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ON NEW SUPRA TOPOLOGICAL SPACES VIA A NEW NOTION OF SOFT SET

[1]M. GILBERT RANI, [2]G. RAM KUMAR AND [3]R. RAJESWARI

[1&2]Assistant Professor, Arul Anandar College, Karumathur, India.
[3]Assistant Professor, Fatima College, Madurai - (T.N.), India.

ABSTRACT

With the help of Modern Mathematics to solve so many complicated problems in engineering, medical science, economics and environmental science. To enrich this path Molodstov[1] introduced the concept of soft set as a new mathematical tool. This paper introduces supra bijective soft topological spaces and its properties.

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1. INTRODUCTION

In this modern world we come across many uncertainty and vagueness problems. Classical Mathematics is not enough to deal with such situations. In 1999 D.Molodstov[1] introduced the notion of soft set as a newly emerging tool.

Based on the work of Molodstov, Maji *et al.* [2] defined equality of two soft sets, subset and super set of soft sets, complement of a soft set, null soft set and absolute soft set with examples. Ke Gong *et al.* [3] proposed bijective soft set and some operations on it. In this paper, we form the topological spaces using bijective soft sets.

The rest of the paper is organized as follows. Section 2 helps recollect all the needed results and definitions. Section 3 introduces supra bijective soft topological spaces and its properties.

2. PRELIMINARIES

Definition 2.1[3]: A subclass $\tau^* \subset P(X)$ is called a supra topology on X if $X \in \tau^*$ and is closed under arbitrary union. (X, τ^*) is called a supra topological space(or supra space). The members of τ^* are called supra open sets.

Definition 2.2 ([1]): Let U be a common universe and let E be a set of parameters. A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U, where F is a mapping given by F: E → P(U).
In other words, the soft set is a parameterized family of subsets of the set U. Every set F(ε) (ε ∈ E), from this family may be considered as the set of ε-elements of the soft sets (F, E), or as the set of ε-approximate elements of the soft set.

Definition 2.3 [6]: The intersection of two soft sets (F, A) and (G, B) over U is the soft set (H, C), where C = A∩B and ∀ε ∈ C, H(ε) = F (ε) or G(ε) (as both are same set). This is denoted by (F, A) $\tilde{\cap}$ (G, B) = (H, C).

Definition 2.4 [6]: The union of two soft sets (F, A) and (G, B) over U is the soft set (H, C), where C = A∪B and ∀ε ∈ C,

$$H(\varepsilon) = \begin{cases} \mathrm{F}(\varepsilon), & \text{if } \varepsilon \in \mathrm{A - B} \\ \mathrm{G}(\varepsilon), & \text{if } \varepsilon \in \mathrm{B - A} \\ \mathrm{F}(\varepsilon) \cup \mathrm{G}(\varepsilon), & \text{if } \varepsilon \in \mathrm{A} \cap \mathrm{B} \end{cases}$$

This is denoted by (F, A) $\tilde{\cup}$ (G, B) = (H, C).

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Definition 2.5[6]: (AND operation on two soft sets) If (F, A) and (G, B) are two soft sets then ''(F, A) AND (G, B)'' denoted by (F, A) ∧ (G, B) is defined by

$(F, A) \wedge (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$, $\forall(\alpha, \beta) \in A \times B$.

Definition 2.6 [6]: (NULL SOFT SET) A soft set (F, A) over U is said to be a NULL soft set denoted by Φ, if ε ∈ A, $F(\varepsilon) = \emptyset$.

Definition 2.7[3]: Let (F, B) be a soft set over a common universe U, where F is a mapping $F: B \to P(U)$ and B is nonempty parameter set. We say that (F, B) is a bijective soft set, if (F, B) such that

(i) $\bigcup_{\varepsilon \in B} F(\varepsilon) = U.$

(ii) For any two parameters $e_i, e_j \in B$, $e_i \neq e_j$, $F(e_i) \cap F(e_j) = \emptyset$.

In other words, Suppose $Y \subseteq P(U)$ and $Y = \{F(e_1), F(e_2), \ldots, F(e_n)\}$, $e_1, e_2, \ldots, e_n \in B$.

That is the mapping $F: B \to P(U)$ can be transformed to the mapping $F: B \to Y$, which is a bijective function. *i.e.* for every y ∈ Y, there is exactly one parameter e in B such that F(e) = y and no unmapped element remains in both B and Y.

3. SOME OPERATIONS ON BIJECTIVE SOFT SET

Definition 3.1: Let (F,A) and (G,B) be two bijective soft sets over U. The intersection of (F,A) and (G,B) denoted by $(F,A) \cap (G,B)$ is defined as $(F \cap G, C)$ where $C = \{e \in A \cap B / F(e) \cap G(e) \neq \varphi\}$ for all $e \in C.(F \cap G)(e) = F(e) \cap G(e)$ In particular, if $A \cap B = \phi$ or $F(e) \cap G(e) = \phi$ for every $e \in A \cap B$, then we see that gg $(F,A) \cap (G, B) = \{\phi\}$

Definition 3.2: Let (F, A) and (G, B) be two bijective soft sets over U. The union of (F, A) and (G, B) denoted by $(F, A) \cup (G, B)$ is defined as $(F \cup G, C)$, where $C = A \cup B$; for all $e \in C$,

$$(F \cup G)(e) = \begin{cases} F(e), & if\ e \in A \setminus B \\ G(e), & if\ e \in B \setminus A \\ F(e) \cup F(e) & otherwise \end{cases}$$

Definition 3.3: Let (F,A) and (G,B) be two bijective soft sets over U. The difference of (F, A) and (G, B) denoted by $(F,A) \setminus (G, B)$ is defined as $(F \setminus G, C)$

Where $C = A \setminus \{e \in A \cap B \setminus F(e) \subseteq G(e), \mathit{for\ all}\ e \in C\}$,

$$(F \setminus G)(e) = \begin{cases} F(e) \setminus G(e) & if\ e \in A \cap B \\ F(e) & otherwise \end{cases}$$

Example 3.4: Suppose $U = \{a_1, a_2, \ldots\ldots, a_8\}$ is a common universe, (F, E) is a soft set over U,

$E = \{e_1, e_2, e_3, e_4, e_5\}$. The mapping is given below

$F(e_1) = \{a_1, a_3, a_5, a_7\}$, $F(e_2) = \{a_2, a_4, a_6, a_8\}$, $F(e_3) = \{a_1, a_2, a_5, a_6, a_8\}$, $F(e_4) = \{a_3, a_4, a_7\}$,

$F(e_5) = \{a_1, a_3, a_4, a_7\}$

From definition 2.7, $(F,\{e_1,e_2\})$, $(F,\{e_3,e_4\})$, $(F,\{e_3,e_5\})$are bijective.

Take $F_1 = (F,\{e_3,e_4\})$ and $F_2 = (F,\{e_3,e_5\})$

$(F, \{e_3,e_4\}) \cup (F, \{e_3,e_5\}) = (F, \{e_3,e_4,e_5\})$ which is a bijective soft set.

$(F, \{e_3,e_4\}) \cap (F, \{e_3,e_5\}) = (F, \{e_3\})$ which is not a bijective soft set.

Remark: The class of all bijective soft sets over U will be denoted by $\mathcal{BS}(\mathcal{U})$.

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4. SUPRA BIJECTIVE SOFT TOPOLIGICAL SPACES

Definition 4.1: Asupra bijective soft topology on (F, E), τ_B, is a collection of bijective soft subsets of (F, E) having the following properties:

(1) $\tilde{\phi}, (F, E) \in \tilde{\tau}_B$

(2) $\{(F, E_i) \subseteq (F,E): i \in I \subseteq N\} \subseteq \tilde{\tau}_B$

The pair (F, E_i, $\tilde{\tau}_B$) is called a supra bijective soft topological space.

Example 4.2: Let U= $\{u_1, u_2, ..., u_7\}$, e= $\{e_1, e_2, e_3, e_4\}$ the mapping is

F(e_1)= $\{u_1, u_2, u_7\}$

F(e_2)= $\{u_2, u_5\}$

F(e_3)= $\{u_1, u_3, u_{4,}, u_6, u_7\}$

F(e_4)= $\{u_3, u_4, u_5, u_6\}$

(F,{e_2,e_3}),(F,{e_1,e_4})are all bijective soft sets. Then

$\tilde{\tau}^1{}_B$ ={ $\tilde{\phi}, (F, E), (F, \{e_2, e_3\})$ }

$\tilde{\tau}^2{}_B$ ={ $\tilde{\phi}, (F, E), (F, \{e_1, e_4\})$ } are supra bijective soft topologies on (F, E).

Definition 4.3: Let (F, E, $\tilde{\tau}_B$) be a supra bijective soft topological space. Then every element of $\tilde{\tau}_B$ is called a supra bijective soft open set. Clearly, $\tilde{\phi}, (F, E)$ are supra bijective soft open sets.

Definition 4.4: Let (F, E, $\tilde{\tau}_B$) be a supra bijective soft topological space and (F, A) $\subseteq$ (F, E). Then (F, A) is said to be supra bijective soft closed if the soft set $(F, A)^c$ is supra bijective soft open sets.

Example 4.5: In example 4.2

$\tilde{\tau}^1{}_B$ ={ $\tilde{\phi}, (F, E), (F, \{e_2, e_3\})$ }

$\tilde{\tau}^2{}_B$ ={ $\tilde{\phi}, (F, E), (F, \{e_1, e_4\})$ } are supra bijective soft topologies on (F, E). Where both $(F, \{e_2, e_3\})$ and $(F, \{e_1, e_4\})$ supra bijective soft open sets as well as supra bijective soft closed sets.

Theorem 4.5: Let (F, E, $\tilde{\tau}_B$) be a supra bijective soft topological space .Then the following hold:

(1) The universal supra bijective soft set (F, E) and $\tilde{\phi}$ are supra bijective soft closed sets.

(2) Arbitrary supra bijective soft intersections of the supra bijective soft closed sets are supra bijective soft closed set.

Definition 4.6: Let (F, E, $\tilde{\tau}_B$) be a supra bijective soft topological spaceand (F, A) $\subseteq$ (F, E). Then the supra bijective soft interior of a supra bijective soft set (F, A) is denoted by $(F, A)^0$ and is defined as the soft union of all supra bijective soft open subsets of (F, A). Thus $(F, A)^0$ is the largest supra bijective soft open set contained in (F, A).

Theorem 4.7: Let (F, E, $\tilde{\tau}_B$) be a supra bijective soft topological space and (F, A), (F, B) $\subseteq$ (F, E) then

(1) $\tilde{\phi}^{s_0} = \phi$

(2) $(F, A)^{s_0} \subset (F, A)$

(3) $((F, A)^{s_0})^{s_0} \subset (F, A)^{s_0}$

(4) (F, B) is a bijective soft open set if and only if $(F, A)^{s_0} = (F, A)$

(5) $(F, A) \subseteq (F, B) \Rightarrow (F, A)^{s_0}) \subseteq (F, B)^{s_0}$

(6) $(F, A)^{s_0}) \cap (F, B)^{s_0} \subset ((F, A) \cap (F, B))^{s_0}$

(7) $(F, A)^{s_0}) \cup (F, B)^{s_0} = ((F, A) \cup (F, B))^{s_0}$

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Definition 4.8: Let (F, E, $\tilde{\tau}_B$) be a supra bijective soft topological space and (F, A) $\subseteq$ (F, E). Then the supra bijective soft closure of (F, A) is denoted by $\overline{(F,A)}^s$ is defined as the soft intersection of all supra bijective soft closed supersets of (F, A). $\overline{(F,A)}^s$ is the smallest supra bijective soft closed set containing F_B.

Theorem 4.9: Let (F, E, $\tilde{\tau}_B$) be a supra bijective soft topological space and (F, A) $\not\subset$ (F, E). (F, A) is a supra bijective soft closed set if and only if $\overline{(F,A)} = (F,A)$

Theorem 4.10: Let (F, E, $\tilde{\tau}_B$) be a supra bijective soft topological space and (F, A), (F, B) $\subseteq$ (F, E). Then

$$(1)\ \overline{\overline{(F,A)}} = \overline{(F,A)}$$

$$(2)\ (F,B) \subset (F,A) \Longrightarrow \overline{(F,B)} \subset \overline{(F,A)}$$

$$(3)\ \overline{(F,A)\cup(F,B)} \supset \overline{(F,A)} \cup \overline{(F,B)}$$

$$(4)\ \overline{(F,A)\cap(F,B)} \supseteq \overline{(F,A)} \cap \overline{(F,B)}$$

Definition 4.11 : Let *(F, E , $\tilde{\tau}_B$)* be a supra bijective soft topological space and *(F, A)* $\subseteq$ *(F, E)*. Then, the supra bijective soft boundary of soft set *(F, A)* is denoted by *(F, A)*^{b} and is defined as $(F,A)^b = \overline{(F,A)} \cap \overline{(F,A)^C}$

Theorem 4.12: Let (F, E , $\tilde{\tau}_B$) be a supra bijective soft topological space and (F, A), (F, B) $\subseteq$ (F, E). Then

$$(1)(F,B)^b \subseteq \overline{(F,B)}$$

$$(2)(F,B)^b = ((F,B)^C)^b$$

$$(3)(F,B)^b = \overline{(F,B)} \setminus (F,B)^O$$

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