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SOME PROPERTIES OF MULTI L – FUZZY COSETS

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ABSTRACT

We introduce the notion of multi L-fuzzy cosets, pseudo multi L-fuzzy cosets, multi L-fuzzy middle cosets of a subgroup A of a group G and discuss some of its properties.

Keywords: *L – fuzzy subgroup, multi L-fuzzy subgroups, L –fuzzy cosets, multi L-fuzzy cosets, multi L-fuzzy normal subgroup, pseudo multi L- fuzzy cosets , multi L- fuzzy middle cosets.*

1. INTRODUCTION

Applying the concept of fuzzy sets introduced by Zadeh [12], Rosenfeld [9] defined fuzzy subgroup of a given group and derived some of their properties. The concept of anti – fuzzy subgroup was introduced by Biswas [4]. S.Sabu and T.V.Ramakrishnan [8] introduce the multi-fuzzy sets. Mukherjee and bhattacharya [7] introduced the fuzzy right cosets and fuzzy left cosets of a group. In all these studies, the closed unit interval [0, 1] is taken as the membership lattice.

2. PRELIMINARIES

Throughout this paper G denotes an arbitrary group with "e" is an identity element and L denotes an arbitrary Lattice with least element 0 and greatest element 1. The join and meet operations in L are denoted by $\vee$ and $\wedge$ respectively. A function A: G → L is called multi L – fuzzy subset of G.

2.1 Definition [12]: Let X be any nonempty set. A function A: X → [0, 1]. is called a fuzzy set A on X.

2.2 Definition [12]: Let (G, **.**) be a group. A fuzzy subset A of G is said to be a fuzzy subgroup (FSG) of G if the following conditions are satisfied:

i. $A(xy) \geq \min\{A(x), A(y)\}$,
ii. $A(x^{-1}) = A(x)$, for all x and $y \in G$.

2.3 Definition [7]: Let (G, **.**) be a group. A fuzzy subgroup A of G is said to be a normal fuzzy subgroup of G if $A(xy) = A(yx)$, for all x and $y \in G$.

2.4 Definition [4]: A fuzzy subset A of G is said to be an anti fuzzy group of G, if for all $x, y \in G$

i. $A(xy) \leq \max\{A(x), A(y)\}$
ii. $A(x^{-1}) = A(x)$.

2.5 Definition [10]: Let X be a non-empty set and L = (L, ≤) be a lattice with least element 0 and greatest element 1. A L-fuzzy subset A of X is a function A : X → L.

2.6 Definition [8]: Let X be a non – empty set. A multi fuzzy set A in X is defined as a set of ordered sequences, A $=\{(x, A_1(x), A_2(x), ..., A_i(x), ...) : x \in X\}$, where $A_i : X \to L$ for all i.

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2.7 Definition [10]: A multi L-fuzzy subset A of G is called a multi L-fuzzy subgroup (MLFS) of G if for every x, y$\in$G,

i. $A(xy) \geq A(x) \wedge A(y)$
ii. $A(x^{-1}) = A(x)$.

2.8 Definition [10]: A multi L-fuzzy subset A of G is called a multi anti L-fuzzy subgroup (MALFS) of G if for every x,y $\in$G,

i. $A(xy) \leq A(x) \vee A(y)$
ii. $A(x^{-1}) = A(x)$.

2.9 Definition: Let A be an multi L-fuzzy subgroup of a group and H = {x $\in$ G/ A(x) = A(e)}, then order of A is defined as O(A) = O(H).

2.10 Definition: Let A be a multi L-fuzzy subgroup of a group G, for any a$\in$G, define $(aA)(x) = A(a^{-1}x)$ for all x$\in$G is called a multi L-fuzzy coset of a multi L-fuzzy group A of the group G determined by the element a$\in$G.

2.11 Definition: Let A be a multi L-fuzzy subgroup of a group G, then for any a,b$\in$G, defined by $(aAb)(x)= A(a^{-1}xb^{-1})$ for every x$\in$G is called a multi L-fuzzy middle coset of a multi L-fuzzy group A of the group G determined by the element a, b$\in$G.

2.12 Definition: Let A be a multi L-fuzzy subgroup of a group G, then for any a$\in$G, the multi L-pseudo fuzzy coset of a multi L-fuzzy subgroup A of the group G determined by the element a$\in$G, denoted by $(aA)^p$ and is defined as $(aA)^p(x) = p(a)A(x))$ for every x$\in$G, p$\in$P, where P = {p(a) / p(a) $\in$[0,1]}

2.13 Definition: Let A and B be any two multi L-fuzzy subgroups of a group G, then for any a $\in$ G, the multi L-pseudo fuzzy double coset of multi L-fuzzy subgroups A and B of the group G determined by the element a$\in$G, denoted by $(AaB)^p$ and is defined as $(AaB)^p = ((aA)^p \cap (aB)^p)(x)= (aA)^p(x) \wedge (aB)^p(x)$ for every x $\in$ G, p$\in$P,
where P = {p(a) / p(a) $\in$[0,1]}.

3. PROPERTIES OF A MULTI L – FUZZY COSETS OF A MULTI L-FUZZY SUBGROUP

In this section, we discuss some of the properties of a multi L-fuzzy coset of a multi L-fuzzy subgroup of a group G.

3.1 Theorem: Let A be a multi L-fuzzy subgroup of a finite group G, then O(A)/O(G).

Proof: Let A be a multi L-fuzzy subgroup of a finite group G with "e" as its identity element.

Clearly H={x$\in$G/ A(x) = A(e)} is a subgroup of the group G. By Lagrange's theorem O(H) / O(G). Hence by the definition of the order of multi L-fuzzy subgroup of a group G, we have O(A)/O(G).

3.2 Theorem: Let A be a multi L-fuzzy subgroup of a group G. The multi L-fuzzy coset aA of a multi L-fuzzy group A of the group G determined by the element a$\in$G is a multi L-fuzzy subgroup of G if $A(y^{-1}a) \wedge A(a) = A(y^{-1}a)$.

Proof: Let A be a multi L-fuzzy subgroup of a group G and $A(a^{-1}y) \wedge A(y) = A(a^{-1}y)$.

Consider the multi L-fuzzy coset aA and for any x, y $\in$ G,

$$\begin{aligned} aA(xy^{-1}) &= A(a^{-1}xy^{-1}) \\ &= A(a^{-1}xy^{-1}aa^{-1}) \\ &\geq A(a^{-1}x) \wedge A(y^{-1}aa^{-1}) \\ &\geq A(a^{-1}x) \wedge \{ A(y^{-1}a) \wedge A(a^{-1}) \} \\ &\geq A(a^{-1}x) \wedge A(y^{-1}a) \\ &= aA(x) \wedge aA(y). \end{aligned}$$

Hence, $aA(xy^{-1}) \geq aA(x) \wedge aA(y)$.

The multi L-fuzzy coset aA of a multi L-fuzzy group A of the group G determined by the element a $\in$ G is a multi L-fuzzy subgroup of G.

3.3 Theorem: If A is a multi L-fuzzy subgroup of a group G, then for any a$\in$G, the multi L-fuzzy middle coset aAa^{-1} of a multi L-fuzzy group A of the group G determined by the element a$\in$G is also a multi L-fuzzy subgroup of a group G.

Proof: Let A be a multi L-fuzzy subgroup of a group G and $a \in G$. for every x, $y \in G$, we have

$$\begin{aligned}(aAa^{-1})(xy^{-1}) &= A(a^{-1}xy^{-1}a),\\ &= A(a^{-1}xaa^{-1}y^{-1}a)\\ &= A((a^{-1}xa)(a^{-1}ya)^{-1})\\ &\geq A(a^{-1}xa) \wedge A((a^{-1}ya)^{-1})\\ &\geq A(a^{-1}xa) \wedge A(a^{-1}ya), \text{ since A is a MLFS of G}\\ &= (aAa^{-1})(x) \wedge (aAa^{-1})(y)\end{aligned}$$

Therefore $(aAa^{-1})(xy^{-1}) \geq (aAa^{-1})(x) \wedge (aAa^{-1})(y)$.

Hence, aAa^{-1} is a multi L-fuzzy subgroup of a group G.

3.4 Theorem: If A is a multi L-fuzzy subgroup of a group G, and aAa^{-1} be a multi L-fuzzy middle coset of a multi L-fuzzy group A of the group G determined by the element $a \in G$, then $O(aAa^{-1}) = O(A)$, for any $a \in G$.

Proof: Let A be a multi L-fuzzy subgroup of a group G and $a \in G$. By Theorem 3.3, the fuzzy middle coset aAa^{-1} is a multi L-fuzzy subgroup of G. Further by the definition of a multi L-fuzzy middle coset of a group G. We have $(aAa^{-1})(x) = A(a^{-1}xa)$, for every $x \in G$.

Hence for any $a \in G$, A and aAa^{-1} are conjugate multi L-fuzzy subgroups of a group G as there exists $a \in G$ such that $(aAa^{-1})(x) = A(a^{-1}xa)$, for every $x \in G$. Hence, $O(aAa^{-1}) = O(A)$, for any $a \in G$.

3.5 Theorem: If A is a multi L-fuzzy subgroup of a group G. Then xA = yA, for x and $y \in G$ if and only if $A(x^{-1}y) = A(y^{-1}x) = A(e)$.

Proof: Let A be a multi L-fuzzy subgroup of a group G. Let xA = yA, for x, $y \in G$.

Then, $xA(x) = yA(x)$, and $xA(y) = yA(y)$, which implies that $A(x^{-1}x) = A(y^{-1}x)$, and $A(x^{-1}y) = A(y^{-1}y)$.

Hence $A(e) = A(y^{-1}x)$, and $A(x^{-1}y) = A(e)$.

Therefore $A(x^{-1}y) = A(y^{-1}x) = A(e)$.

Conversely, let $A(x^{-1}y) = A(y^{-1}x) = A(e)$, for x and $y \in G$.

For every $g \in G$ and we have,

$$\begin{aligned}xA(g) &= A(x^{-1}g)\\ &= A(x^{-1}yy^{-1}g)\\ &\geq A(x^{-1}y) \wedge A(y^{-1}g)\\ &= A(e) \wedge A(y^{-1}g)\\ &= A(y^{-1}g)\\ &= yA(g).\end{aligned}$$

Therefore, $xA(g) \geq yA(g)$ and,

$$\begin{aligned}yA(g) &= A(y^{-1}g)\\ &= A(y^{-1}xx^{-1}g)\\ &\geq A(y^{-1}x) \wedge A(x^{-1}g) \}\\ &= A(e) \wedge A(x^{-1}g) \}\\ &= A(x^{-1}g)\\ &= xA(g).\end{aligned}$$

Therefore, $yA(g) \geq xA(g)$

Hence, $xA(g) = yA(g)$ and g is arbitrary,

We get, xA = yA.

3.6 Theorem: If A is a multi L-fuzzy subgroup of a group G and xA= yA, for x and $y \in G$, then A(x) = A(y).

Proof: Let A be a multi L-fuzzy subgroup of a group G and xA = yA, for x and $y \in G$.

Now,

$$\begin{aligned}A(x) &= A(yy^{-1}x)\\ &\geq A(y) \wedge A(y^{-1}x)\\ &= A(y) \wedge A(e), \text{ by Theorem 3.5}\\ &= A(y).\end{aligned}$$

Therefore $\quad A(x) \geq A(y)$ and, $A(y) = A(xx^{-1}y)$

$\geq A(x) \wedge A(x^{-1}y)$

$= A(x) \wedge A(e)$, by Theorem 3.5

$= A(x)$.

Therefore $\quad A(y) \geq A(x)$

Hence, we get, $\quad A(x) = A(y)$.

3.7 Theorem: If A be a multi L-fuzzy subgroup of a group G, then the multi L-pseudo fuzzy coset $(aA)^p$ is a multi L-fuzzy subgroup of a group G for every $a \in G$.

Proof: Let A be a multi L-fuzzy subgroup of a group G. for every $x, y \in G$, we have

i. $\quad (aA)^p(xy) = p(a)\, A(xy)$

$\geq p(a)\{ A(x) \wedge A(y)\}$

$= p(a)A(x) \wedge p(a)\, A(y)$

$= (aA)^p(x) \wedge (aA)^p(y)$

Therefore, $\quad (aA)^p(xy) \geq (aA)^p(x) \wedge (aA)^p(y)$

ii. $\quad (aA)^p(x^{-1}) = p(a)\, A(x^{-1})$

$= p(a)\, A(x)$

$= (aA)^p(x)$

Therefore, $\quad (aA)^p(x^{-1}) = (aA)^p(x)$.

Hence, $(aA)^p$ is a multi L-fuzzy subgroup of a group G for every $a \in G$.

3.8 Theorem: Let A and B be any two multi L-fuzzy subgroup of a group G, then the multi L-pseudo fuzzy double coset $(AaB)^p$ is a multi L-fuzzy subgroup of a group G for every $a \in G$.

Proof: Let A and B be any two multi L-fuzzy subgroup of a group G.

For every $x, y \in G$, we have

$(aAB)^p(xy^{-1}) = ((aA)^p \cap (aB)^p)(xy)$

$= (aA)^p(xy) \wedge (aB)^p(xy)$

$= p(a)A(xy^{-1}) \wedge p(a)B(xy^{-1})$

$\geq p(a)\{ A(x) \wedge A(y)\} \wedge p(a)\{ B(x) \wedge B(y)\}$

$\geq \{p(a)A(x) \wedge p(a)\, B(x)\} \wedge \{p(a)A(y) \wedge p(a)\, B(y)\}$

$= ((aA)^p \cap (aB)^p)(x) \wedge ((aA)^p \wedge (aB)^p)(y)$

$= (AaB)^p(x) \wedge (AaB)^p(y)$

That is, $\quad (aAB)^p(xy^{-1}) \geq (AaB)^p(x) \wedge (AaB)^p(y)$.

Hence, $(aAB)^p$ is a multi L-fuzzy subgroup of G.

3.9 Theorem: Let A be a multi L-fuzzy normal subgroup of a group G. The multi L-fuzzy coset aA of a multi L-fuzzy normal sub group A of the group G determined by the element $a \in G$ is also a multi L-fuzzy normal subgroup of G if $A(a^{-1}y) \wedge A(y) = A(a^{-1}y)$.

Proof: By Theorem 3.2, the multi L-fuzzy coset aA is a of a multi L-fuzzy normal group A of the group G determined by the element $a \in G$ is also a multi L-fuzzy subgroup of G.

Now, $\quad aA(xy) = A(a^{-1}xy)$

$= A(a^{-1}yx)$, since A is a MLFNS of G.

$= aA(yx)$

$aA(xy) = aA(yx)$.

Hence, the multi L-fuzzy coset aA of a multi L-fuzzy normal group A of the group G determined by the element $a \in G$ is also a multi L-fuzzy normal subgroup of G.

3.10 Theorem: If A is a multi L-fuzzy normal subgroup of a group G, then the set $G/A = \{xA: x \in G\}$ is a group with the operation $(xA)(yA) = (xy)A$.

Proof: Let x and y in G, xA and yA in G /A.

Clearly, y^{-1} in G. Therefore, $y^{-1}A$ in G /A.

Now, $(xA)(y^{-1}A) = (xy^{-1})A$ in G /A.

Hence G /A is a group.

3.11 Theorem: If A be a multi L-fuzzy normal subgroup of a group G, then the multi L-pseudo fuzzy coset $(aA)^p$ of a multi L-fuzzy normal group A of the group G determined by the element $a\in G$ is also a multi L-fuzzy normal subgroup of a group G for every $a\in G$.

Proof: By Theorem 3.7, the multi L-pseudo fuzzy coset $(aA)^p$ of a multi L-fuzzy normal sub group A of the group G determined by the element $a\in G$ is also a multi L-fuzzy subgroup of a group G. For any $x,y\in G$,

$(aA)^p(xy) = p(a)A(xy)$

$= p(a)A(yx)$, since A is MLFNS of G,

$= (aA)^p(yx)$.

$(aA)^p(xy) = (aA)^p(yx)$.

Hence, the multi L-pseudo fuzzy coset $(aA)^p$ of a multi L-fuzzy normal group A of the group G determined by the element $a\in G$ is also a multi L-fuzzy normal subgroup of a group G for every $a\in G$.

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