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HOMOMORPHISM AND ANTI HOMOMORPHISM OF PSEUDO FUZZY COSET OF A HX RING

R. MUTHURAJ[1]
PG & Research Department of Mathematics,
H.H.The Rajah's College, Pudukkottai- 622001, Tamilnadu, India.

N. RAMILA GANDHI[2]
Department of Mathematics,
PSNA College of Engineering and Technology, Dindigul-624622, Tamilnadu, India.

E-mail: rmr1973@yahoo.co.in[1], satrami@yahoo.com[2]

ABSTRACT

In this paper, we introduce the notion of pseudo fuzzy cosets of a fuzzy HX ring and reviewed its properties. We introduce the concept of an image, pre-image of pseudo fuzzy coset of a HX ring and discuss the properties of homomorphic and anti homomorphic images and pre images of pseudo fuzzy coset of a fuzzy HX ring of a HX ring $\Re$. Also, we introduce the notion of pseudo fuzzy cosets of a fuzzy normal HX ring and discuss the concept of an image, pre-image of pseudo fuzzy coset of a HX ring and discuss the properties of homomorphic and anti homomorphic images and pre images of pseudo fuzzy coset of a fuzzy normal HX ring of a HX ring $\Re$.

Keywords- *HX ring, fuzzy HX ring, pseudo fuzzy coset of a HX ring.*

1. INTRODUCTION

In 1965, Lotfi.A.Zadeh [11] introduced the concept of fuzzy set. In 1982 Wang-jin Liu [3] introduced the concept of fuzzy subring and fuzzy ideal. In 1988, Professor Li Hong Xing [4] proposed the concept of HX ring and derived some of its properties, then Professor Zhong [1, 2] gave the structures of HX ring on a class of ring. W. B. Vasantha kandasamy [9] introduced the concept of fuzzy cosets. In this paper we define pseudo fuzzy coset of a fuzzy HX ring and investigate some related properties under homomorphism and anti homomorphism.

2. PSEUDO FUZZY COSET OF A FUZZY HX RING

In this section, we introduce the notion of pseudo fuzzy cosets of a fuzzy HX ring and discuss its properties.

2.1 Definition: Let μ be a fuzzy set defined on R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. Let λ^μ be a fuzzy HX subring of a HX ring $\Re$ and $A \in \Re$. Then the pseudo fuzzy coset of a fuzzy HX ring λ^μ of a HX ring $\Re$ determined by the element $A \in \Re$ is denoted as $(A + \lambda^\mu)^P$ and is defined by $(A + \lambda^\mu)^P (X) = p(A)\, \lambda^\mu (X)$ for every $X \in \Re$ and for some $p \in P$, where $P = \{p(X) / p(X) \in [0, 1]$ for all $X \in \Re\}$.

2.2 Theorem: Let λ^μ be a fuzzy HX subring of a HX ring $\Re$, then the pseudo fuzzy coset $(A + \lambda^\mu)^P$ is a fuzzy HX subring λ^μ of a HX ring $\Re$ determined by the element $A \in \Re$.

Proof: Let λ^μ be a fuzzy HX subring of a HX ring $\Re$.

For every X, Y, $A \in \Re$ we have

i.
$$\begin{aligned}(A + \lambda^\mu)^P (X-Y) &= p(A)\, \lambda^\mu (X-Y)\\ &\geq p(A) \min \{ \lambda^\mu (X), \lambda^\mu (Y)\}\\ &= \min \{p(A)\lambda^\mu (X), p(A)\, \lambda^\mu (Y)\}\\ &= \min\{(A + \lambda^\mu)^P(X),(A + \lambda^\mu)^P (Y)\}\end{aligned}$$

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Therefore,
$(A + \lambda^{\mu})^P(X–Y) \geq \min\{(A + \lambda^{\mu})^P(X), (A + \lambda^{\mu})^P(Y)\}$

ii. $(A + \lambda^{\mu})^P(XY) = p(A)\lambda^{\mu}(XY)$
$\geq p(A)\min\{\lambda^{\mu}(X), \lambda^{\mu}(Y)\}$
$= \min\{p(A)\lambda^{\mu}(X), p(A)\lambda^{\mu}(Y)\}$
$= \min\{(A + \lambda^{\mu})^P(X), (A + \lambda^{\mu})^P(Y)\}$

Therefore,
$(A + \lambda^{\mu})^P(XY) \geq \min\{(A + \lambda^{\mu})^P(X), (A + \lambda^{\mu})^P(Y)\}$

Hence, the pseudo fuzzy coset $(A + \lambda^{\mu})^P$ determined by the element $A \in \mathfrak{R}$ is a fuzzy HX subring of a HX ring $\mathfrak{R}$.

2.3 Theorem: Let μ and η be any two fuzzy sets defined on R. Let $(A + \lambda^{\mu})^P$ and $(A + \gamma^{\eta})^P$ be any two pseudo fuzzy coset of a fuzzy HX subrings λ^{μ} and γ^{η} of a HX ring $\mathfrak{R}$ determined by the element $A \in \mathfrak{R}$.Then $(A + \lambda^{\mu})^P \cap (A + \gamma^{\eta})^P$ is a fuzzy HX subring of $\mathfrak{R}$.

Proof: Let $(A + \lambda^{\mu})^P$ and $(A + \gamma^{\eta})^P$ be any two pseudo fuzzy coset of a fuzzy HX subring λ^{μ} of a HX ring $\mathfrak{R}$ determined by the element $A \in \mathfrak{R}$.

By Theorem 2.2, $(A + \lambda^{\mu})^P$ and $(A+ \gamma^{\eta})^P$ are fuzzy HX subrings of a HX ring $\mathfrak{R}$ determined by the element $A \in \mathfrak{R}$.
Hence, $(A+ \lambda^{\mu})^P \cap (A + \gamma^{\eta})^P$ is a fuzzy HX subring of $\mathfrak{R}$ determined by the element $A \in \mathfrak{R}$.

2.4 Theorem: Let μ and η be any two fuzzy sets defined on R. Let $(A+ \lambda^{\mu})^P$ and $(A+\gamma^{\eta})^P$ be any two pseudo fuzzy coset of a fuzzy HX subrings λ^{μ} and γ^{η} of a HX ring $\mathfrak{R}$ determined by the element $A \in \mathfrak{R}$.Then $(A + \lambda^{\mu})^P \cup (A + \gamma^{\eta})^P$ is a fuzzy HX subring of $\mathfrak{R}$.

Proof: Let $(A + \lambda^{\mu})^P$ and $(A + \gamma^{\eta})^P$ be any two pseudo fuzzy coset of a fuzzy HX subring λ^{μ} of a HX ring $\mathfrak{R}$ determined by the element $A \in \mathfrak{R}$.

By Theorem 2.2, $(A + \lambda^{\mu})^P$ and $(A + \gamma^{\eta})^P$ are fuzzy HX subrings of a HX ring $\mathfrak{R}$ determined by the element $A \in \mathfrak{R}$.
Hence, $(A + \lambda^{\mu})^P \cup (A + \gamma^{\eta})^P$ is a fuzzy HX subring of $\mathfrak{R}$ determined by the element $A \in \mathfrak{R}$.

2.5 Theorem: Let μ and η be any two fuzzy sets defined on R. Let $(A + \lambda^{\mu})^P$ and $(A + \gamma^{\eta})^P$ be any two pseudo fuzzy coset of a fuzzy HX subrings λ^{μ} and γ^{η} of a HX ring $\mathfrak{R}$ determined by the element $A \in \mathfrak{R}$.Then $(A + \lambda^{\mu})^P \times (A + \gamma^{\eta})^P$ is a fuzzy HX subring of $\mathfrak{R}\times\mathfrak{R}$.

Proof: Let $(A + \lambda^{\mu})^P$ and $(A + \gamma^{\eta})^P$ be any two pseudo fuzzy coset of a fuzzy HX subring λ^{μ} of a HX ring $\mathfrak{R}$ determined by the element $A \in \mathfrak{R}$.

By Theorem 2.2, $(A + \lambda^{\mu})^P$ and $(A + \gamma^{\eta})^P$ are fuzzy HX subrings of a HX ring $\mathfrak{R}$ determined by the element $A \in \mathfrak{R}$.
Hence, $(A + \lambda^{\mu})^P \times (A + \gamma^{\eta})^P$ is a fuzzy HX subring of $\mathfrak{R}\times\mathfrak{R}$ determined by the element $A \in \mathfrak{R}$.

3. HOMOMORPHISM AND ANTI HOMOMORPHISM OF A PSEUDO FUZZY COSET OF A FUZZY HX SUBRING OF A HX RING

In this section, we introduce the concept of an image, pre-image of pseudo fuzzy coset of a HX ring and discuss the properties of homomorphic and anti homomorphic images and pre images of pseudo fuzzy coset of a fuzzy HX ring λ^{μ} of a HX ring $\mathfrak{R}$ determined by the element $A \in \mathfrak{R}$.

3.1 Theorem: Let $\mathfrak{R}_1$ and $\mathfrak{R}_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let f: $\mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ be a homomorphism onto HX rings. Let $(A+\lambda^{\mu})^P$ be the pseudo fuzzy coset of a fuzzy HX ring λ^{μ} of a HX ring $\mathfrak{R}_1$ determined by the element $A \in \mathfrak{R}_1$.Then $f((A + \lambda^{\mu})^P)$ is the pseudo fuzzy coset of a fuzzy HX ring $f(\lambda^{\mu})$ of a HX ring $\mathfrak{R}_2$ determined by the element $f(A) \in \mathfrak{R}_2$ and $f((A + \lambda^{\mu})^P) = (f(A) + f(\lambda^{\mu}))^p$, if λ^{μ} has a supremum property and λ^{μ} is f-invariant.

Proof: Let f : $\mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ be a homomorphism onto HX rings.

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Let $(A+\lambda^{\mu})^P$ be the pseudo fuzzy coset of a fuzzy HX ring λ^{μ} of a HX ring $\mathfrak{R}_1$ determined by the element $A\in \mathfrak{R}_1$. $f(\lambda^{\mu})$ is a fuzzy HX subring of a HX ring $\mathfrak{R}_2$.Then $f((A+\lambda^{\mu})^P)$ is the pseudo fuzzy coset of a fuzzy HX ring $f(\lambda^{\mu})$ of a HX ring $\mathfrak{R}_2$ determined by the element $f(A)\in \mathfrak{R}_2$.

Let $A, X \in \mathfrak{R}_1$, then $f(A), f(X)\in \mathfrak{R}_2$.

Now, $(f(A) + f(\lambda^{\mu}))^p f(X) = p(f(A))(f(\lambda^{\mu})(f(X)))$
$= p(A)\, \lambda^{\mu}(X)$
$= (A + \lambda^{\mu})^P(X)$
$= f((A + \lambda^{\mu})^P)f(X)$.
$(f(A) + f(\lambda^{\mu}))^p f(X) = f((A + \lambda^{\mu})^P)f(X)$, for any $f(X)\in \mathfrak{R}_2$.

Hence, $f((A+\lambda^{\mu})^P) = (f(A) + f(\lambda^{\mu}))^p$.

3.2 Theorem: Let $\mathfrak{R}_1$ and $\mathfrak{R}_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let f: $\mathfrak{R}_1 \to \mathfrak{R}_2$ be a homomorphism on HX rings. Let $(B+\eta^{\alpha})^P$ be the pseudo fuzzy coset of a fuzzy HX ring η^{α} of a HX ring $\mathfrak{R}_2$ determined by the element $B\in \mathfrak{R}_2$.Then $f^{-1}((B+\eta^{\alpha})^P)$ is the pseudo fuzzy coset of a fuzzy HX ring $f^{-1}(\eta^{\alpha})$ of a HX ring $\mathfrak{R}_1$ determined by the element $f^{-1}(B)\in \mathfrak{R}_1$ and $f^{-1}((B+\eta^{\alpha})^P) = (f^{-1}(B) + f^{-1}(\eta^{\alpha}))^p$.

Proof: Let f : $\mathfrak{R}_1 \to \mathfrak{R}_2$ be a homomorphism on HX rings.

Let $(B+\eta^{\alpha})^P$ be the pseudo fuzzy coset of a fuzzy HX ring η^{α} of a HX ring $\mathfrak{R}_2$ determined by the element $B\in \mathfrak{R}_2$.
$f^{-1}(\eta^{\alpha})$ is a fuzzy HX subring of a HX ring $\mathfrak{R}_1$.Then, $f^{-1}((B+\eta^{\alpha})^P)$ is the pseudo fuzzy coset of a fuzzy HX ring $f^{-1}(\eta^{\alpha})$ of a HX ring $\mathfrak{R}_1$ determined by the element $f^{-1}(B)\in \mathfrak{R}_1$.

Let $X \in \mathfrak{R}_1$ and $B\in \mathfrak{R}_2$.

Now, $(f^{-1}(B) + f^{-1}(\eta^{\alpha}))^p (X) = p(f^{-1}(B))(f^{-1}(\eta^{\alpha}))(X)$
$= p(B)(\eta^{\alpha}(f(X))$
$= (B + \eta^{\alpha})^P f(X)$
$= f^{-1}((B + \eta^{\alpha})^P)(X)$.
$(f^{-1}(B) + f^{-1}(\eta^{\alpha}))^p (X) = f^{-1}((B + \eta^{\alpha})^P)(X)$.

Hence, $f^{-1}((B + \eta^{\alpha})^P) = (f^{-1}(B) + f^{-1}(\eta^{\alpha}))^p$.

3.3 Theorem: Let $\mathfrak{R}_1$ and $\mathfrak{R}_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let f: $\mathfrak{R}_1 \to \mathfrak{R}_2$ be an anti homomorphism onto HX rings. Let $(A+\lambda^{\mu})^P$ be the pseudo fuzzy coset of a fuzzy HX ring λ^{μ} of a HX ring $\mathfrak{R}_1$ determined by the element $A\in \mathfrak{R}_1$.Then $f((A+\lambda^{\mu})^P)$ is the pseudo fuzzy coset of a fuzzy HX ring $f(\lambda^{\mu})$ of a HX ring $\mathfrak{R}_2$ determined by the element $f(A)\in \mathfrak{R}_2$ and $f((A+\lambda^{\mu})^P) = (f(A) + f(\lambda^{\mu}))^p$, if λ^{μ} has a supremum property and λ^{μ} is f-invariant.

Proof: Let f : $\mathfrak{R}_1 \to \mathfrak{R}_2$ be an anti homomorphism onto HX rings.

Let $(A+\lambda^{\mu})^P$ be the pseudo fuzzy coset of a fuzzy HX ring λ^{μ} of a HX ring $\mathfrak{R}_1$ determined by the element $A\in \mathfrak{R}_1$.
$f(\lambda^{\mu})$ is a fuzzy HX ring of a HX subring $\mathfrak{R}_2$.Then $f((A+\lambda^{\mu})^P)$ is the pseudo fuzzy coset of a fuzzy HX ring $f(\lambda^{\mu})$ of a HX ring $\mathfrak{R}_2$ determined by the element $f(A)\in \mathfrak{R}_2$.

Let $A, X \in \mathfrak{R}_1$, then $f(A), f(X)\in \mathfrak{R}_2$.

Now, $(f(A) + f(\lambda^{\mu}))^p f(X) = p(f(A))(f(\lambda^{\mu})(f(X))$
$= p(A)\, \lambda^{\mu}(X)$
$= (A + \lambda^{\mu})^P(X)$
$= f((A + \lambda^{\mu})^P)f(X)$.
$(f(A) + f(\lambda^{\mu}))^p f(X) = f((A + \lambda^{\mu})^P)f(X)$, for any $f(X)\in \mathfrak{R}_2$.

Hence, $f((A + \lambda^{\mu})^P) = (f(A) + f(\lambda^{\mu}))^p$.

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3.4 Theorem: Let $\mathfrak{R}_1$ and $\mathfrak{R}_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let f: $\mathfrak{R}_1 \to \mathfrak{R}_2$ be an anti homomorphism on HX rings. Let $(B+ \eta^{\alpha})^P$ be the pseudo fuzzy coset of a fuzzy HX ring η^{α} of a HX ring $\mathfrak{R}_2$ determined by the element $B\in \mathfrak{R}_2$. Then $f^{-1}((B + \eta^{\alpha})^P)$ is the pseudo fuzzy coset of a fuzzy HX ring $f^{-1}(\eta^{\alpha})$ of a HX ring $\mathfrak{R}_1$ determined by the element $f^{-1}(B)\in \mathfrak{R}_1$ and $f^{-1}((B + \eta^{\alpha})^P) = (f^{-1}(B) + f^{-1}(\eta^{\alpha}))^p$.

Proof: Let f: $\mathfrak{R}_1 \to \mathfrak{R}_2$ be an anti homomorphism on HX rings.

Let $(B + \eta^{\alpha})^P$ be the pseudo fuzzy coset of a fuzzy HX ring η^{α} of a HX ring $\mathfrak{R}_2$ determined by the element $B\in \mathfrak{R}_2$.
$f^{-1}(\eta^{\alpha})$ is a fuzzy HX subring of a HX ring $\mathfrak{R}_1$.Then $f^{-1}((B + \eta^{\alpha})^P)$ is the pseudo fuzzy coset of a fuzzy HX ring $f^{-1}(\eta^{\alpha})$ of a HX ring $\mathfrak{R}_1$ determined by the element $f^{-1}(B)\in \mathfrak{R}_1$.

Let $X \in \mathfrak{R}_1$ and $B\in \mathfrak{R}_2$.

$$\begin{aligned} \text{Now, } (f^{-1}(B) + f^{-1}(\eta^{\alpha}))^p (X) &= p(f^{-1}(B))(f^{-1}(\eta^{\alpha}))(X) \\ &= p(B)(\eta^{\alpha}(f(X)) \\ &= (B + \eta^{\alpha})^P f(X) \\ &= f^{-1}((B + \eta^{\alpha})^P)(X). \\ (f^{-1}(B) + f^{-1}(\eta^{\alpha}))^p (X) &= f^{-1}((B + \eta^{\alpha})^P)(X). \end{aligned}$$

Hence, $f^{-1}((B + \eta^{\alpha})^P) = (f^{-1}(B) + f^{-1}(\eta^{\alpha}))^p$.

4. PSEUDO FUZZY COSET OF A FUZZY NORMAL HX SUBRING OF A HX RING

In this section, we introduce the notion of pseudo fuzzy cosets of a fuzzy normal HX ring and discuss its properties.

4.1 Definition: Let μ be a fuzzy set defined on R. Let $\mathfrak{R} \subset 2^R - \{\phi\}$ be a HX ring. Let λ^{μ} be a fuzzy normal HX subring of a HX ring $\mathfrak{R}$ and $A\in \mathfrak{R}$. Then the pseudo fuzzy coset of a fuzzy normal HX ring λ^{μ} of a HX ring $\mathfrak{R}$ determined by the element $A\in \mathfrak{R}$ is denoted as $(A + \lambda^{\mu})^P$ and is defined by $(A + \lambda^{\mu})^P (X) = p(A)\, \lambda^{\mu} (X)$ for every $X\in \mathfrak{R}$ and for some $p\in P$, where $P =\{p(X) / p(X) \in[0, 1]$ for all $X\in \mathfrak{R}\}$.

4.2 Theorem: Let λ^{μ} be a fuzzy normal HX subring of a HX ring $\mathfrak{R}$, then the pseudo fuzzy coset $(A + \lambda^{\mu})^P$ is a fuzzy normal HX subring λ^{μ} of a HX ring $\mathfrak{R}$ determined by the element $A\in \mathfrak{R}$.

Proof: Let λ^{μ} be a fuzzy normal HX subring of a HX ring $\mathfrak{R}$.

By Theorem 2.2, $(A + \lambda^{\mu})^P$ is a fuzzy HX subring λ^{μ} of a HX ring $\mathfrak{R}$.

For $X, Y\in \mathfrak{R}$, we have

$$\begin{aligned} (A + \lambda^{\mu})^P (XY) &= p(A)\, \lambda^{\mu}(XY) \\ &= p(A)\, \lambda^{\mu}(YX) \\ &= (A+\lambda^{\mu})^P (YX). \\ \text{Therefore, } (A+\lambda^{\mu})^P (XY) &= (A+\lambda^{\mu})^P (YX). \end{aligned}$$

Hence, the pseudo fuzzy coset $(A + \lambda^{\mu})^P$ determined by the element $A\in \mathfrak{R}$ is a fuzzy normal HX subring of a HX ring $\mathfrak{R}$.

4.3 Theorem: Let μ and η be any two fuzzy sets defined on R. Let $(A + \lambda^{\mu})^P$ and $(A + \gamma^{\eta})^P$ be any two pseudo fuzzy coset of a fuzzy normal HX subrings λ^{μ} and γ^{η} of a HX ring $\mathfrak{R}$ determined by the element $A\in \mathfrak{R}$.Then
$(A + \lambda^{\mu})^P \cap (A + \gamma^{\eta})^P$ is a fuzzy normal HX subring of $\mathfrak{R}$.

Proof: Let $(A + \lambda^{\mu})^P$ and $(A + \gamma^{\eta})^P$ be any two pseudo fuzzy coset of a fuzzy normal HX subrings of a HX ring $\mathfrak{R}$ determined by the element $A\in \mathfrak{R}$.

By Theorem 4.2, $(A + \lambda^{\mu})^P$ and $(A + \gamma^{\eta})^P$ are fuzzy normal HX subrings of a HX ring $\mathfrak{R}$ determined by the element $A\in \mathfrak{R}$.
Hence, $(A + \lambda^{\mu})^P \cap (A + \gamma^{\eta})^P$ is a fuzzy normal HX subring of $\mathfrak{R}$ determined by the element $A\in \mathfrak{R}$.

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4.4 Theorem: Let μ and η be any two fuzzy sets defined on R. Let $(A + \lambda^{\mu})^P$ and $(A + \gamma^{\eta})^P$ be any two pseudo fuzzy coset of a fuzzy normal HX subrings λ^{μ} and γ^{η} of a HX ring $\Re$ determined by the element $A \in \Re$.Then
$(A + \lambda^{\mu})^P \cup (A + \gamma^{\eta})^P$ is a fuzzy normal HX subring of $\Re$.

Proof: Let $(A + \lambda^{\mu})^P$ and $(A + \gamma^{\eta})^P$ be any two pseudo fuzzy coset of a fuzzy normal HX subrings of a HX ring $\Re$ determined by the element $A \in \Re$.

By Theorem 4.2, $(A + \lambda^{\mu})^P$ and $(A + \gamma^{\eta})^P$ are fuzzy normal HX subrings of a HX ring $\Re$ determined by the element $A \in \Re$.
Hence, $(A + \lambda^{\mu})^P \cup (A + \gamma^{\eta})^P$ is a fuzzy normal HX subring of $\Re$ determined by the element $A \in \Re$.

4.5 Theorem: Let μ and η be any two fuzzy sets defined on R. Let $(A+ \lambda^{\mu})^P$ and $(A + \gamma^{\eta})^P$ be any two pseudo fuzzy coset of a fuzzy normal HX subrings λ^{μ} and γ^{η} of a HX ring $\Re$ determined by the element $A \in \Re$. Then
$(A+ \lambda^{\mu})^P \times (A + \gamma^{\eta})^P$ is a fuzzy normal HX subring of $\Re \times \Re$.

Proof: Let $(A + \lambda^{\mu})^P$ and $(A + \gamma^{\eta})^P$ be any two pseudo fuzzy coset of a fuzzy HX subring λ^{μ} of a HX ring $\Re$ determined by the element $A \in \Re$.

By Theorem 4.2, $(A + \lambda^{\mu})^P$ and $(A + \gamma^{\eta})^P$ are fuzzy normal HX subrings of a HX ring $\Re$ determined by the element $A \in \Re$.
Hence, $(A + \lambda^{\mu})^P \times (A + \gamma^{\eta})^P$ is a fuzzy normal HX subring of $\Re \times \Re$ determined by the element $A \in \Re$.

5. HOMOMORPHISM AND ANTI HOMOMORPHISM OF A PSEUDO FUZZY COSET OF A FUZZY NORMAL HX SUBRING OF A HX RING

In this section, we introduce the concept of an image, pre-image of pseudo fuzzy coset of a HX ring and discuss the properties of homomorphic and anti homomorphic images and pre images of pseudo fuzzy coset of a fuzzy normal HX ring λ^{μ} of a HX ring $\Re$ determined by the element $A \in \Re$.

5.1 Theorem: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let $f : \Re_1 \rightarrow \Re_2$ be a homomorphism onto HX rings. Let $(A + \lambda^{\mu})^P$ be the pseudo fuzzy coset of a fuzzy normal HX ring λ^{μ} of a HX ring $\Re_1$ determined by the element $A \in \Re_1$.Then $f((A + \lambda^{\mu})^P)$ is the pseudo fuzzy coset of a fuzzy normal HX ring $f(\lambda^{\mu})$ of a HX ring $\Re_2$ determined by the element $f(A) \in \Re_2$ and $f((A + \lambda^{\mu})^P) = (f(A) + f(\lambda^{\mu}))^p$, if λ^{μ} has a supremum property and λ^{μ} is f-invariant.

Proof: Let $f: \Re_1 \rightarrow \Re_2$ be a homomorphism onto HX rings.

Let $(A + \lambda^{\mu})^P$ be the pseudo fuzzy coset of a fuzzy normal HX subring λ^{μ} of a HX ring $\Re_1$ determined by the element $A \in \Re_1$. $f(\lambda^{\mu})$ is a fuzzy normal HX ring of a HX ring $\Re_2$.

By Theorem 3.1, $f((A + \lambda^{\mu})^P)$ is the pseudo fuzzy coset of a fuzzy normal HX ring $f(\lambda^{\mu})$ of a HX ring $\Re_2$ determined by the element $f(A) \in \Re_2$ and $f((A + \lambda^{\mu})^P) = (f(A) + f(\lambda^{\mu}))^p$.

5.2 Theorem: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let $f: \Re_1 \rightarrow \Re_2$ be a homomorphism on HX rings. Let $(B + \eta^{\alpha})^P$ be the pseudo fuzzy coset of a fuzzy normal HX ring η^{α} of a HX ring $\Re_2$ determined by the element $B \in \Re_2$.Then $f^{-1}((B + \eta^{\alpha})^P)$ is the pseudo fuzzy coset of a fuzzy normal HX ring $f^{-1}(\eta^{\alpha})$ of a HX ring $\Re_1$ determined by the element $f^{-1}(B) \in \Re_1$ and $f^{-1}((B + \eta^{\alpha})^P) = (f^{-1}(B) + f^{-1}(\eta^{\alpha}))^p$.

Proof: Let $f : \Re_1 \rightarrow \Re_2$ be a homomorphism on HX rings.

Let $(B + \eta^{\alpha})^P$ be the pseudo fuzzy coset of a fuzzy normal HX ring η^{α} of a HX ring $\Re_2$ determined by the element $B \in \Re_2$. $f^{-1}(\eta^{\alpha})$ is a fuzzy normal HX ring of a HX ring $\Re_1$.

By Theorem 3.2, $f^{-1}((B + \eta^{\alpha})^P)$ is the pseudo fuzzy coset of a fuzzy normal HX ring $f^{-1}(\eta^{\alpha})$ of a HX ring $\Re_1$ determined by the element $f^{-1}(B) \in \Re_1$ and $f^{-1}((B + \eta^{\alpha})^P) = (f^{-1}(B) + f^{-1}(\eta^{\alpha}))^p$.

5.3 Theorem: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let f: $\Re_1 \rightarrow \Re_2$ be an anti homomorphism onto HX rings. Let $(A + \lambda^{\mu})^P$ be the pseudo fuzzy coset of a fuzzy normal HX ring λ^{μ} of a HX ring $\Re_1$ determined by the element $A \in \Re_1$.Then $f((A + \lambda^{\mu})^P)$ is the pseudo fuzzy coset of a fuzzy normal HX ring $f(\lambda^{\mu})$ of a HX ring $\Re_2$ determined by the element $f(A) \in \Re_2$ and $f((A + \lambda^{\mu})^P) = (f(A) + f(\lambda^{\mu}))^p$, if λ^{μ} has a supremum property and λ^{μ} is f-invariant.

Proof: Let f : $\Re_1 \rightarrow \Re_2$ be an anti homomorphism onto HX rings.

Let $(A + \lambda^{\mu})^P$ be the pseudo fuzzy coset of a fuzzy normal HX ring λ^{μ} of a HX ring $\Re_1$ determined by the element $A \in \Re_1$.$f(\lambda^{\mu})$ is a fuzzy normal HX ring of a HX ring $\Re_2$.

By Theorem 3.3, $f((A + \lambda^{\mu})^P)$ is the pseudo fuzzy coset of a fuzzy normal HX ring $f(\lambda^{\mu})$ of a HX ring $\Re_2$ determined by the element $f(A) \in \Re_2$ and $f((A + \lambda^{\mu})^P) = (f(A) + f(\lambda^{\mu}))^p$.

5.4 Theorem: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let f : $\Re_1 \rightarrow \Re_2$ be an anti homomorphism on HX rings. Let $(B + \eta^{\alpha})^P$ be the pseudo fuzzy coset of a fuzzy normal HX ring η^{α} of a HX ring $\Re_2$ determined by the element $B \in \Re_2$.Then $f^{-1}((B + \eta^{\alpha})^P)$ is the pseudo fuzzy coset of a fuzzy normal HX ring $f^{-1}(\eta^{\alpha})$ of a HX ring $\Re_1$ determined by the element $f^{-1}(B) \in \Re_1$ and $f^{-1}((B + \eta^{\alpha})^P) = (f^{-1}(B) + f^{-1}(\eta^{\alpha}))^p$.

Proof: Let f : $\Re_1 \rightarrow \Re_2$ be an anti homomorphism on HX rings.

Let $(B + \eta^{\alpha})^P$ be the pseudo fuzzy coset of a fuzzy normal HX ring η^{α} of a HX ring $\Re_2$ determined by the element $B \in \Re_2$. $f^{-1}(\eta^{\alpha})$ is a fuzzy normal HX ring of a HX ring $\Re_1$.

By Theorem 3.4, $f^{-1}((B + \eta^{\alpha})^P)$ is the pseudo fuzzy coset of a fuzzy normal HX ring $f^{-1}(\eta^{\alpha})$ of a HX ring $\Re_1$ determined by the element $f^{-1}(B) \in \Re_1$ and $f^{-1}((B + \eta^{\alpha})^P) = (f^{-1}(B) + f^{-1}(\eta^{\alpha}))^p$.

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