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SUPRA SEMI NORMAL SPACES AND SOME SUPRA MAPS

G. RAMKUMAR[1], M. GILBERT RANI[2] AND P. AROCKIADAS[3]

[1,2&3]Department of Mathematics,
Arul Anandar College, Karumathur, Madurai District, Tamil Nadu, India.

E-mail: [1]ramanujam_1729@yahoo.com, [2]gil.anto@yahoo.co.in, and [3]aga.arul@gmail.com.

ABSTRACT

In this paper, We obtain the characterizations of supra semi normal space by using supra semi generalized-open sets (supra sg- open sets). Moreover, inorder to obtain preservation theorems of supra semi normal spaces, we introduce the concepts of supra pre sg- continuous maps and supra pre sg –closed maps and also investigate several properties of new notions

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1. INTRODUCTION

In 1983, Mashhour *et al.* [6] introduced supra topological spaces and studied S –continuous maps and S* - continuous maps. In 2008, Devi *et al*. [3] introduced and studied a class of sets called supra α-open and a class of maps called sα-continuous between topological spaces, respectively. Ravi *et al.* [10] introduced and studied a class of sets called supra g-closed and a class of maps called supra g-continuous and supra g-closed respectively. Kamaraj *et al.* [4] introduced the concepts of supra sg -closed sets and supra gs-closed sets and study their basic properties. Also, introduced the concepts of supra normal spaces and supra-s-normal spaces. In this paper, we obtain the characterizations of supra semi normal space by using supra semi generalized-open sets (supra sg- open sets). Moreover, inorder to obtain preservation theorems of supra semi normal spaces, we introduce the concepts of supra pre sg- continuous maps and supra pre sg –closed maps and also investigate several properties of new notions

2. PRELIMINARIES

Throughout this paper (X, τ), (Y, σ) and (Z, ν) (or simply, X, Y and Z) denote topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of (X, τ), the closure and the interior of A in X with respect to τ are denoted by cl(A) and int(A) respectively. The complement of A is denoted by X\A.

Definition 2.1 [6, 11]: Let X be a non-empty set. The subfamily $\mu \subseteq P(X)$ where P(X) is the power set of X is said to be a supra topology on X if $X \in \mu$ and μ is closed under arbitrary unions. (X, μ) is called a supra topological space.

The elements of μ are said to be supra open in (X, μ).

The complement of supra open set is called supra closed set.

Definition 2.2 [3]: Let A be a subset of X. Then

(i) The supra closure of a set A is, denoted by $cl^{\mu}(A)$, defined as $cl^{\mu}(A) = \cap \{B : B$ is a supra closed and $A \subseteq B\}$.

(ii) the supra interior of a set A is, denoted by $int^{\mu}(A)$, defined as $int^{\mu}(A) = \cup\{B : B$ is a supra open and $A \supseteq B\}$.

Definition 2.3 [6]: Let (X, τ) be a topological space and μ be a supra topology on X. We call μ is a supra topology associated with τ if $\tau \subseteq \mu$.

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Definition 2.4: Let (X, μ) be a supra topological space. A subset A of X is called
(i) supra semi-open [3] if $A \subseteq cl^{\mu}(int^{\mu}(A))$;
(ii) supra α-open [3,12] if $A \subseteq int^{\mu}(cl^{\mu}(int^{\mu}(A)))$;
(iii) supra b-open [11] if $A \subseteq cl^{\mu}(int^{\mu}(A)) \cup int^{\mu}(cl^{\mu}(A))$;
(iv) supra β-open [9] if $A \subseteq cl^{\mu}(int^{\mu}(cl^{\mu}(A)))$;
(v) supra pre-open [12] if $A \subseteq int^{\mu}(cl^{\mu}(A))$.
The complements of the above mentioned open sets are called their respective closed sets.

Definition 2.5 [4]: Let A be a subset of (X, μ). Then
(i) the supra semi-closure of A is, denoted by $scl^{\mu}(A)$, defined as $scl^{\mu}(A) = \cap \{B : B$ is a supra semi-closed and $A \subseteq B\}$.
(ii) the supra semi-interior of A is, denoted by $sint^{\mu}(A)$, defined as $sint^{\mu}(A) = \cup\{G : G$ is a supra semi-open and $A \supseteq G\}$.

Definition 2.6[4]: Let (X, μ) be a supra topological space. A subset A of X is called Suprasg-closed if $scl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra semi-open in (X, μ).

Definition 2.7[5]: Let (X, τ) and (Y,σ) be two topological spaces with $\tau \subseteq \mu$ and $\sigma \subseteq \lambda$. A map $f : (X, \mu) \to (Y,\lambda)$ is called
(i) suprasg-continuous if the inverse image of each open set of Y is a supra sg-open set in X.
(ii) Supra sg-irresoluate if the inverse image of each supra sg-closed set of Y is a supra sg-closed set in X.
(iii) Supra pre semi-closed (resp. supra pre semi-open) if the image of each semi-closed (resp. semi-open) set of X is supra semi-closed (resp. supra semi-open) in Y.

Definition 2.8[4]: A space X is called supra normal if for any pair of disjoint supra closed subsets A and B of X, there exist disjoint supra open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Definition 2.9[4]: A space X is called supras-normal if for any pair of disjoint supraclosed subsets A and B of X, there exist disjoint suprasemi-open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

3. Supra semi-normal space

Definition 3.1: A space X is called supra semi normal if for each pair of disjoint supra semi-closed sets A and B, there exist disjoint supra-semi open sets U and V such that $A \subseteq U$ and $B \subseteq U$.

Definition 3.2: A space X is called supra $T_{½}$ if every supra g-closed set of X is supra closed in X.

Definition 3.3: A space X is called supra semi $T_{½}$ if every supra sg-closed set of X is supra semi-closed in X.

Theorem 3.4: Let (X, μ) be supra topological space. The following properties are equivalent.
a) X is supra semi-normal.
b) For each pair of disjoint supra semi-closed sets A and B, there exist disjoint supra sg-open sets U and V such that $A \subseteq U$ and $B \subseteq U$.
c) For each supra semi-closed set A and each supra semi-open set containing A, there exist a supra sg-open set G such that $A \subseteq G \subseteq scl^{\mu}(G) \subseteq U$.
d) For each supra semi-closed set A and each supra sg-open set U containing A, there exist supra semi-open set G, such that $A \subseteq G \subseteq scl^{\mu}(G) \subseteq sint^{\mu}(G)$.
e) For each supra sg-closed set A and each supra semi-open set U containing A, there exist supra semi open set G, such that $A \subseteq scl^{\mu}(A) \subseteq G \subseteq scl^{\mu}(G) \subseteq U$.
f) For each supra semi-closed set A of X and supra semi open set U containing A, there exists $G \in$ S-SO(X) ∩ S-SC(X) such that $A \subseteq G \subseteq U$.

Proof:
$(a) \Rightarrow (b)$: It is obvious, since every supra semi-open set is supra sg-open set.

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$(b) \Rightarrow (c)$: Let A$\in$S-SC(X) and U$\in$S-SO(X) containing A. Then A∩(X\U) = φ and X\U$\in$SC(X). There exist supra sg-open sets G and V such that $A \subseteq G$, X\U$\subseteq V$, G∩V=φ. Therefore, we have $A \subseteq G \subseteq X \setminus U \subseteq V$ and hence $scl^{\mu}(G) \subseteq scl^{\mu}(X \setminus V) \subseteq U$ since X\V is supra sg-closed and U$\in$S-SO(X). Consequently, we obtain A$\subseteq$G$\subseteq scl^{\mu}$(G)$\subseteq$ U.

$(c) \Rightarrow (d)$: Let A$\in$S-SC(X) and U be a supra sg-open set containing A. We have $A \subseteq sint^{\mu}(U)$ and $sint^{\mu}(U) \in$S-SO(X). There exist a supra sg-open set V such that A$\subseteq$V$\subseteq sint^{\mu}$(V)$\subseteq sint^{\mu}$(U). Put G = $sint^{\mu}$(V), then we obtain G$\in$S – SO(X) and A$\subseteq$G$\subseteq sint^{\mu}$(G)$\subseteq sint^{\mu}$(U).

$(d) \Rightarrow (e)$: Let A be supra sg-closed set and U$\in$S-SO(X) containing A. Then, we have $scl^{\mu}(A) \subseteq U$ and scl^{μ}(A)$\in$S-SC(X). Since every supra semi open set is supra sg-open, there exist G$\in$S-SO(X) such that A$\subseteq scl^{\mu}$(A)$\subseteq$G$\subseteq scl^{\mu}$(G)$\subseteq$U.

$(e) \Rightarrow (f)$: Let A$\in$S-SC(X) and U$\in$S-SO(X) containing A. There exist V$\in$S-SO(X) such that A$\subseteq$V$\subseteq scl^{\mu}$(V)$\subseteq$U. Put G = scl^{μ}(V), then G is supra semi-open and supra semi-closed and $A \subseteq G \subseteq U$.

$(f) \Rightarrow (a)$: Let A and B be any pair of disjoint supra semi-closed sets. Then we have $A \subseteq$ X\B$\in$S-SO(X) and there exist U$\in$S-SO(X) ∩ S-SC(X) such that $A \subseteq U \subseteq$X\B. Now, put V = X\U then we obtain $A \subseteq U$, $B \subseteq V \in$S-SO(X) and U∩V=φ. This shows that X is supra semi-normal.

4. Supra pre sg-continuous maps

In this section, we introduce a new class of functions called supra pre sg-continuous maps.

Definition 4.1: Let (X, τ) and (Y, σ) be two supra topological space and τ$\subseteq$μ. A map f:(X, μ)→(Y, σ) is said to be supra pre sg-continuous if the inverse image of each semi-closed set in Y is supra sg-closed set in X.

Definition 4.2: Let (X, τ) and (Y, σ) be two supra topological space and τ$\subseteq$μ, σ$\subseteq$λ. A map f:(X, μ)→(Y, λ) is called supra pre sg*- continuous if the inverse image each supra semi-closed set in Y is supra sg-closed in X.

Remark 4.3: From the above definition we have the following implications

$$\begin{tikzcd}
 & & \text{suprasg-irresoluteness} \arrow[d] \\
 & \text{supra irresoluteness} \arrow[r] \arrow[d] & \text{supra pre sg-continuous} \arrow[d] \\
\text{supra continuity} \arrow[r] & \text{supra semi-continuity} \arrow[r] & \text{supra sg-continuous}
\end{tikzcd}$$

None of the implications are reversible

Theorem 4.4: Let (X, τ) and (Y, σ) be two supra topological space with τ$\subseteq$μ, σ$\subseteq$λ. A mapf:(X, μ)→(Y, λ) is called supra pre sg- continuous and supra pre semi-closed then f is supra sg-irresolute.

Proof: Let K be any supra sg-closed set of Y and U be supra semi open set containing f^{-1}(V). Since f is supra pre semi-closed, there exist supra semi open set V in Y such that K$\subseteq$V and f^{-1}(V)$\subseteq$U. Since K is supra sg-closed in Y, scl^{μ}(V)$\subseteq$V and hence $f^{-1}(scl^{\mu}(V)) \subseteq f^{-1}(V) \subseteq$U. Since f is supra pre sg-continuous. $f^{-1}(scl^{\mu}(V))$ is supra sg-closed in X and hence $scl^{\mu}(f^{-1}(K)) \subseteq scl^{\mu}(f^{-1}(scl^{\mu}(V))) \subseteq$U. This shows that f^{-1}(K) supra sg-closed in X. Hence f is supra sg-irresolute.

Corollary 4.5: Every supra irresolute, supra pre-semi closed function is supra sg-irresolute.

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Corollary 4.6: Let (X, τ) and (Y, σ) be topological space with $\tau \subseteq \mu$, $\sigma \subseteq \lambda$. If a space X is supra semi $T_{½}$ and f:(X, μ)→(Y, λ) is onto supra irresolute and supra pre semi-closed then the space (Y, σ) is supra semi $T_{½}$.

Proof: Let A be supra sg-closed set in Y. we have that $f^{-1}(A)$ is supra sg-closed in X [In 5 Theorem 3.6]. since X is supra semi $T_{½}$, $f^{-1}(A)$ is supra semi-closed and hence A is supra semi closed. Hence Y is supra $T_{½}$ space.

Theorem 4.7: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu$, $\sigma \subseteq \lambda$. Let X be supra semi $T_{½}$ space. A map f: X→Y is supra pre sg^*- continuous if and only if f is supra semi-irresolute.

Proof: Suppose that f is supra pre sg^*-continuous. Let K be any supra semi closed set of Y. Then $f^{-1}(K)$ is supra sg-closed in X. Since X is supra semi $T_{½}$, $f^{-1}(K)$ is supra semi-closed. Hence f is supra semi-irresolute. The converse is obvious.

Corollary 4.8: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu$, $\sigma \subseteq \lambda$. A map f :(X, μ)→(Y, λ) is supra sg-irresolute and X is supra semi $T_{½}$ then f is supra semi irresolute.

Proof: Let V be supra semi-closed in Y. Every supra semi-closed set is supra sg-closed. Since f is supra sg-irresolute then $f^{-1}(V)$ is supra sg-closed in X. Since X is supra semi $T_{½}$.$f^{-1}(V)$ is supra semi-closed in X. Hence f is supra semi-irresolute.

Definition 4.9: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu$, $\sigma \subseteq \lambda$. A map f:(X, μ)→(Y, λ) is called supra pre $semi^*$-closed if the image of each supra semi-closed in X is supra semi-closed in Y.

Theorem 4.10: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu$, $\sigma \subseteq \lambda$. If a map f:(X, μ)→(Y, λ) is supra pre sg^*-continuous, supra pre $semi^*$-closed injection and Y is supra semi normal space, then X is supra semi normal.

Proof: Let A and B be any two disjoint supra semi-closed sets of X. Since f is supra pre $semi^*$-closed injection, f(A) and f(B) are disjoint supra semi-closed sets of Y. By supra semi normality of Y, there exit supra semi-open sets U and V in Y such that $f(A) \subseteq U$ and $f(B) \subseteq V$. Since f is supra pre sg^*-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint supra sg-closed sets containing A and B respectively. By theorem 3.4, X is supra semi normal.

Corollary 4.11: The inverse image of supra semi normal space under an supra irresolute and supra pre $semi^*$-closed injection is supra semi-normal.

5. Supra pre sg-closed maps

In this section, we introduce the new class of maps called supra pre sg-closed and supra pre sg^*-closed maps.

Definition 5.1: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu$, $\sigma \subseteq \lambda$. If a map f:(X, μ)→(Y, λ) is called supra pre sg-closed if the image of each semi closed set in X is supra sg-closed in Y.

Definition 5.2: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu$, $\sigma \subseteq \lambda$. A map f:(X, μ)→(Y, λ) is called supra sg^*-closed if image of each supra semi-closed in X is supra sg-closed in Y.

Theorem 5.3: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu$, $\sigma \subseteq \lambda$. A map f: (X, μ)→(Y, λ) is an supra semi irresolute and supra pre sg^*-closed map and A is a supra sg-closed of X, then f(A) is a supra sg-closed in Y.

Proof: Let A be a supra sg-closed set of X. Let $f(A) \subseteq U$ where U is supra semi-open set of Y. Then $A \subseteq f^{-1}(U)$ and $f^{-1}(U)$ is supra semi-open in X because f is supra semi irresolute. Since f is supra pre sg^*-closed $scl^{\mu}(A)$ is supra semi closed in X and $f(scl^{\mu}(A))$ is supra sg-closed in Y and $f(scl^{\mu}(A)) \subseteq U$. Therefore we have
$scl^{\lambda}f(A) \subseteq scl^{\lambda}(f(scl^{\mu}(A)))=f(scl^{\mu}(A)) \subseteq U$. Hence f(A) is supra sg-closed in Y.

Proposition 5.4: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu$, $\sigma \subseteq \lambda$. A surjective map f:(X, μ)→(Y, λ) is supra pre sg^*-closed if and only if for each subset B of Y and each supra open set U of X contain $f^{-1}(B)$, there exist a supra sg-open set V of Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq U$.

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Proof:
Necessity: Suppose that f is supra pre sg*-closed. Let B be any subset of Y and U is supra semi open set containing $f^{-1}(B)$. V=Y\f(X\U) then V is supra sg-open inY, $B \subseteq V$ and $f^{-1}(V) \subseteq U$.

Sufficient: Let F be any supra semi-closed set of X. Put B =Y\f(F) then we have $f^{-1}(B) \subseteq X\backslash F$ supra semi open in X. There exist a supra sg-open set V of Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq X\backslash F$. Therefore we obtain f(F) = Y-V and hence f(F) is supra sg-closed in Y. Hence f is supra pre sg*-closed.

Theorem 5.5: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu$, $\sigma \subseteq \lambda$. A surjective map f: $(X, \mu) \rightarrow (Y, \lambda)$ is supra pre sg*-closed and supra semi-irresolute surjection and X is supra semi-normal space, then Y is supra semi normal.

Proof: Let A and B be any pair of disjoint supra semi-closed sets of Y. Since f is supra semi-irresolute, $f^{-1}(A)$ and $f^{-1}(B)$are disjoint semi closed sets of X. By semi normality of X, there exist supra semi open set U and V of X such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$ and $U \cap V = \varphi$. By proposition 5.4 there exist supra sg-open set G and H such that $A \subseteq G$ and $B \subseteq H$, $f^{-1}(G) \subseteq U$ and $f^{-1}(H) \subseteq V$, since f is surjective and $U \cap V = \varphi$. We have $G \cap H = \varphi$. By theorem 3.4 we have Y is supra semi-normal.

Corollary 5.6: Supra semi normality is preserved under supra pre semi*-closed and irresolute map.

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