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OPTIMIZATION OF UNBALANCED TRAPEZOIDAL FUZZY ASSIGNMENT PROBLEM

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ABSTRACT

The existence of uncertainty is ubiquitous and it acts as a prime hurdle for the managerial people in decision making process. The manufacturing sector is a combination of several activities involving multi functions. The ultimate aim of these sectors is to maximize the profit with minimum input which is possible only if suitable assignment of tasks is accomplished, which is a much challenging mission for them. The Hungarian method is the most common method used to determine the optimal solution to the problem of assignment if the data matrix contains crisp values. But to reflect the present imprecise situations, in this paper the data matrix is devised with trapezoidal fuzzy number values and different methods of ranking is applied to convert fuzzy data matrix to crisp data matrix to determine the optimal assignment.

Keywords: *Optimization, Unbalanced Fuzzy Assignment problem (UFAP), Trapezoidal Fuzzy number, ranking.*
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1. INTRODUCTION

Assignment problem (AP) is one of the thrust areas in Operations Research; it is a sub-class of linear programming problem. Researchers have proposed various algorithms for solving assignment problem, but the Hungarian algorithm seems to be highly adaptable and compatible. To deal with the vagueness of the data the concept of Fuzzy Assignment problem came into existence. Few decades back, the pollsters have presented their profound investigations on Fuzzy Assignment problems in which different methods of ranking the fuzzy numbers were discussed.

The generalized fuzzy assignment problem can be discussed under other dimension which is the unbalanced fuzzy assignment problem. In generalized AP the number of assignments and to be assigned will be equal. In reality the assignments and the number to be assigned will not be balanced, at that juncture UFAP comes into existence. Addition of dummy rows or columns converts UFAP to FAP. The method of ranking the fuzzy numbers is used in solving FAP. Initially Triangular fuzzy unbalanced assignment problems were solved using ranking methods and it has been extended to other types of fuzzy numbers. In this paper we make an attempt to solve Trapezoidal fuzzy unbalanced assignment problem using various method of ranking.

The paper is organized as follows: section 2 contains the elementary inceptions of the concepts; section 3 presents the methodology of the ranking methods; section 4 consists of the applications of the methods to the trapezoidal data matrix of UFAP; section 5 presents the results and discussion and the last section concludes the paper.

2. ELEMENTARY INCEPTIONS

This section presents the basic definitions of the concepts used in this paper.

Fuzzy Set: A fuzzy set A is a mapping from X to [0, 1], where X is the universal set.

Fuzzy Number: A fuzzy set A becomes as fuzzy number if it is a convex set; if there is only one x_0 that satisfies $f_A(x_0) = 1$ and $f_A(x)$ is continuous in an interval.

Trapezoidal Fuzzy Number: Let $\tilde{A} = (a, b, c, d)$, $a < b < c < d$, be a fuzzy set on $R = (-\infty, \infty)$. It is called a trapezoid fuzzy number, if its membership function is

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$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, if\ a \le x \le b \\ 1, \quad if\ b \le x \le c \\ \frac{d-x}{d-c}, if\ c \le x \le d \\ 0, \quad otherwise \end{cases}$$

Mathematical Formulation of Fuzzy Assignment Problem

Minimize Z = $\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{C}_{ij}x_{ij}$

Subject to the constraints

$$\sum_{i=1}^{n} x_{ij} = 1, i = 1,2,...m$$

$$\sum_{j=1}^{m} x_{ij} = 1, j = 1,2,...n$$

$$x_{ij} = \begin{cases} 1 & \text{if the } i^{th} \text{ person is assigned } j^{th} \text{ job} \\ 0 & \text{otherwise} \end{cases}$$

3. DIFFERENT RANKING METHODS OF TRAPEZOIDAL FUZZY NUMBERS (TFN)

There are various methods that can be applied in ordering TFN, but in this paper the most compatible and feasible methods are taken into consideration. The methods of median, centroid and α – cut are used for ranking.

Method of Median

Let $\tilde{A} = (a, b, c, d)$ be a trapezoidal fuzzy number, then $M_{\tilde{A}} = \frac{a+b+c+d}{4}$

Method of Centroid

Let $\tilde{A} = (a, b, c, d)$ be a trapezoidal fuzzy number, then $C_{\tilde{A}} = \frac{c^2 + d^2 + cd - a^2 - b^2 - ab}{3(c+d-a-b)}$

Method of α – cut

Let $\tilde{A} = (a, b, c, d)$ be a trapezoidal fuzzy number, then

R(A) = $\frac{1}{2}\left[\alpha(a+d) + 2(1-\alpha)(b+c)\right] \alpha \in [0,1]$

4. APPLICATIONS OF THE METHODS TO THE TRAPEZOIDAL DATA MATRIX OF UFAP

Let us consider an UFAP whose cost data matrix is presented as below. The objective is to determine the optimal assignment and the minimum cost.

	J1	J2	J3	J4
A	(2,4,5,6)	(4,7,10,11)	(8,9,10,14)	(4,7,8,10)
B	(6,7,9,10)	(2,4,5,6)	(5,7,9,11)	(4,5,8,9)
C	(5,7,9,11)	(1,4,5,6)	(7,10,12,13)	(3,5,6,8)

The UFAP is converted to FAP with the addition of dummy row of devoid cost.

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	J1	J2	J3	J4
A	(2,4,5,6)	(4,7,10,11)	(8,9,10,14)	(4,7,8,10)
B	(6,7,9,10)	(2,4,5,6)	(5,7,9,11)	(4,5,8,9)
C	(5,7,9,11)	(1,4,5,6)	(7,10,12,13)	(3,5,6,8)
D	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)

The mathematical formulation of FAP is

Minimize {R (2,4,5,6) x_{11} + R(4,7,10,11)x_{12} + R(8,9,10,14)x_{13}+ R(4,7,8,10)x_{14} + R(6,7,9,10)x_{21}+ R(2,4,5,6)x_{22} + R(5,7,9,11)x_{23} + R(4,5,8,9)x_{24} + R(5,7,9,11)x_{31} + R(1,4,5,6) x_{32} + R(7,10,12,13)x_{33} + R (3,5,6,8)x_{34} + R(0,0,0,0)x_{41} + R(0,0,0,0)x_{42} + R(0,0,0,0)x_{43} + R(0,0,0,0)x_{44}

Subject to the constraints

$x_{11} + x_{12} + x_{13} + x_{14} = 1$

$x_{21} + x_{22} + x_{23} + x_{24} = 1$

$x_{31} + x_{32} + x_{33} + x_{34} = 1$

$x_{41} + x_{42} + x_{43} + x_{44} = 1$

$x_{ij} \in [0,1]$

Table-5.1

<table>
<tr><th>Methods</th><th>The Defuzzified Cost Data Matrix</th><th>Optimal Assignment</th><th>Optimal Cost</th></tr>
<tr><td>Method of Median</td><td><table><tr><td></td><td>J1</td><td>J2</td><td>J3</td><td>J4</td></tr><tr><td>A</td><td>4.25</td><td>8</td><td>10.25</td><td>7.25</td></tr><tr><td>B</td><td>8</td><td>4.25</td><td>8</td><td>6.25</td></tr><tr><td>C</td><td>8</td><td>4</td><td>10.5</td><td>5.5</td></tr><tr><td>D</td><td>0</td><td>0</td><td>0</td><td>0</td></tr></table></td><td><table><tr><td>A</td><td>J1</td></tr><tr><td>B</td><td>J2</td></tr><tr><td>C</td><td>J4</td></tr><tr><td>D</td><td>J3</td></tr></table></td><td>14</td></tr>
<tr><td>Method of Centroid</td><td><table><tr><td></td><td>J1</td><td>J2</td><td>J3</td><td>J4</td></tr><tr><td>A</td><td>4.2</td><td>7.9</td><td>10.4</td><td>7.1</td></tr><tr><td>B</td><td>8</td><td>4.2</td><td>8</td><td>6.5</td></tr><tr><td>C</td><td>8</td><td>3.8</td><td>10.4</td><td>5.5</td></tr><tr><td>D</td><td>0</td><td>0</td><td>0</td><td>0</td></tr></table></td><td><table><tr><td>A</td><td>J1</td></tr><tr><td>B</td><td>J2</td></tr><tr><td>C</td><td>J4</td></tr><tr><td>D</td><td>J3</td></tr></table></td><td>13.9</td></tr>
<tr><td>Method of α – cut</td><td><table><tr><td></td><td>J1</td><td>J2</td><td>J3</td><td>J4</td></tr><tr><td>A</td><td>6.5</td><td>12.25</td><td>15</td><td>11</td></tr><tr><td>B</td><td>12</td><td>6.5</td><td>12</td><td>9.75</td></tr><tr><td>C</td><td>12</td><td>6.25</td><td>16</td><td>8.25</td></tr><tr><td>D</td><td>0</td><td>0</td><td>0</td><td>0</td></tr></table></td><td><table><tr><td>A</td><td>J1</td></tr><tr><td>B</td><td>J2</td></tr><tr><td>C</td><td>J4</td></tr><tr><td>D</td><td>J3</td></tr></table></td><td>21.25</td></tr>
</table>

5. RESULTS AND DISCUSSION

The Table 4.1 shows that the methods applied for ranking the trapezoidal fuzzy numbers have yielded the same optimal assignment and the optimal costs is closer for the first two methods and it differs for the third method. It is fair to conclude that the first two methods are compatible than the method of α – cut. The representations of Trapezoidal data matrix can be converted to crisp data matrix using the methods of median and centroid which appears to be feasible in the context of consistency and reliability.

CONCLUSION

This paper scatters light on the aspect of optimizing an unbalanced fuzzy assignment problem with the cost matrix being represented as trapezoidal fuzzy number. In this research article three different methods of ranking TFN are used and a comparative analysis is made which duly assists in determining the compatible and feasible methods. The research work can be extended by using different types of fuzzy numbers such as Hexagonal, Octagonal and so on.

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