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L-FUZZY CONNECTED ON L- FUZZY TOPOLOGICAL TM- SYSTEMS

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ABSTRACT

In 2010, Tamilarasi and Megalai introduced a new class of algebras called as TM-algebras. In this paper, we discuss the notion of L-Fuzzy connected on L-Fuzzy Topological TM-systems.

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Key words: *BCK/BCI Algebra, TM-Algebra, Fuzzy set, Fuzzy Topology.*

1. INTRODUTION

In 1996, Y.Imai and Iseki [6] introduced two classes of algebras originated from the classical and non-classical propositional logic. These algebras are known as BCK and BCI algebras. It is known that the notion of BCI-algebra is a generalization of BCK-algebras in such a way that the class of BCK algebras is a subclass of the class of BCI algebras [7]. Recently in 2010, Tamilarasi and Manimegalai introduced a new class of algebras called TM-algebras [9ss].

In 1965, L.A.Zadeh [12] introduced the notion of fuzzy sets, to evaluate the modern concept of uncertainty in real physical world. In the notion of fuzzy sets, the boundaries are not crisp or sharp but flexible. In 1967, J.A.Goguen [5] introduced the concept of L-Fuzzy sets.

The theory of fuzzy topological spaces is developed by Chang [4], Wong [11], Lowen [8] and others. In [1], we studied L- fuzzy topological TM-system. In [2], we studied L–fuzzy topological TM subsystem. In [3], we studied fuzzy connected on fuzzy topological TM- system. In 2007, Qutaiba Ead Hassan, Marid [10] introduced the notion of fuzzy connected spaces. In this paper, we discuss the notion of L- fuzzy connected on L-fuzzy topological Tm-Systems and investigate some of simple but elegant results.

2. PRELIMINARIES

In this section we recall some basic definitions that are required in the sequel.

Definition 2.1: Let X be a non-empty set. A mapping $\mu: X \to L$ is called an L- fuzzy set of X, where L is a complete lattice, with sup 1 and inf 0.

Definition 2.2: Let A and B be any two fuzzy sets in a non-empty set X.

(1) The union of A and B denoted by, AUB is defined to be the L-Fuzzy set $(AUB)(x)=\mu_A(x) V \mu_B(x)$ for all $x \in X$.
(2) The intersection of A and B, denoted by, $A\cap B$ is defined to be the L- fuzzy set $(A\cap B)(x) = \mu_A(x) \Lambda \mu_B(x)$ for all $x \in X$.
(3) $A \subset B \Rightarrow A(x) \leq (B)x$ for all $x \in X$.
(4) The Complement of A is defined to be A' (x) =1- A(x) for all $x \in X$

Definition 2.3: A lattice is a partially ordered set in which any two elements have a least upper bound and a greatest lower bound.

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Definition 2.4: A lattice L is called a complete lattice if every subset A={a_α} of L has a sup denoted by $1\equiv\wedge a_\alpha$ and an inf denoted by $0 \equiv\vee a_\alpha$

Definition 2.5: A TM-Algebra (X,*,0) is a non-empty set X with a constant 0 and a binary operation * satisfying the following axioms;

(1) x*0 =x
(2) (x*y)*(x*z)=z*y for all x, y, z $\in$ X

Definition 2.6: L – Fuzzy TM-Subalgebra. Let L be a complete lattice with sup 1 and inf 0. An L-fuzzy subset μ of a TM-Algebra (X,*,0) is called an L –fuzzy TM-Subalgebra of X if, for all x, y ∈X, $\mu(x*y) \geq \mu(x) \wedge \mu(y)$.

Definition 2.7: Let (X,*) be a TM-Algebra. X is said to be a Fuzzy Topological TM-System if there is a family T of fuzzy subalgebras in X which satisfies the following conditions

(1) ϕ, X∈T
(2) If A,B ∈T then A∩B∈T
(3) If A_i∈T for each i∈I then $U_I A_i$∈T where I is an indexing set.

Any element in T is called a T – open fuzzy subalgebra in the TM system X.

Definition 2.8: A fuzzy set μ is said to be proper if $\mu \neq 0$ and $\mu\neq1$

Definition 2.9: A fuzzy set μ which is both fuzzy open and fuzzy closed is called fuzzy clopen set

3. L- FUZZY CONNECTED ON L-FUZZY TOPOLOGICAL TM-SYSTEMS

Definition 3.1: Let (X,*) be a TM-algebra. X is said to be a L-fuzzy topological TM-system if there is a family L_T of fuzzy subalgebras in X which satisfies the following conditions

i) φ, X ε L_T
ii) If A,B ε L_T then $A\wedge B \varepsilon L_T$
iii) If A_i ε L_T for each iεI then $\vee_I A_i \varepsilon L_T$ where I is an indexing set.

Any element in L_T is called a L_T -open fuzzy subalgebra in the TM-algebra.

Remark 3.2: If X is a set with an L-fuzzy topological then (X, L_T) is called an L-fuzzy topological TM – System and any element in L_T is called an L_{T-} open fuzzy subalgebra in X

Example 3.3: Consider the set X = {0, 1, 2, 3, 4, 5} with the following Cayley table

*	0	1	2	3	4	5
0	0	3	4	1	2	5
1	1	0	2	3	5	4
2	2	4	0	5	1	3
3	3	1	5	0	4	2
4	4	5	3	2	0	1
5	5	2	1	4	3	0

Then (X,*) is a TM-algebra. Let L be a complete lattice with $\sup(L) \equiv 1$ and $\inf(L) \equiv 0$.

Let $t_1, t_2, t_3, t_4, t_5, t_6, t_7 \in L$ such that $0\leq t_1 \leq t_2 \leq t_3 \leq t_4 \leq t_5 \leq t_6 \leq t_7 \leq 1$

Let the L- fuzzy subalgebras $\mu_i : X \rightarrow L$, i=1,2,3,4,5,6,7,8,9,10 be given by

$$\mu_1(x)=\begin{cases} 1 & if\ x=0 \\ t_5 & if\ x=1,3 \\ t_3 & if\ x=2,4,5 \end{cases} \quad \mu_2(x)=\begin{cases} t_7 & if\ x=0 \\ 0 & if\ x=1,2,3,4 \\ t_3 & if\ x=5 \end{cases} \quad \mu_3(x)=\begin{cases} t_3 & if\ x=0,5 \\ 0 & if\ x=1,2,3,4 \end{cases}$$

$$\mu_4(x)=\begin{cases} 1 & if\ x=0 \\ t_4 & if\ x=1,3 \\ t_3 & if\ x=2,4,5 \end{cases} \quad \mu_5(x)=\begin{cases} 1 & if\ x=0 \\ t_2 & if\ x=1,2,3,4 \\ t_3 & if\ x=5 \end{cases} \quad \mu_6(x)=\begin{cases} 1 & if\ x=0 \\ t_6 & if\ x=1,3 \\ t_4 & if\ x=2,4,5 \end{cases}$$

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$\mu_7(x)=\begin{cases} 1 & if\ x=0 \\ t_1 & if\ x=1,2,3,4 \\ t_3 & if\ x=5 \end{cases}$ $\mu_8(x)=\begin{cases} 1 & if\ x=0 \\ t_5 & if\ x=1,3 \\ t_4 & if\ x=2,4,5 \end{cases}$ $\mu_9(x)=\begin{cases} t_6 & if\ x=0 \\ 0 & if\ x=12,3,4 \\ t_3 & if\ x=5 \end{cases}$

$\mu_{10}(x)=\begin{cases} 1 & if\ x=0 \\ t_7 & if\ x=1,3 \\ t_4 & if\ x=2,4,5 \end{cases}$

Then the collection $L_T=\{\phi, X, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6, \mu_7, \mu_8, \mu_9, \mu_{10}\}$ is an L-fuzzy topology on X. Hence (X,L_T) is an L - fuzzy topological TM-system.

Definition 3.4: Let (X, *) be a TM algebra. Let (X,T) be a fuzzy topological TM-systems an X respectively. A fuzzy topological TM-System (X,T) is called fuzzy connected TM-System if it has no proper fuzzy clopen subalgebras.

Definition 3.5: Let (X,*) be a TM algebra. Let (X, L_T) be a fuzzy topological TM-systems on X respectively. An L- fuzzy topological TM-System (X, L_T) is called L-fuzzy connected TM-System if it has no proper L-fuzzy clopen subalgebras.

Example 3.6: Considet the set X = {0, 1, 2, 3, 4} with the following Cayley table

*	0	1	2	3	4
0	0	4	3	2	1
1	1	0	4	3	2
2	2	1	0	4	3
3	3	2	1	0	4
4	4	3	2	1	0

Then (X,*) is a TM-algebra. Let L be a complete lattice with $\sup(L) \equiv 1$ and $\inf(L) \equiv 0$.

Let $t_1, t_2, t_3, t_4, t_5, t_6, t_7 \in L$ such that $0 \le t_1 \le t_2 \le t_3 \le t_4 \le t_5 \le t_6 \le t_7 \le 1$

Let the L- fuzzy subalgebras $\mu_i : X \rightarrow L$, i=1,2,3,4,5,6 ,7,8,9 be given by

$\mu_1(x)=\begin{cases} 1 & if\ x=0 \\ t_4 & if\ x=1,2 \\ t_5 & if\ x=3,4 \end{cases}$ $\mu_2(x)=\begin{cases} 1 & if\ x=0 \\ 0 & if\ x=1,2 \\ t_2 & if\ x=3,4 \end{cases}$ $\mu_3(x)=\begin{cases} 1 & if\ x=0 \\ t_2 & if\ x=1,2 \\ t_4 & if\ x=3,4 \end{cases}$

$\mu_4(x)=\begin{cases} t_5 & if\ x=0 \\ 0 & if\ x=1,2 \\ t_4 & if\ x=3,4 \end{cases}$ $\mu_5(x)=\begin{cases} 1 & if\ x=0 \\ t_5 & if\ x=1,2 \\ t_7 & if\ x=3,4 \end{cases}$ $\mu_6(x)=\begin{cases} 1 & if\ x=0 \\ t_3 & if\ x=1,2 \\ t_4 & if\ x=3,4 \end{cases}$

$\mu_7(x)=\begin{cases} 0 & if\ x=0,1,2,3,4 \end{cases}$ $\mu_8(x)=\begin{cases} 1 & if\ x=0 \\ t_1 & if\ x=1,2 \\ t_6 & if\ x=3,4 \end{cases}$ $\mu_9(x)=\begin{cases} 1 & if\ x=0 \\ t_3 & if\ x=1,2 \\ t_7 & if\ x=3,4 \end{cases}$

Then the collection $L_T=\{\phi, X, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6, \mu_7, \mu_8, \mu_9\}$ is an L-fuzzy topology on X. Hence (X, L_T) is an L-fuzzy topological TM-system. An L-fuzzy topological TM-system (X, L_T) has no proper L-fuzzy clopen subalgebras. Hence L - fuzzy topological TM-system is L - fuzzy connected TM-system.

Definition 3.7: Let (X,*) be a TM Algebra. Let (X, L_T) be an L – fuzzy topological TM-system. Consider a T-open L-fuzzy subalgebra $\breve{\mu}$ in (X, L_T). An L – fuzzy interior TM-system of $\breve{\mu}$ is the union of all T-open L – fuzzy subalgebras contained in $\breve{\mu}$. It is denoted by $(\dot{\breve{\mu}})$.

Definition 3.8: Let (X,*) be a TM Algebra. Let (X,L_T) be an L – fuzzy topological TM-system. Consider a T-open L-fuzzy subalgebra $\breve{\mu}$ in (X, L_T). An L – fuzzy closure TM-system of $\breve{\mu}$ is the intersection of all L – fuzzy closed subalgebras containing $\breve{\mu}$. It is denoted by$\overline{(\breve{\mu})}$.

Definition 3.9: Let (X,*) be a TM Algebra. Let (X,L_T) be an L – fuzzy topological TM-system. An L- fuzzy TM-subalgebra $\breve{\mu}$ in (X, L_T) is called as L-fuzzy regular open TM-system if $\overline{(\dot{\breve{\mu}})}$.

Example 3.10: Considet the set X = {0, 1, 2, 3} with the following Cayley table

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then (X,*) is a TM-algebra. Let L be a complete lattice with $\sup(L) \equiv 1$ and $\inf(L) \equiv 0$.

Let $t_1, t_2, t_3, t_4, t_5, t_6 \in L$ such that $0 \le t_1 \le t_2 \le t_3 \le t_4 \le t_5 \le t_6 \le 1$

Let the L- fuzzy subalgebras $\mu_i : X \to L$, i=1,2,3,4,5,6 be given by

$\mu_1(x)=\begin{cases} 1 & if\ x = 0 \\ t_5 & if\ x = 1,2,3 \end{cases}$ $\mu_2(x)=\begin{cases} t_4 & if\ x = 0 \\ t_3 & if\ x = 1,2 \\ 0 & if\ x = 3 \end{cases}$ $\mu_3(x)=\begin{cases} t_2 & if\ x = 0 \\ t_1 & if\ x = 1,2 \\ 0 & if\ x = 3 \end{cases}$

$\mu_4(x)=\begin{cases} 1 & if\ x = 0 \\ t_3 & if\ x = 1,2 \\ t_1 & if\ x = 3 \end{cases}$ $\mu_5(x)=\begin{cases} t_6 & if\ x = 0 \\ t_3 & if\ x = 1,2 \\ 0 & if\ x = 3 \end{cases}$ $\mu_6(x)=\begin{cases} 1 & if\ x = 0 \\ t_6 & if\ x = 1,2 \\ t_5 & if\ x = 3 \end{cases}$

Then the collection $L_T=\{\phi, X, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6\}$ is an L-fuzzy topology on X. Hence (X, L_T) is an L- fuzzy topological TM-system.

Consider $\check{\mu} = \mu_2 \Rightarrow \overline{\mu_2} = \mu'_1 \Rightarrow (\mu^{i}{}_1) = \mu_2$

Hence $\left((\dot{\overline{\mu_2}})\right) = \mu_2$

Definition 3.11: Let (X,*) be a TM Algebra. Let (X, L_T) be an L – fuzzy topological TM-system. An L – fuzzy topological TM-system (X, L_T) is called as L- fuzzy super connected TM-System if it has no proper L-fuzzy regular open subalgebra

Example 3.12: Consider the TM – algebra from example 3.3 Let L be a complete lattice with $\sup(L) \equiv 1$ and $\inf(L) \equiv 0$.

Let $t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8 \in L$ such that $0 \le t_1 \le t_2 \le t_3 \le t_4 \le t_5 \le t_6 \le t_7 \le t_8 \le 1$

Let the L- fuzzy subalgebras $\mu_i : X \to L$, i=1,2,3,4,5 be given by

$\mu_1(x)=\begin{cases} 1 & if\ x = 0 \\ t_5 & if\ x = 1,3 \\ t_1 & if\ x = 2,4,5 \end{cases}$ $\mu_2(x)=\begin{cases} t_8 & if\ x = 0 \\ t_6 & if\ x = 1,3 \\ 0 & if\ x = 2,4,5 \end{cases}$ $\mu_3(x)=\begin{cases} 1 & if\ x = 0 \\ t_3 & if\ x = 1,3 \\ t_2 & if\ x = 2,4,5 \end{cases}$

$\mu_4(x)=\begin{cases} 1 & if\ x = 0,1,2,3,4,5 \end{cases}$ $\mu_5(x)=\begin{cases} t_7 & if\ x = 0 \\ t_4 & if\ x = 1,3 \\ 0 & if\ x = 2,4,5 \end{cases}$

Then the collection $L_T=\{\phi, X, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5\}$ is an L-fuzzy topology on X. Hence (X, L_T) is an L-fuzzy topological TM-system. Each L-fuzzy TM Subalgebra on (X, L_T) is not satisfied the condition of regular open subalgebra. Hence L-fuzzy topological TM-system has no proper L-fuzzy regular open subalgebras. Therefore L-fuzzy topological TM-system is L-fuzzy super connected TM – system.

Theorem 3.13: Let (X,*) be a TM Algebra. Let (X, L_T) be an L - fuzzy topological TM-system. If (X, L_T) is L-fuzzy super connected TM-system then L- fuzzy topological TM-system is L-fuzzy connected TM-system.

Proof: Let (X, L_T) be an L - fuzzy topological TM-system. (X, L_T) is L-fuzzy super connected TM-system. That is L - fuzzy topological TM-system has no proper L-fuzzy regular open subalgebras. Suppose (X, L_T) is not L-fuzzy connected TM-system. Then (X,L_T) has proper clopen subalgebras.

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Consider $\check{\mu}$ is proper clopen subalgebra in (X, L_T).

$$\overline{(\dot{\check{\mu}})} = \wedge\{\mu' : \mu' \supseteq \check{\mu}, \mu' \text{ is } L_T - closed\}$$
$$= (\dot{\mu}')$$
$$= (\dot{\check{\mu}}) \text{ [Since } \check{\mu} \text{ is proper L-fuzzy clopen subalgebra]}$$
$$= \check{\mu} \text{ [Since } \check{\mu} \text{ is proper L-fuzzy clopen subalgebra]}$$

Therefore $\check{\mu}$ is proper L-fuzzy regular open subalgebra which is a contradiction to our hypothesis. Hence (X, L_T) is an L-fuzzy connected TM-system.

Theorem 3.14: Let (X,*) be a TM Algebra. Let (X, L_T) be an L - fuzzy topological TM-system. If any two L - fuzzy topological TM-system is L-fuzzy connected TM-system then the intersection of that L-fuzzy connected TM-system is again a L-fuzzy connected TM-system.

Proof: Let (X, L_T), (X, L_U) be two L - fuzzy connected TM-systems. Consider $\mu_i \in$ (X, L_T), $\sigma_i \in$ (X, L_U). We prove the theorem by two cases.

Case -i):

$$\mu_i \wedge \sigma_i = \min\{\mu_i(x), \ \sigma_i(x)\}$$
$$= \begin{cases} \mu_i(x) \ \ if \ \mu_i(x) \leq \ \sigma_i(x) \\ \sigma_i(x) \ \ if \ \sigma_i(x) \leq \ \mu_i(x) \end{cases}$$

Either $\mu_i \wedge \sigma_i \in$ (X, L_T) if $\mu_i(x) \leq \sigma_i(x)$ or $\mu_i \wedge \sigma_i \in$ (X, L_U) if $\sigma_i(x) \leq \mu_i(x)$

In both cases intersection of two L-fuzzy subalgebras is belong to L-fuzzy connected TM-system.

Case- ii):

$$\mu_i \wedge \sigma_i = \min\{\mu_i(x), \ \sigma_i(x)\}$$
$$= \gamma_i(x)$$

In this case, $\gamma_i(x)$ is minimum value of two subalgebras, which is also satisfy the L-fuzzy connected TM-system.

Therefore both cases, we get L-fuzzy connected TM-system.

Hence any two L - fuzzy topological TM-system is L-fuzzy connected TM-system then the intersection of that L-fuzzy connected TM-system is again a L-fuzzy connected TM-system.

Theorem 3.15: Let (X,*) be a TM Algebra. Let (X, L_T) be an L - fuzzy topological TM-system. If any two L - fuzzy topological TM-system is L-fuzzy connected TM-system then the union of that L-fuzzy connected TM-system is again a L-fuzzy connected TM-system

Proof: Let (X, L_T), (X, L_U) be two L - fuzzy connected TM-systems. Consider $\mu_i \in$ (X, L_T), $\sigma_i \in$ (X, L_U). We prove the theorem by two cases.

Case -i):

$$\mu_i \vee \sigma_i = \max\{\mu_i(x), \ \sigma_i(x)\}$$
$$= \begin{cases} \mu_i(x) \ \ if \ \mu_i(x) \geq \ \sigma_i(x) \\ \sigma_i(x) \ \ if \ \sigma_i(x) \geq \ \mu_i(x) \end{cases}$$

Either $\mu_i \vee \sigma_i \in$ (X, L_T) if $\mu_i(x) \geq \sigma_i(x)$ or $\mu_i \vee \sigma_i \in$ (X, L_U) if $\sigma_i(x) \geq \mu_i(x)$

In both cases union of two L-fuzzy subalgebras is belong to L-fuzzy connected TM-system.

Case-ii):

$$\mu_i \vee \sigma_i = \max\{\mu_i(x), \ \sigma_i(x)\}$$
$$= \gamma_i(x)$$

In this case, $\gamma_i(x)$ is maximum value of two subalgebras, which is also satisfy the L-fuzzy connected TM-system.

Therefore both cases, we get L-fuzzy connected TM-system.

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Hence any two L - fuzzy topological TM-system is L-fuzzy connected TM-system then the union of that L-fuzzy connected TM-system is again a L-fuzzy connected TM-system.

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