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PERFECT DOMINATION IN BIPOLAR FUZZY GRAPH

R. MUTHURAJ[1], P. J. JAYALAKSHMI[2] AND S. REVATHI[3]

[1]Assistant Professor, PG & Research Department of Mathematics,
H. H. The Rajah's College, Pudukkottai – 622 001, Tamilnadu, India.

[2]Associate Professor, Department of Mathematics,
Sri Meenakshi Vidiyal Arts and Science College, Trichy – 621 305, Tamilnadu, India.

[3]Assistant Professor, Department of Mathematics,
Saranathan College of Engineering, Trichy – 620 012, Tamilnadu, India.

E-mail: rmr1973@yahoo.co.in, saijayalakshmi1977@gmail.com revathi.soundar@gmail.com.

ABSTRACT

In this paper, we introduced the concept of perfect domination in bipolar fuzzy graph. We defined the perfect domination number for various classes of bipolar fuzzy graph and we also determined some properties for bipolar fuzzy graph.

Keywords: *Bipolar fuzzy graph, Dominating set in bipolar fuzzy graph, Domination number in bipolar fuzzy graph, Perfect dominating set in bipolar fuzzy graph and perfect domination number in bipolar fuzzy graph.*

Mathematical Classification: *03E72, 05C07, 05C69, 05C72, 05C76.*

I. INTRODUCTION

The concept of fuzzy graph was proposed by Zadeh.L. A [8]. Although, In 1975, Rosenfeld [7] introduced another elaborated concept, including fuzzy vertex and fuzzy edges and several fuzzy analogues of graph theoretic concepts such as paths, cycles, connectedness and etc. In the year 2003, A.Nagoor Gani and M. Basheer Ahamed[6] investigated Order and Size in fuzzy graph. In 2011, Muhammad Akram introduced Bipolar fuzzy graphs [1]. In 2013, Revathi .S, *et.al*. [3, 4] introduced Perfect Dominating Sets in Fuzzy Graph and Intuitionistic Fuzzy graph. In 2013, Akram.M, Karunambigai. M.G [5] introduced Domination in Bipolar Fuzzy Graphs.

2. BASIC DEFINITIONS

In this section, Some basic definitions are discussed.

2.1 Definition: Let X be a non-empty set. A **bipolar fuzzy set** B in X is an object having the form $B = \{(x, \mu_B^P(x), \mu_N^P(x)) / x \in X\}$ where $\mu_B^P : X \to [0,1]$ *and* $\mu_B^N : X \to [-1, 0]$ are mappings.

We use the positive membership degree $\mu^P(x)$ to denote the satisfaction degree of an element x to the property corresponding to a bipolar fuzzy set B, and the negative membership degree $\mu^N(x)$ to denote the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar fuzzy set B. If $\mu^P(x) \neq 0$ and $\mu^N(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for B. It is possible for an element x to be such $\mu^P(x) \neq 0$ and $\mu^N(x) \neq 0$, when the membership function of the property overlaps that of its ounter property overlaps that of its counter property over some portion of x. For the sake of simplicity, the symbol B=(μ^P,μ^N) is used for the bipolar fuzzy set $B = \{(x, \mu_B^P(x), \mu_B^N(x)) : x \in X\}$

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2.2 Definition: By a **bipolar fuzzy graph**, we mean a pair G = (A, B) where $A=(\mu_A^P,\mu_A^N)$ is a bipolar fuzzy set in V and $B=(\mu_B^P,\mu_B^N)$ is a bipolar relation on V such that $\mu_B^P(\{x,y\})\le \min(\mu_A^P(x),\mu_A^P(y))$ and $\mu_B^N(\{x,y\})\ge \max(\mu_A^N(x),\mu_A^N(y))$ for all {x, y} ∈ E. We call A the bipolar fuzzy vertex set of V, B the bipolar fuzzy edge set of E, respectively. Note that B is symmetric bipolar fuzzy relation on A. We use the notation xy for an element of E. Thus, G = (A, B) is a bipolar graph of G* = (V, E) if $\mu_B^P(xy)\le \min(\mu_A^P(x),\mu_A^P(y))$ and $\mu_B^N(xy)\ge \max(\mu_A^N(x),\mu_A^N(y))$ for all {xy} ∈ E.

2.3 Definition: Let G = (A, B) be a bipolar fuzzy graph where $A=(\mu_A^P,\mu_A^N)$ and $B=(\mu_B^P,\mu_B^N)$ be two bipolar fuzzy sets on a non-empty finite set V and E ⊆ V×V respectively. The **positive degree of a vertex** is $d(\mu_A^P(x)) = \sum_{xy\in E}\mu_B^P(xy).$ Similarly, the **negative degree of a vertex** is $d(\mu_A^N(x)) = \sum_{xy\in E}\mu_B^N(xy).$ The **degree of a vertex μ** is d(μ) = (d^P(μ), d^N(μ)).

2.4 Definition: Let G = (A,B) be a bipolar fuzzy graph. The **order of a bipolar fuzzy graph ,** denoted O(G), is defined as O(G) = (O^P(G), O^N(G)) , where $O^P(G)=\sum_{x\in V}\mu_A{}^P(x), O^N(G)=\sum_{x\in V}\mu_A{}^N(x)$. Similarly, the size of bipolar fuzzy graph, denoted by S(G), is defined as S(G) = (S^P(G), S^N(G)), where The **size of a bipolar fuzzy graph** is $S^P(G) = \sum_{xy\in V}\mu_A{}^P(xy), S^N(G) = \sum_{xy\in V}\mu_A{}^N(xy)$. Here, the vertex cardinality of is

$$p=|V|\sum_{y\in V}\frac{1+\mu_A{}^P(y)+\mu_A{}^N(y)}{2}$$, and Edge cardinality of $$q=|E|=\sum_{xy\in V}\frac{1+\mu_A{}^P(xy)+\mu_A{}^N(xy)}{2}$$.

2.5 Definition: A bipolar fuzzy graph G = (A, B) is called **strong bipolar fuzzy graph**, if $\mu_B^P(xy) = \min(\mu_A^P(x),\mu_A^P(y))$ and $\mu_B^N(xy) = \min(\mu_A^N(x),\mu_A^N(y))$ for all xy ∈ E.

2.6 Definition: Let G = (A, B) be a bipolar fuzzy graph. Let x, y∈V. We say that x dominates y in G if $\mu_B^P(xy) = \min(\mu_A^P(x),\mu_A^P(y))$ and $\mu_B^N(xy) = \min(\mu_A^N(x),\mu_A^N(y))$ for all xy ∈ E. A subset D of V is called a **dominating set** in G if for every y∉D, there exists x∈D such that x dominates y.

2.7 Definition: Let G = (A, B) be a bipolar fuzzy graph. Let x, y∈V. The vertex x dominates the vertex y in G if (x, y) is a strong arc or strong edge. A subset P of A is called a **perfect dominating set** of G if for each vertex y is not in P and y is dominated by exactly one vertex of P.

2.8 Definition: A perfect dominating set P of a bipolar fuzzy graph G is said to be a **minimal perfect dominating set,** if for each vertex y in P, P-{y} is not a perfect dominating set of a bipolar fuzzy graph G.

2.9 Definition: The minimum fuzzy cardinality of a minimal perfect dominating set of a bipolar fuzzy graph G is called the **perfect domination number** of a bipolar fuzzy graph G. It is denoted by γ_{Pb}(G).

2.10 Definition: The maximum fuzzy cardinality of a minimal perfect dominating set of a bipolar fuzzy graph G is called the **upper perfect domination number** of a bipolar fuzzy graph G. It is denoted by Γ_{Pb}(G) .

3. MAIN RESULTS

3.1 Theorem: Let G = (A, B) be a bipolar fuzzy graph without isolated vertices. Let P_b be a minimal perfect dominating set of G. Then V-P_b is not a perfect dominating set of G.

Proof: Let P be a minimal perfect dominating set in bipolar fuzzy graph. Let x be any vertex of P. Since G has no isolated vertices, there exists a vertex y∈N(x). y must be dominated by atleast one vertex in P-{x}, (i.e) P-{x} is a dominating set. Therefore, every vertex in P is dominated by atleast one vertex in V-P and V-P is a dominating set. But, every vertex in P is not dominated by exactly one vertex in V-P. So, V-P_b is not a perfect dominating set.

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3.2 Theorem: For a bipolar fuzzy graph G = (A, B)

(i) $\gamma_{P_b}(G) \le O^P(G) - \Delta_N^P(G) \le O^P(G) - \Delta_E^P(G)$ (ii) $\gamma_{P_b}(G) \le O^P(G) - \delta_N^P(G) \le O^P(G) - \delta_E^P(G)$

Proof: Let x, y∈V and let G be any bipolar fuzzy graph. we know that $O^P(G) - \Delta_N^P(G)$ is the sum of the membership values of vertices excluding the maximum degree of a vertex. It is clear that

$$\gamma_{P_b}(G) \le O^P(G) - \Delta_N^P(G) \qquad (1).$$

Similarly, $O^P(G) - \delta_N^P(G)$ is the sum of the membership values of vertices excluding the minimum degree of a vertex. It is clear that

$$\gamma_{P_b}(G) \le O^P(G) - \delta_N^P(G) \qquad (2)$$

and further $\Delta_E^P(G) \le \Delta_N^P(G)$ and $\delta_E^P(G) \le \delta_E^P$ which implies that

$$O^P(G) - \Delta_E^P(G) \ge O^P(G) - \Delta_N^P(G) \qquad (3)$$

and

$$O^P(G) - \delta_E^P(G) \ge O^P(G) - \delta_E^P \qquad (4)$$

From (1) and (3), we get $\gamma_{P_b}(G) \le O^P(G) - \Delta_N^P(G) \le O^P(G) - \Delta_E^P(G)$.

From (2) and (4), we get $\gamma_{P_b}(G) \le O^P(G) - \Delta_N^P(G) \le O^P(G) - \Delta_E^P(G)$.

3.3 Theorem: For any complete bipolar fuzzy graph G, then perfect domination number $\gamma_{P_b}(G)$ = min{ $|x_i|$ } where $|x_i|$ is the vertex cardinality of bipolar graph G.

Proof: Let G be a complete bipolar fuzzy graph. In this complete bipolar fuzzy graph, every edge in G is an effective edge and each vertex dominates all other vertices. It satisfies every vertex $y \in V - P$, y dominated by exactly one vertex x of P. Perfect dominating set P = {x}, for all $x \in V$ which is the minimum vertex cardinality of G.
Therefore, perfect domination number = min{ $|x_i|$ } for all $x_i \in V$.

3.4 Theorem: Perfect domination number does not exists for any complement of complete bipolar fuzzy graph G.

Proof: Let G = (V, E) be a complete bipolar fuzzy graph. So, every edge in G has an effective edge. By the definition of complement of complete bipolar fuzzy graph, a bipolar fuzzy graph $\overline{G}$ becomes isolated vertices. It is clear that an isolated vertex does not dominate any other vertex in G. Hence, Perfect domination number does not exists for any complement of complete bipolar fuzzy graph G.

3.5 Theorem: For any complete bipolar fuzzy graph without isolated vertices, $\gamma_{P_b}(G) \le \frac{O^P(G)}{2}$.

Proof: Any complete bipolar fuzzy graph without isolated vertices has atleast one disjoint perfect dominating sets.

Therefore, $\gamma_{P_b}(G) \le \frac{O^P(G)}{2}$.

3.6 Theorem: For any complete bipolar fuzzy graph, $\gamma_{P_b}(G) \ge O^P(G) - \Delta_N^P(G)$.

Proof: Let y be any vertex in complete bipolar fuzzy graph G= (V, E). Assume that $|N(y)| = \Delta$.
Then V-N(y) is a perfect dominating set of G. (i.e) y is a perfect dominating set in complete bipolar fuzzy graph. So,
$\gamma_{P_b}(G) \ge |V - N(y)| = |V| - |N(y)| = O^P(G) - \Delta_N^P(G)$.

3.7 Theorem: For any complete bipartite bipolar fuzzy graph G, then perfect domination number
$\gamma_{P_b}(G)$ = min { $|x_i|$} + min{ $|y_i|$ for all $x_i \in V_1$ and $y_i \in V_2$.

Proof: Since {x, y} is a perfect dominating set of a bipolar fuzzy graph G with minimum fuzzy cardinality for $x \in V_1$ and $y \in V_2$. Then $\gamma_{P_b}(G)$ = min { $|x_i|$} + min{ $|y_i|$ }.

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