1$  and  $R_2$  respectively. Let  $f: \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$  be an anti homomorphism on HX rings. Let  $G$  be an intuitionistic fuzzy subset of  $R_2$ . Let  $\gamma_G$  be an intuitionistic anti-fuzzy HX bi-ideal of  $\mathfrak{R}_2$ , then  $f^{-1}(\gamma_G)$  is an intuitionistic anti-fuzzy HX bi-ideal of  $\mathfrak{R}_1$ .

**Proof:** Let  $G = \{\langle y, \alpha(y), \beta(y) \rangle / y \in R_2\}$  be an intuitionistic fuzzy sets defined on a ring  $R_2$ . Then,  $\gamma_G = \{\langle Y, \gamma_\alpha(Y), \gamma_\beta(Y) \rangle / Y \in \mathfrak{R}_2\}$  be an intuitionistic anti-fuzzy HX bi-ideal of a HX ring  $\mathfrak{R}_2$ . Then,  $f^{-1}(\gamma_G) = \{\langle X, f^{-1}(\gamma_\alpha)(X), f^{-1}(\gamma_\beta)(X) \rangle / X \in \mathfrak{R}_1\}$ .

For any  $X, Y \in \mathfrak{R}_1, f(X), f(Y) \in \mathfrak{R}_2$ ,

$$\begin{aligned}
 \text{i. } (f^{-1}(\gamma_\alpha))(X-Y) &= \gamma_\alpha(f(X-Y)) \\
 &= \gamma_\alpha(f(X) - f(Y)) \\
 &\leq \max \{\gamma_\alpha(f(X)), \gamma_\alpha(f(Y))\} \\
 &= \max \{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\} \\
 &\leq \max \{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\}. \\
 \text{ii. } (f^{-1}(\gamma_\alpha))(XY) &= \gamma_\alpha(f(XY)) \\
 &= \gamma_\alpha(f(Y)f(X)) \\
 &\leq \max \{\gamma_\alpha(f(Y)), \gamma_\alpha(f(X))\} \\
 &= \max \{\gamma_\alpha(f(X)), \gamma_\alpha(f(Y))\} \\
 &= \max \{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\} \\
 &\leq \max \{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\} \\
 \text{iii. } (f^{-1}(\gamma_\alpha))(XYZ) &= \gamma_\alpha(f(XYZ)) \\
 &= \gamma_\alpha(f(Z)f(Y)f(X)) \\
 &\leq \max \{\gamma_\alpha(f(Z)), \gamma_\alpha(f(X))\} \\
 &= \max \{\gamma_\alpha(f(X)), \gamma_\alpha(f(Z))\} \\
 &= \max \{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Z)\} \\
 &\leq \max \{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Z)\} \\
 \text{iv. } (f^{-1}(\gamma_\alpha))(XYZ) &= \gamma_\beta(f(X-Y)) \\
 &= \gamma_\beta(f(X) - f(Y)) \\
 &\geq \min \{\gamma_\beta(f(X)), \gamma_\beta(f(Y))\} \\
 &= \min \{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\} \\
 &\geq \min \{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\}. \\
 \text{v. } (f^{-1}(\gamma_\beta))(XY) &= \gamma_\beta(f(XY)) \\
 &= \gamma_\beta(f(Y)f(X)) \\
 &\geq \min \{\gamma_\beta(f(Y)), \gamma_\beta(f(X))\} \\
 &= \min \{\gamma_\beta(f(X)), \gamma_\beta(f(Y))\} \\
 &= \min \{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\} \\
 &\geq \min \{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\}. \\
 \text{vi. } (f^{-1}(\gamma_\beta))(XYZ) &= \gamma_\beta(f(XYZ)) \\
 &= \gamma_\beta(f(Z)f(Y)f(X)) \\
 &\geq \min \{\gamma_\beta(f(Z)), \gamma_\beta(f(X))\} \\
 &= \min \{\gamma_\beta(f(X)), \gamma_\beta(f(Z))\} \\
 &= \min \{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Z)\} \\
 &\geq \min \{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Z)\}.
 \end{aligned}$$

Hence,  $f^{-1}(\gamma_G)$  is an intuitionistic anti-fuzzy HX bi-ideal of  $\mathfrak{R}_1$ .

## ACKNOWLEDGEMENTS

The authors would like to express their sincere thanks to all our friends for their help to make this paper a successful one.

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*Source of support: Proceedings of National Conference March 1<sup>st</sup> - 2018, On "Recent Advances in Pure and Applied Mathematics (RAPAM - 2018)", Organized by Department of Mathematics, Arul Anandar College (Autonomous), Madurai. Tamilnadu, India.*