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VERTEX EQUITABLE LABELING OF ALTERNATE SNAKE GRAPHS

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ABSTRACT

Let G be a graph with p vertices and q edges and A = {0, 1, 2,..., $\left\lceil \frac{q}{2} \right\rceil$*} A vertex labeling f: V (G)* $\rightarrow$ *A induces an edge labeling* f^* *defined by* $f^*(uv) = f(u) + f(v)$ *for all edges uv. For* $a \in A$*, let* $v_f(a)$ *be the number of vertices v with f(v) = a. A graph G is said to be vertex equitable if there exists a vertex labeling f such that for all a and b in A,* $\left|v_f(a) - v_f(b)\right| \le 1$ *and the induced edge labels are 1, 2, 3,…, q. In this paper, we investigate some new families of vertex equitable graphs.*

Key words: *Vertex equitable labeling, vertex equitable graph.*

AMS Classification (2010): *05C78.*

1. INTRODUCTION

All graphs considered here are simple, finite, connected and undirected .We follow the basic notations and terminologies of graph theory as in [1]. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. There are several types of labeling and a detailed survey of graph labeling can be found in [2]. The vertex set and the edge set of a graph are denoted by $V(G)$ and $E(G)$ respectively. The concept of vertex equitable labeling was due to Lourdusamy and Seenivasan in [3] and further discussed in [4-11]. Let G be a graph with p vertices and q edges and A = {0,1,2, ..., $\left\lceil \frac{q}{2} \right\rceil$}. A graph G is said to be vertex equitable if there exists a vertex labeling $f : V(G) \rightarrow A$ that induces an edge labeling f^* defined by $f^*(uv) = f(u) + f(v)$ for all edges uv such that for all a and b in A, $\left|v_f(a) - v_f(b)\right| \le 1$ and the induced edge labels are 1, 2, 3,…, q, where $v_f(a)$ be the number of vertices v with $f(v) = a$ for $a \in A$. The vertex labeling f is known as vertex equitable labeling. A graph G is said to be a vertex equitable if it admits vertex equitable labeling. In this paper we extend our study on vertex equitable labeling and prove that the graphs $S(Q_n)$, $A(Q_{4m}) \odot nK_1$ and $A(TL_{4m})$ are vertex equitable.

In this paper main results follows after some definitions.

Theorem 1.1 [9]: Let $G_1(p_1,q_1)$, $G_2(p_2,q_2)$,…, $G_m(p_m,q_m)$ be vertex equitable graphs with q_i's even (i=1,2,…,m) and u_i, v_i be the vertices of G_i $(1 \le i \le m)$ labeled by 0 and $\frac{q_i}{2}$. Then the graph G obtained by identifying v_1 with u_2 and v_2 with u_3 and v_3 with u_4 and so on until we identify v_{m-1} with u_m is also a vertex equitable graph.

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Theorem 1.2 [9]: Let $G_1(p_1, q)$, $G_2(p_2, q)$,…, $G_m (p_m , q)$ be vertex equitable graphs with q odd and u_i, v_i be vertices of G_i $(1 \le i \le m)$ labeled by 0 and $\left\lceil \frac{q}{2} \right\rceil$. Then the graph G obtained by joining v_1 with u_2 and v_2 with u_3 and v_3 with u_4 and so on until joining v_{m-1} with u_m by an edge is also a vertex equitable graph.

Definition 1.3: The corona $G_1 \odot G_2$ of the graphs G_1 and G_2 is defined as a graph obtained by taking one copy of G_1 (with p vertices) and p copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex of the i^{th} copy of G_2.

Definition 1.4: A Triangular ladder $TL_n, n \ge 2$ is a graph obtained from the ladder $L_n = P_n \times P_2$ by adding the edges $u_i v_{i+1}, 1 \le i \le n-1$ .Such a graph has 2n vertices with 4n-3 edges.

Definition 1.5: Let G be a graph. The subdivision graph $S(G)$ is obtained from G by subdividing each edge of G with a vertex.

Definition 1.6: An alternate quadrilateral snake $A(Q_n)$ consists of alternate quadrilateral snakes that have a common path. That is, a alternate quadrilateral snake is obtained from a path $u_1, u_2, ..., u_n$ by joining u_i and u_{i+1} (alternatively) to the two new vertices v_i and w_i respectively and adding the edges $v_i w_i$ for i=1,2,…,n-1.That is every alternative edge of a path is replaced by a cycle C_4.

2. MAIN RESULTS

Theorem 2.1: The graph $S(Q_n)$ is a vertex equitable graph.

Proof: Let $G_i = S(Q_2)$ $1 \le i \le n-1$ and u_i , v_i be the vertices of degree 2.
The vertex equitable labeling of $G_i = S(Q_2)$ is given in Figure 2.2.

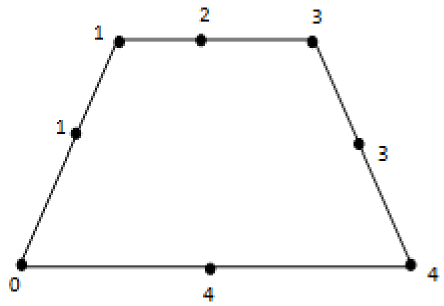

Figure-2.2

The vertex equitable labeling of u_i and v_i are 0 and $\frac{q_i}{2} = 2$ respectively. By Theorem 1.1, $S(Q_n)$ is vertex equitable labeling.

Theorem 2.3: The graph $A(Q_4) \odot nK_1$ is a vertex equitable graph for $n \ge 1$.

Proof: Let $G = A(Q_4) \odot nK_1$ .Let $V(G)$={u_1,u_2,u_3,u_4,v,w,x,y} $\cup \{ u_{ij} : 1 \le i \le 4, 1 \le j \le n \} \cup \{ v_i, w_i, x_i, y_i : 1 \le i \le n \}$ and $E(G)$={$u_1u_2,u_2u_3,u_3u_4,u_1v$, vw, wu_2,u_3x,u_4y,xy} $\cup \{ u_i u_{ij} : 1 \le i \le 4, 1 \le j \le n \} \cup \{ vv_i, ww_i, xx_i, yy_i : 1 \le i \le n \}$. Here $|V(G)|$ =8(n+1) and $|E(G)| = 8n + 9$.

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Let $A = \{0, 1, 2,..., \left\lceil \frac{8n+9}{2} \right\rceil\}$.Define a vertex labeling $f: V(G) \rightarrow A$ as follows. For $1 \le i \le n$

$f(u_{1i}) = f(v_i) = i$, $f(u_{2i}) = 2n+3-i$, $f(u_{3i}) = 2n+2+i$, $f(u_{4i}) = f(y_i) = 3n+4+i$, $f(x_i) = 2n+i+2$, $f(w_i) = n+1+i$, $f(u_1) = 0$, $f(u_2) = 2n+2$, $f(u_3) = 2n+3$, $f(u_4) = 4n+5$, $f(v) = n+1$, $f(w) = n+2$, $f(x) = 3n+3$, $f(y) = 3n+4$. It can be verified that the induced edge labels of $A(Q_4) \odot nK_1$ are 1, 2,…, 8n+9 and $|v_f(a) - v_f(b)| \le 1$ for all $a, b \in A$. Hence f is a vertex equitable labeling of $A(Q_4) \odot nK_1$.

An example for the vertex equitable labeling of $A(Q_4) \odot 4K_1$ is shown in Figure 2.4.

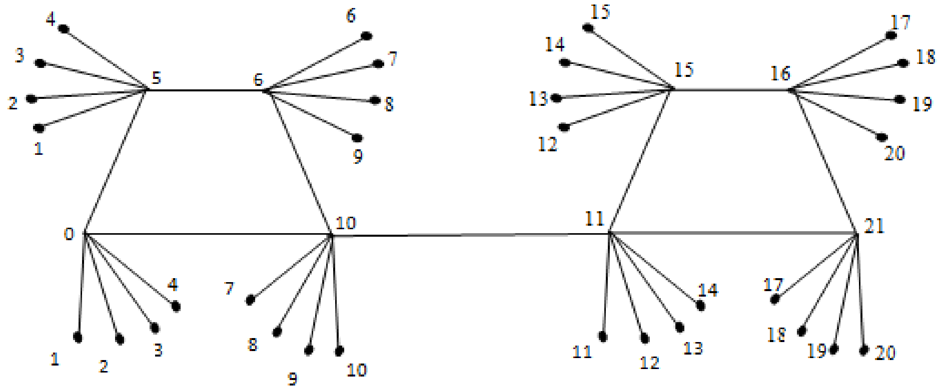

Figure-2.4

Theorem 2.5: The graph $A(Q_{4m}) \odot nK_1$ is a vertex equitable graph for $m \ge 2, n \ge 1$

Proof: By Theorem 2.3, $A(Q_4) \odot nK_1$ is a vertex equitable graph for $n \ge 1$. Let $G_i = A(Q_4) \odot nK_1$ for $1 \le i \le m-1$. Since each G_i has 8*n*+9 edges, by Theorem 1.2, $A(Q_{4m}) \odot nK_1$ admits vertex equitable labeling.

Theorem 2.6: The graph $A(TL_4)$ is a vertex equitable graph.

Proof: Let $V(A(TL_4))=\{u_1,u_2,u_3,u_4,v,w,x,y\}$ and $E(A(TL_4))=\{u_1u_2,u_2u_3,u_3u_4,u_1v,u_1w,vw, wu_2,u_3x,u_3y,u_4y,xy\}$. Here $|V(A(TL_4))| = 8$ and $|E(A(TL_4))| = 11$. Let $A = \{0, 1, 2,...,6\}$. Define a vertex labeling $f: V(G) \rightarrow A$ as follows. $f(u_i) = 2i-2$ *if* $1 \le i \le 4$, $f(v) = 1$, $f(w) = f(x) = 3$, $f(y) = 5$. The vertex equitable labeling of $A(TL_4)$ is given in Figure 2.7.

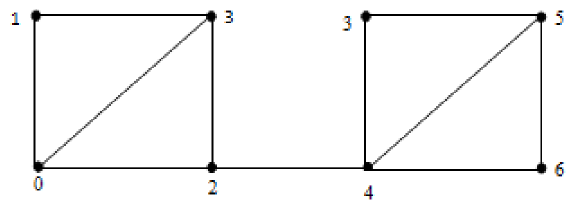

Figure-2.7

Theorem 2.8: The graph $A(TL_{4m})$ is a vertex equitable graph for $m \ge 2$.

Proof: By Theorem 2.6, $A(TL_4)$ is a vertex equitable graph. Let $G_i = A(TL_4)$ for $1 \le i \le m-1$. Since each G_i has 11 edges, by Theorem 1.2, $A(TL_{4m})$ admits vertex equitable labeling.

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