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NON-SPLIT TOTAL STRONG (WEAK) DOMINATION IN BIPOLAR FUZZY GRAPH

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ABSTRACT

In this paper, we define Non-Split Total strong (weak) domination in Bipolar Fuzzy Graph and its various classifications. Size, Order and Degree of Non-Split Total strong (weak) domination in Bipolar Fuzzy Graph is derived with some examples. Some basic parametric conditions are introduced with suitable examples. The properties of total strong (weak) domination number and Non-Split total strong (weak) domination number in Bipolar Fuzzy Graph are discussed.

Keywords: *Bipolar Fuzzy Graph, Dominating set in BFG, strong (weak) dominating set in BFG, total strong (weak) dominating set in BFG, Non-Split total strong (weak) dominating set in BFG.*

Mathematical Classification: *03E72, 05C07, 05C69, 05C72, 05C76.*

I. INTRODUCTION

The concept of fuzzy graph was proposed by Zadeh.L. A [8]. Although, In 1975, Rosenfeld introduced another elaborated concept, including fuzzy vertex and fuzzy edges and several fuzzy analogues of graph theoretic concepts such as paths, cycles, connectedness and etc. In the year 2003, A.Nagoor Gani and M. Basheer Ahamed[6] investigated Order and Size in fuzzy graph. In 2010, C.Natarajan and S.K. Ayyasamy[5] introduced On strong (weak) domination in fuzzy graph. In 2011, Muhammad Akram introduced Bipolar fuzzy graphs [1]. In 2012, P.J. Jayalakshmi [3] *et.al.* introduced total strong (weak) domination in fuzzy graph. In 2013, Akram.M, Karunambigai. M.G [4] introduced Domination in Bipolar Fuzzy Graphs.

2. BASIC DEFINITIONS

In this section, Some basic definitions are discussed.

2.1 **Definition:** Let X be a non-empty set. A **bipolar fuzzy set** B in X is an object having the form
$B=\{(x,\mu_B^P(x),\mu_N^P(x))/x\in X\}$ where $\mu_B^P: X\to[0,1]$ *and* $\mu_B^N: X\to[-1,0]$ are mappings.

We use the positive membership degree $\mu^P(x)$ to denote the satisfaction degree of an element x to the property corresponding to a bipolar fuzzy set B, and the negative membership degree $\mu^N(x)$ to denote the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar fuzzy set B. If $\mu^P(x)\neq 0$ and $\mu^N(x)=0$, it is the situation that x is regarded as having only positive satisfaction for B. It is possible for an

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element x to be such $\mu^P(x) \neq 0$ and $\mu^N(x) \neq 0$, when the membership function of the property overlaps that of its counter property overlaps that of its counter property over some portion of x. For the sake of simplicity, the symbol B = (μ^P,μ^N) is used for the bipolar fuzzy set $B = \{(x, \mu_B^P(x), \mu_B^N(x)) : x \in X\}$

2.2 Definition: By a **bipolar fuzzy graph**, we mean a pair G = (A, B) where $A = (\mu_A^P, \mu_A^N)$ is a bipolar fuzzy set in V and $B = (\mu_B^P, \mu_B^N)$ is a bipolar relation on V such that $\mu_B^P(\{x,y\}) \le \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(\{x,y\}) \ge \max(\mu_A^N(x), \mu_A^N(y))$ for all {x, y} ∈ E. We call A the bipolar fuzzy vertex set of V, B the bipolar fuzzy edge set of E, respectively. Note that B is symmetric bipolar fuzzy relation on A. We use the notation xy for an element of E. Thus, G = (A, B) is a bipolar graph of G* = (V, E) if $\mu_B^P(xy) \le \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(xy) \ge \max(\mu_A^N(x), \mu_A^N(y))$ for all {xy} ∈ E.

2.3 Definition: Let G = (A, B) be a bipolar fuzzy graph where $A = (\mu_A^P, \mu_A^N)$ and $B = (\mu_B^P, \mu_B^N)$ be two bipolar fuzzy sets on a non-empty finite set V and E ⊆ V×V respectively. The **positive degree of a vertex** is $d(\mu_A^P(x)) = \sum_{xy \in E} \mu_B^P(xy)$. Similarly, the **negative degree of a vertex** is $d(\mu_A^N(x)) = \sum_{xy \in E} \mu_B^N(xy)$. The **degree of a vertex μ** is d(μ) = $(d^P(\mu), d^N(\mu))$.

2.4 Definition: Let G = (A,B) be a bipolar fuzzy graph. The **order of a bipolar fuzzy graph,** denoted O(G), is defined as O(G) = $(O^P(G), O^N(G))$, where $O^P(G) = \sum_{x \in V} \mu_A{}^P(x), O^N(G) = \sum_{x \in V} \mu_A{}^N(x)$. Similarly, the size of bipolar fuzzy graph, denoted by S(G), is defined as S(G) = $(S^P(G), S^N(G))$, where The **size of a bipolar fuzzy graph** is $S^P(G) = \sum_{xy \in V} \mu_A{}^P(xy), S^N(G) = \sum_{xy \in V} \mu_A{}^N(xy)$.

Here, the vertex cardinality of is $p = |V| \sum_{y \in V} \frac{1 + \mu_A{}^P(y) + \mu_A{}^N(y)}{2}$, and Edge cardinality of

$$q = |E| = \sum_{xy \in V} \frac{1 + \mu_A{}^P(xy) + \mu_A{}^N(xy)}{2}.$$

2.5 Definition: Let G = (A, B) be a bipolar fuzzy graph. Let x, y ∈ V. We say that x dominates y in G if $\mu_B^P(xy) = \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(xy) = \min(\mu_A^N(x), \mu_A^N(y))$ for all xy ∈ E. A subset D of V is called a **dominating set** in G if for every y∉D, there exists x∈D such that x dominates y.

2.6 Definition: A bipolar fuzzy graph G = (A, B) is called **strong bipolar fuzzy graph**, if $\mu_B^P(xy) = \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(xy) = \min(\mu_A^N(x), \mu_A^N(y))$ for all xy ∈ E.

2.7. Definition: Let G be a bipolar fuzzy graph. The **closed neighbourhood degree of a vertex x in G** is defined by $\deg[x] = (\deg_A[x], \deg_B[x])$ where $\deg_A[x] = \sum_{y \in N(x)} \left[\mu_A{}^P(y) + \mu_A{}^P(x)\right]$ and

$$\deg_B[x] = \sum_{y \in N(x)} \left[\mu_A{}^N(y) + \mu_A{}^N(x)\right].$$

2.8 Definition: Let G = (A, B) be a bipolar fuzzy graph. Let x and y be any two vertices. Then **x totally strong dominates y (y totally weak dominates x)** if

i. $\mu_B^P(xy) = \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(xy) = \min(\mu_A^N(x), \mu_A^N(y))\sqrt{b^2 - 4ac}$ for all xy ∈ E.
ii. $d_N(x) \ge d_N(y)$ for all x ∈ T , y ∈ V - T and
iii. every vertex in G dominates x.

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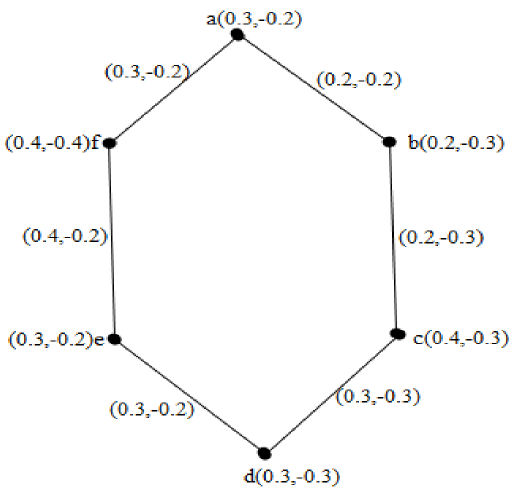
2.9 Definition: Let G = (A, B) be a bipolar Fuzzy Graph. T_b is said to be **total strong (weak) dominating bipolar set of G** if $d_N(x) \geq d_N(y)$ for all $x \in T_b$, $y \in V - T_b$ and x dominates y.

2.10 Definition: A total strong (weak) dominating bipolar fuzzy set T_b is called **minimal total strong (weak) dominating bipolar fuzzy set** of G, if for every vertex $x \in T_b$, T_b - {x}is not a total strong (weak) dominating bipolar fuzzy set of G.

2.11 Definition: The minimum fuzzy cardinality among all minimal total strong (weak) dominating bipolar fuzzy set T_b is called **total strong (weak) dominating bipolar fuzzy set of G** and its total strong (weak) domination bipolar fuzzy number is denoted by $\gamma_{T_b}(G)$.

2.12 Example: Let G be a bipolar fuzzy graph.



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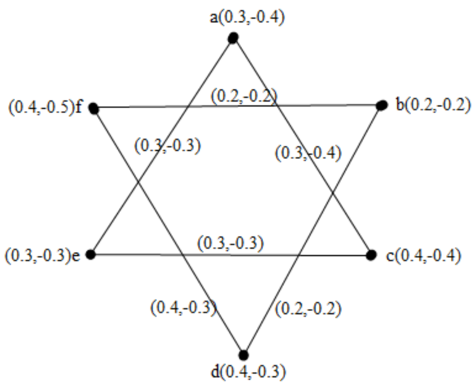
Here, Total strong (weak) dominating set on bipolar fuzzy graph, T_b = {a, b, d, e}
O(G) = (1.9,-1.7), S(G) = (1.7,-1.4), $\delta_N(G)$ = (0.5,-0.7), $\Delta_N(G)$ = (0.7,-0.4), $\delta_E(G)$ = (0.4,-0.6), $\Delta_E(G)$ = (0.7,-0.4) and $\gamma_{T_b}(G) = 2.05$

2.13 Example: Let G be a Total strong (weak) dominating set of a bipolar fuzzy graph.



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Here, total strong (weak) dominating set on bipolar fuzzy graph, $T_b = \{a, b, c, d\}$
O(G) = (2,-2.1), S(G) = (1.7,-1.7), $\delta_N(G)$ = (0.6,-0.5), $\Delta_N(G)$ = (0.8,-0.8), $\delta_E(G)$ = (0.4,-0.4), $\Delta_E(G)$ = (0.6,-0.7) and $\gamma_{T_b}(G) = 2$

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2.14 Proposition: For any total strong (weak) domination bipolar fuzzy graph,
(i) $\gamma_{T_b}(G) \geq O^P(G) \geq S^P(G)$ (ii) $O^N(G) \leq S^N(G) \leq \gamma_{T_b}(G)$

2.15 Proposition: For any total strong (weak) domination bipolar fuzzy graph,
(i) $\gamma_{T_b}(G) \geq \Delta_N{}^P(G) \geq \Delta_E{}^P(G)$ (ii) $\Delta_N{}^N(G) \leq \Delta_E{}^N(G) \leq \gamma_{T_b}(G)$

3. MAIN RESULTS

3.1 Definition: Let G be a Total *strong* (weak) domination on bipolar Fuzzy Graph. NS_{T_b} is said to be **Non-Split total strong (weak) dominating bipolar fuzzy set of G** if the induced sub graph $< V - NS_{T_b} >$ is connected.

3.2 Definition: A non - split total strong (weak) dominating bipolar fuzzy set NS_{T_b} is called **minimal non - split total strong (weak) dominating bipolar fuzzy set** of G, if $y \in NS_{T_b}, NS_{T_b} - \{y\}$ is not a non – split total strong (weak) dominating bipolar fuzzy set of G.

3.3 Definition: The minimum fuzzy cardinality among all minimal non - split total strong (weak) dominating bipolar fuzzy set is called **non - split total strong (weak) dominating bipolar fuzzy number** and its non - split total strong (weak) dominating bipolar fuzzy number and it is denoted by $\gamma_{NS_{T_b}}(G)$.

3.4 Theorem: For any Non-Split total strong (weak) domination bipolar fuzzy graph,
(i) $\gamma_{NS_{T_b}}(G) \geq \delta_N{}^P(G) \geq \delta_E{}^P(G)$ (ii) $\delta_N{}^N(G) \leq \delta_E{}^N(G) \leq \gamma_{NS_{T_b}}(G)$

Proof:
(i) Let NS_{T_b} be a Non-Split total strong (weak) domination in bipolar fuzzy graph in G. Let $\gamma_{NS_{T_b}}(G)$ be the minimum non-split total strong (weak) domination in bipolar fuzzy graph in G. Then, the scalar cardinality of $V - NS_{T_b}$ is less than or equal to the scalar cardinality of X x X.

Obviously, $\delta_N{}^P(G) \leq \gamma_{NS_{T_b}}(G)$

That is $\gamma_{NS_{T_b}}(G) \geq \delta_N{}^P(G)$ (1)

Now, Let A be the vertex with minimum effective incident positive degree $\delta_E{}^P(G)$. Clearly, $NS_{T_b} - \{v\}$ is a Non – Split total strong (weak) domination in bipolar fuzzy graph and hence $\delta_N{}^P(G) \geq \delta_E{}^P(G)$ (2)

From (1) & (2), we get $\gamma_{NS_{T_b}}(G) \geq \delta_N{}^P(G) \geq \delta_E{}^P(G)$.
(ii) as same as above said.

3.5 Theorem: For any Non – Split total (weak) domination bipolar fuzzy graph of G,

$$\gamma_{T_b}(G) \leq \gamma_{NS_{T_b}}(G) \leq p - \Delta_N(G)$$

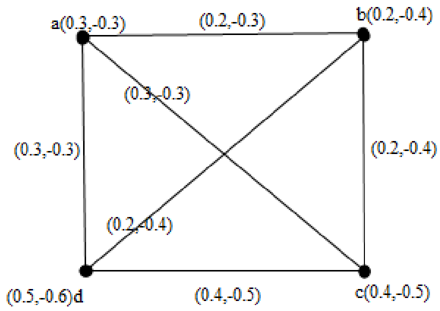
Proof: Since every Non-split total strong (weak) domination bipolar fuzzy graph is a total strong (weak) domination bipolar fuzzy set of G, we have $\gamma_{T_b}(G) \leq \gamma_{NS_{T_b}}(G)$. Let u $\in$A. If $d_N(u) = \Delta_N$, then V – N(u) is a Non- split total strong (weak) domination bipolar fuzzy set but not minimal. Therefore $\gamma_{NS_{T_b}}(G) \leq |V - N(u)| = p - \Delta_N(G)$.

3.6 Theorem: For any Non – Split total (weak) domination bipolar fuzzy graph of G,

$$\gamma_{T_b}(G) \leq \gamma_{NS_{T_b}}(G) \leq p - \Delta_N(G) \leq q - \Delta_E(G)$$

Proof: Since every Non-split total strong (weak) domination bipolar fuzzy graph is a total strong (weak) domination bipolar fuzzy set of G, we have $\gamma_{T_b}(G) \leq \gamma_{NS_{T_b}}(G)$. Let u $\in$A. If $d_N(u) = \Delta_N$, then V – N(u) is a Non- split total strong (weak) domination bipolar fuzzy set but not minimal. Therefore $\gamma_{NS_{T_b}}(G) \leq |V - N(u)| = p - \Delta_N(G)$. Let u $\in$A. If dE(u) = ΔE, then V – N(u) is a non-split total strong (weak) domination bipolar fuzzy set but not minimal. Therefore, $\gamma_{NS_{T_b}}(G) \leq |V - N(u)| = p - \Delta_E(G)$. Hence the result.

3.5 Example: Let G be Non-Split total strong (weak) dominating bipolar fuzzy set.



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$NS_{T_b} = \{a,b\}$, $\gamma_{NS_{T_b}}(G) = 0.9$, $\delta_N(G) = (0.7,-1)$, $\delta_E(G) = (0.8,-1.2)$, $\Delta_N(G) = (0.7,-0.8)$, $\Delta_E(G) = (0.9,-0.9)$
O(G) = (1.4,-1.8), S(G) = (1.7,-2.2), p = 1.8, q = 3.2.

4. CONCLUSION

In this paper, the effective degree of a vertex and degree of a vertex in bipolar fuzzy graph are examined and some proposition and theorem are introduced. In our future investigation, various types of total strong (weak) domination bipolar fuzzy graph are introduced. We have extended our research work to other areas by using bipolar fuzzy graph.

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