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FIXED POINTS IN MULTIPLICATIVE METRIC SPACE FOR NON COMPATIBLE MAPS

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ABSTRACT

In this paper we obtain common fixed point theorems in multiplicative metric space for a pair of self maps by using the notion of pointwise R-weakly commutativity but without assuming the completeness of the space or continuity of the mapping involved.

Key words: *Common fixed point, commute, multiplicative metric space, expansive.*

MSC: *47H10, 54 E50, 54 H25.*

INTRODUCTION

It is well known that R_+ is not complete according to the usual metric. To over come this problem, in 2008, Bashirov *et al.* [5] introduced the notion of multiplicative metric spaces and studied the concept of multiplicative calculus and proved the elementary theorem of multiplicative calculus. In Özavşar and Çevikel [15] investigate multiplicative metric spaces by remarking its topological properties, and introduced concept of multiplicative contraction mapping and proved some fixed point theorems of multiplicative contraction mappings on multiplicative spaces. Recently, He *et al.* [20] proved common fixed point theorems for four self-mappings in multiplicative metric space. Very recently, Abbas *et al.* [2] Proved some common fixed point results of quasi-weak commutative mappings on a closed ball in the framework of multiplicative metric space. Kang *et al.* [9] introduced the notions of compatible mappings and its variants in multiplicative metric spaces, and proved some common fixed point theorems for these mappings. In this paper we discuss the notion of R-weakly commuting maps of type (A_g) and the property (E. A) in the multiplicative metric space and then prove common fixed point theorems for a pair of selfmaps.

Definition 1.1: Let X be a nonempty set. Multiplicative metric is a mapping
$d : X\times X \rightarrow R^+$ satisfying the following conditions:

(i) $d(x, y)\geq 1$ for all $x, y \in X$ and $d(x, y) = 1$ if and only if $x = y$,
(ii) $d(x, y) = d(y, x)$ for all $x, y \in X$,
(iii) $d(x, y) \leq d(x, z) . d(z, y)$ for all $x, y, z \in X$ (multiplicative triangle inequality).

Although the multiplicative metric was announced as a new distance notion, we note that composition of the multiplicative metric with a logarithmic function yields a standard metric. Hence, all fixed point results in the context of multiplicative metric spaces can easily be concluded from the corresponding existing famous fixed point results in the context of the standard metric.

Theorem1.2: Let X be a non-empty set. A mapping $d: X \times X \rightarrow [0, \infty)$ is a multiplicative metric. Then the mapping $d^* : X \times X \rightarrow[0,\infty)$ with
$d^*(x, y) = \ln(d (x, y))$ forms a metric.

Example 1.3: Let R_+^n be the collection of all n-tuples of positive real numbers. Let $d : R_+^n \times R_+^n \rightarrow R$ be defined as

$d(x, y) = \left(\left|\frac{x_1}{y_1}\right|, \left|\frac{x_2}{y_2}\right|, \ldots\ldots\ldots \left|\frac{x_n}{y_n}\right|\right)$, where $x=(x_1,x_2,\ldots\ldots,x_n)$, $y=(y_1,y_2,\ldots\ldots,y_n)\in R_+^n$.

and $|.|: R_+ \rightarrow R_+$ is defined by

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$$|a| = \begin{cases} a & if a \geq 1 \\ \frac{1}{a} & if a < 1 \end{cases}$$

Then it is obvious that all conditions of multiplicative metric are satisfied.

Example 1.4: Let d : $R \times R \to [1, \infty)$ be defined by d(x, y) = $a^{|x-y|}$ where x, y $\in$ R and a > 1.Then d is multiplicative metric.

Remark 1.5: We note that the Example 1.3 is valid for positive real numbers and Example 1.4 is valid for all real numbers.

Definition 1.6: Let (X, d) be a multiplicative metric space. Then a sequence $\{x_n\}$ in X said to be
(1) a multiplicative convergent to x if for every multiplicative open ball $B_\in(x) = \{y ; d(x, y) < \in\}$, $\in > 1$; there exists a natural number N such that n $\geq$ N; then $x_n \in B_\in(x)$, that is, $d(x_n, x) \to 1$ as $n \to \infty$.
(2) a multiplicative Cauchy sequence if for all $\in > 1$; there exists a natural number N such that $d(x_n, x_m) < \in$ for all m, n > N, that is, $d(x_n, x_m) \to 1$ as $n \to \infty$.
(3) We call a multiplicative metric space complete if every multiplicative Cauchy sequence in it is multiplicative convergent to x $\in$ X,

Definition1.7: Let f and g map from a multiplicative metric space (X, d) into itself. The maps f and g are said to be compatible, if

$$\lim_{n\to\infty} (d\,(fgx_n, gfx_n)=1$$

Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} g\, x_n$ =t for some $t \in X$.

From this definition it is inferred that f and g are noncompatible maps from a multiplicative metric space (X, d) into itself if $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} g\, x_n$ =t for some $t \in X$ but either $\lim_{n\to\infty} (d\,(fgx_n, gfx_n)) \neq 1$or the limit does not exist.

Definition1.8: Let f and g be two mappings of a multiplicative metric space (X, d) into itself. Then f and g are said to be

(i) R-weakly commuting if there exists some R > 0 such that d(fgx, gfx) $\leq$ d^R(fx, gx) for all x $\in$X.
(ii) R-weakly commuting of type (A_g) if there exists some R>0 such that d(fgx, ggx) $\leq$ d^R(fx, gx) for all x $\in$X.
(iii) R-weakly commuting of type (A_f) if there exists some R>0 such that d(ffx, gfx) $\leq$ d^R(fx, gx) for all x $\in$X.

Definition1.9: Let f and g be two self mappings of a multiplicative metric space (X, d). We say that f and g satisfy the (E.A) if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} g\, x_n$ =t for some $t \in X$.

RESULTS

Theorem 2.1: Let f and g be pointwise R-weakly commuting self mappings of a multiplicative metric space (X, d) satisfying the property (E.A) and

(i) f(X) $\subset$g(X)
(ii) d(fx, fy)$\leq$d(gx, gy)
(iii) d(fx,f^2x) < max{d(gx,gfx), d(fx, gx), d(f^2x, gfx), d(fx, gfx), d(gx, ffx)} whenever fx$\neq$ f^2x.
if the range of f or g is a complete subspace of X, then f and g have a common fixed point.

Proof: Since f and g are satisfy the (E.A), there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} g\, x_n$ =t for some $t \in X$. Since t $\in$ f(X) and f(X) $\subset$g(X), there exists some point u in X such that t= gu where t=$\lim_{n\to\infty} g\, x_n$. If fu $\neq$ gu, the inequality

$$d(fx_n, fu) \leq d(\, gx_n, gu)$$

On letting $n \to \infty$ yields

$$d(gu, fu) \leq d(\, gu, gu) =1$$

Hence fu= gu.

Since f and g are R-weak commutating there exists R>0 such that

$$d(\, fgu, gfu) \leq d^R(fu, gu) =1$$

that is, fgu = gfu and ffu = fgu = gfu = ggu. If fu $\neq$ ffu, using (iii), we get

$$\begin{aligned} d(fu, ffu) &< \max\{d(gu, gfu), d(fu, gu), d(ffu, gfu), d(fu, gfu), d(gu, ffu)\} \\ &= \max\{d(gu, gfu), d(fu, gfu), d(gu, ffu)\} \\ &= d(fu, ffu) \end{aligned}$$

A contradiction. Hence fu = ffu = fgu = gfu = ggu.

Hence fu is a common fixed point of f and g. The case when fX is a complete subspace of X is similar to the above case since fX $\subset$ gX. Hence we have the theorem.

Example 2.2: Let X= [2, 20] and define the mapping d: $X \times X \to R^+$ by d(x, y) = $a^{|x-y|}$ for all x, y $\in$ X. Clearly (X, d) is a complete multiplicative metric space.
Consider f, g: X $\to$ X as

$$fx = \begin{cases} 2 & if\ x = 2\, or > 5 \\ 6 & if\ \ 2 < x \le 5 \end{cases}$$

$$gx = \begin{cases} 2 & if\ x = 2 \\ x + 4 & if\ 2 < x < 5 \\ \dfrac{4x+10}{5} & if\ x \ge 5 \end{cases}$$

Clearly fX={2,6} and gX={2} $\cup$ [6,18] thus fX $\subset$ g(X).it can be verified that f and g are pointwise R-weakly commuting maps and satisfy the (E.A) property and also satisfy all the conditions of the above theorems. Also f and g have common fixed points at x=2 and 5.

Example 2.3: Let X= [2, ∞). Define d: $X \times X \to R^+$ by d(x, y) = $\left|\dfrac{x}{y}\right|$. Then (X, d) is a multiplicative metric space.

Define f, g: X $\to$ X by fx=x and gx=x-1 for all x $\in$ X. Then d(fx, gx) = $\dfrac{x}{x-1}$, d(fgx, gfx) = 1, d(fgx, ggx) = $\dfrac{x-1}{x-2}$, d(ffx, gfx) = $\dfrac{x}{x-1}$, d(ffx, ggx) = $\dfrac{x}{x-2}$, d(fx, ffx) = 1, d(gx, gfx) =1,d(fx, gfx) = $\dfrac{x}{x-1}$, d(gx.ffx) = $\dfrac{x}{x-1}$

Then clearly d(fgx, gfx) $\le$ d^R(fx, gx) for all x in X and for R > 0, implies that f and g are R- weakly commuting mappings. Clearly f and g are also R-weakly commuting mappings and satisfies all three properties of the theorem but not satisfy (E.A) property. Also f and g have no common fixed points.

Theorem has been proved by using the concept of (E.A) property. It may, however be observed that by using the notion of noncompatible maps in place of (E.A) property, we can not only prove the theorem, but also to show that maps are discontinuous at their common fixed point.

Theorem 2.4: Let f and g be noncompatible pointwise R-weakly commuting self mappings of type (A_g) of a multiplicative metric space (X, d) and satisfying

(i) f(X) $\subset$ g(X)
(ii) d(fx, fy) $\le$ d(gx, gy)
(iii) d(fx, f^2x) < max{d(gx, gfx), d(fx, gx), d(ffx, gfx), d(fx, gfx), d(gx, ffx)} whenever fx $\neq$ f^2x.

If the range of f or g is a complete subspace of X, then f and g have a common fixed point and the point is the point of discontinuity.

Proof: Since f and g are noncompatible maps, there exists a sequence {x_n} in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} g\,x_n = t$ for some t $\in$ X, but either $\lim_{n\to\infty} d(fgx_n, gfx_n) \neq 1$or the limit does not exist. Since t $\in$ f(X) and f(X) $\subset$ g(X), there exists some point u in X such that t= gu where t= $\lim_{n\to\infty} g\,x_n$. if fu $\neq$ gu , the inequality

$$d(fx_n, fu) \le d(gx_n, gu)$$

On letting n $\to \infty$ yields

$$d(gu, fu) \le d(gu, gu) = 1$$

Hence fu= gu.

Since f and g are R-weak commutating of type (A_g) there exists R>0 such that

$$d(ffu, gfu) \leq d^R(fu, gu) = 1$$

that is, ffu = gfu and ffu = fgu = gfu = ggu. If fu $\neq$ ffu, using (iii), we get

$$\begin{aligned} d(fu, ffu) &< \max\{d(gu, gfu), d(fu, gu), d(ffu, gfu), d(fu, gfu), d(gu, ffu)\} \\ &= \max\{d(gu, gfu), d(fu, gfu), d(gu, ffu)\} \\ &= d(fu, ffu) \end{aligned}$$

a contradiction. Hence fu = ffu = fgu = gfu = ggu.

Hence fu is a common fixed point of f and g. the case when fX is a complete subspace of X is similar to the above case since fX $\subset$ gX. Hence we have the theorem. We now show that f and g are discontinuous at the common fixed point t = fu = gu. If possible, suppose f is continuous. Then considering the sequence $\{x_n\}$, we get $\lim_{n\to\infty} ffx_n = ft = t$. R-weak commutativity of type (A_g) implies that $d(ffx_n, gfx_n) \leq d^R(fx_n, gx_n) = 1$ which on letting n $\to \infty$ this yields $\lim_{n\to\infty} gfx_n = ft = t$. this yields $\lim_{n\to\infty} d(fgx_n, gfx_n) = 1$. This contradicts the fact that $\lim_{n\to\infty} d(fgx_n, gfx_n)$ is either not one or nonexistent for the sequence $\{x_n\}$.

Hence f is discontinuous at the fixed point. Next, Suppose that g is continuous. Then for the sequence $\{x_n\}$, we get $\lim_{n\to\infty} gfx_n = gt = t$ and $\lim_{n\to\infty} ggx_n = gt = t$. In view of these limits, the inequality $d(fx_n, fgx_n) \geq d(gx_n, ggx_n)$ yields a contradiction unless $\lim_{n\to\infty} fgx_n = ft = gt$. But $\lim_{n\to\infty} fgx_n = gt$ and $\lim_{n\to\infty} gfx_n = gt$ contradicts the fact that $\lim_{n\to\infty} d(fgx_n, gfx_n)$ is either non one or non existent. Thus both f and g are discontinuous at their common fixed point. Hence the theorem.

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