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EFFECT OF RADIATION AND THERMODIFFUSION ON CONVECTIVE HEAT AND MASS TRANSFER FLOW THROUGH A POROUS MEDIUM IN HORIZONTAL CHANNEL BOUNDED BY NON-UNIFORMLY HEATED WAVY WALLS

M. SIVA SANKARA REDDY*, V. RAGHAVENDRA PRASAD* AND D. R.V. PRASAD RAO

*Assistant Professor, Dept. of Basic Sciences,
G.Pulla Reddy Engineering College, (Autonomous), Kurnool, Andhra Pradesh, India.

^Professor, Dept. of Mathematics,
S. K. University, Anantapur, Andhra Pradesh, India.

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ABSTRACT

In this paper we investigate the radiation effect on the mixed convective heat transfer of viscous fluid through a porous medium confined in horizontal wavy channels which are maintained at non-uniform temperature in the presence of heat sources. The equations governing the flow, heat and mass transfer are solved using the perturbation technique with δ the slope of boundary as perturbation parameter. The velocity, temperature and concentration are analyzed for different variations of the governed parameters G, D^{-1}, N and Sc. The shear stress and the rate of heat and mass transfer have been evaluated for different variations.

Key Words: *Radiation, Thermodiffusion Convective Heat Transfer, Porous Medium.*

1. INTRODUCTION

Thermal and solutal transport by fluid flowing through a porous matrix is a phenomenon of great interest from both the theory and application point of view. Heat transfer studies with internal heat sources have been proposed for earth's mantle [6, 18] and for the outer region of star interiors [1]. The volumetric rate of heat generation has been assumed to be either constant [2, 11, 15] or a function of space variable [3,9]. Some authors have consider directly the viscous dissipation and the expansion effect [8, 14]. The analysis of temperature field as modified by the generation of heat in moving fluids is important in engineering processes pertaining to flows in which a fluid supports an exothermic chemical or nuclear reaction and in problems connected with dissociating fluids [10]. For class of problems related to geothermal energy systems there is a need for including constant heat sources or at times heat generating in porous media. Such studies have been made by several authors suitably Gabsser and Kazmi [7], Dhir and Catton [4] and Hardee and Nelson [10].

Flow through a channel or pipe with either non-uniform boundaries or boundaries maintained at non-uniform temperature have drawn the attention due to their practical applications in different technological problems [14, 15]. Likewise flow through ducts of non-uniform cross sections has applications in the study of membrane oxygenerators and heat exchange in biomedical apparatus. From the observations made experimentally by Gagen *et al.* [5] regarding the augmentation of heat transfer in fluid and heat transfer through non-uniform pipes/channels, the theoretical analysis of this aspect has been attempted by a few authors [11,12,15]. Rao *et al.*, [16] have investigated free convection in a vertical wavy channel in the presence of a constant heat source/sink. Using a mathematical method similar to that of Vajravelu and Sastri [19] the zeroth order, the first order and the total solution of the problem are evaluated numerically for different values of heat source, wall wavy ness parameter and the free convection parameter, Krishna *et al.*, [11] have analyzed the effect of temperature dependent heat generating sources on the free and forced convective flow of a viscous incompressible fluid in a vertical wavy channel. Some of these papers [14, 20] dealt with perturbation methods in which the perturbation is over the slope of the non-uniform wall which is assumed to be small. Analytical solutions were obtained for arbitrary shapes of the boundary walls as well as arbitrarily chosen non-uniform wall temperatures. The theoretical analysis confirms the augmentation of the heat transfer as observed experimentally.

Corresponding Author: M. Siva Sankara Reddy,*
**Assistant Professor, Dept. of Basic Sciences,*
G.Pulla Reddy Engineering College, (Autonomous), Kurnool, Andhra Pradesh, India.

In the context of space technology and in processes involving high temperatures the effects of radiation are of vital importance. Recent developments in hypersonic flights, missile recentry, rocket combustion chambers, power plants for inter planetary flight and gas cooled nuclear reactors, have focused attention on thermal radiation as a mode of energy transfer, and emphasize the need for improved understanding of radiative transfer in these processes. Mansour [23] studied the radiative and free convective effects on the oscillatory flow past a vertical plate. Prasad *et al.*, [30] considered the radiation and mass transfer effects on two-dimensional flow past an impulsively started isothermal vertical plate.

2. FORMULATION OF THE PROBLEM

We analyze the steady flow of a viscous, incompressible fluid through a porous medium confined in a horizontal channel bounded by two wavy walls which are maintained at non-uniform temperature. The concentration is maintained uniform on the boundaries. The Boussinesq approximation is used so that the density variations will be considered only in the buoyancy force. The viscous dissipation is neglected in comparison to the transport of heat by conduction and convection. The flow occurs at low concentration difference so that the thermo-diffusion effects and the inertial effects and the inertial velocity due to the mass diffusion can be neglected. Also the kinematic viscosity and the thermal conductivity are treated as constants. We choose the Cartesian coordinate system 0(x, y) with x-axis in the horizontal direction and y-axis normal to the walls. The walls of the channel are at y = $\pm$ Lf. The governing equations of the steady flow of heat and mass transfer are

Momentum equation

$$\rho_e\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\left(\frac{\mu}{k_1}\right)u$$

$$\rho_e\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\rho g-\left(\frac{\mu}{k_1}\right)u \tag{2.1}$$

Equation Continuity

$$\frac{\partial v}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.2}$$

Energy equation

$$\rho_e C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=K\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)-\frac{\partial(q_r)}{\partial y} \tag{2.3}$$

Diffusion equation

$$\left(u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right)=D_1\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+k_{11}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{2.4}$$

Equation of State

$$\rho=\rho_e[1-\beta_0(T-T_e)-\beta^*(C-C_e)] \tag{2.5}$$

Where ρ_e, T_e, C_e are the density, temperature and the concentration in the equilibrium state, (u, v) are velocity components along 0(x, y) directions, P is the pressure, T is the temperature, C is the concentration in the flow region, ρ is the density of the fluid, μ is the coefficient of viscosity, C_p is the Specific heat at constant pressure, k_1 is the coefficient of permeability, β_0 is the coefficient of thermal expansion, β^* is the volume coefficient of expansion with mass fraction and D_1 is the molecular diffusivity.

In the equilibrium state

$$0=-\frac{\partial P_e}{\partial y}-P_e g \tag{2.6}$$

Where $P=P_d+P_e$, P_d is the hydromagnetic pressure (2.7)

The boundary conditions are

$$u = v = o, (T - T_e) = \gamma\left(\frac{\delta x}{L}\right), C = C_1 \quad \text{at} \quad y = Lf\left(\frac{\delta x}{L}\right)$$

$$u = v = o, (T - T_e) = \gamma\left(\frac{\delta x}{L}\right), C = C_2 \quad \text{at} \quad y = -Lf\left(\frac{\delta x}{L}\right) \tag{2.8}$$

Where f is chosen twice differential function, δ is small parameter characterizing the slope of the boundary. The flow is maintained by a constant imposed flux for which a characteristic velocity q is defined as

$$q = -\frac{1}{L}\int_{-Lf}^{Lf} u\,dy \tag{2.9}$$

Invoking Rosseland approximation the radiative flux q_r is

$q_r = \dfrac{-4\sigma_s}{3K_e}\dfrac{\partial T^{1^4}}{\partial y}$, and lineralized by expanding T^{1^4} about Te by Taylor series, which after neglecting higher order terms and takes the form $T^{1^4} \cong 4T_e^{1^3}T^1 - 3T_e^{1^4}$

The corresponding non-dimensional variables are

$$x^* = \frac{x}{L},\quad y^* = \frac{y}{L},\quad (u^*, u^*) = \frac{(u,v)}{q},\quad P^* = \frac{P_d}{\rho_e q^2},\quad \theta^* = \frac{T - T_e}{\Delta T_e},\quad Q_1^* = \frac{Q_1}{A},$$

$$\gamma^* = \frac{\gamma}{\Delta T_e},\qquad C^* = \frac{C - C_e}{C_1 - C_e}$$

$$\Delta T_e = T_e(L) - T_e(-L) \tag{2.10}$$

Substitute the above non-dimensional variables in the equations (2.1 to 2.5) and (2.8) and eliminate the pressure term from the momentum equations in terms of dimensionless stream function ψ reduces to (on dropping the stars)

$$R\frac{\partial(\psi, \nabla^2\psi)}{\partial(x,y)} = \nabla^4\psi + \frac{G}{R}(\theta_x + NC_x) - D^{-1}\nabla^2\psi \tag{2.11}$$

The energy and the Diffusion equations in the non-dimensional from are

$$P_1(u\theta_x + v\theta_y) = \left(1 + \frac{4}{3N}\right)\frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial x^2} \tag{2.12}$$

$$S_C(uC_x + vC_y) = \nabla^2 C + \frac{S_C S_O}{N}\nabla^2\theta \tag{2.13}$$

Where $u = -\dfrac{\partial\psi}{\partial y}, v = \dfrac{\partial\psi}{\partial x}$ $G = \dfrac{g\beta(Aa)a^3}{\gamma^2}$ (Grashoffnumber) $D^{-1} = \dfrac{L^2}{k_1}$ (Darcy parameter)

$R = \dfrac{qL}{v}$ (Reynolds number) $P_r = \dfrac{\rho C_p \gamma}{\lambda}$ (Prandtl number) $P_1 = \dfrac{3N_1 P}{3N_1 + 4}$

$S_C = \dfrac{\nu}{D}$ (Schmidt number) $N_1 = \dfrac{4\sigma^* T_e^3}{\beta R\lambda}$ (Radiationparameter) $\alpha_1 = \dfrac{3N_1\alpha}{3N_1 + 4}$

The boundary conditions in non-dimensional form are

$$\psi(+1) - \psi(-1) = -1,\ \frac{\partial\psi}{\partial y} = 0\ \ \frac{\partial\psi}{\partial x} = 0, \theta(x,y) = \gamma(\delta x) \text{ on } y = \pm f(\delta x)$$

$$C = m \quad \text{on } y = -f(\delta x \quad C = 1 \quad \text{on } y = +f(\delta x) \tag{2.14}$$

where $m = \dfrac{C_2 - C_e}{C_1 - C_e}$ is the wall concentration ratio.

The value of ψ on the boundary assumes the constant volumetric flow in consistent with the hypothesis. Also the temperature varies in the axial direction in accordance with the prescribed arbitrary function γ .

3. ANALYSIS OF THE FLOW

The main aim of the analysis is to discuss the perturbations created over a combined free and forced convection flow due to non-uniform slowly varying temperature imposed on the boundaries. We introduce the following transformation

$$\bar{x} = \delta x \qquad \frac{\partial}{\partial \bar{x}} = \delta \frac{\partial}{\partial x}$$

With this introduction equations

$$\delta R \frac{\partial(\psi, F^2\psi)}{\partial(\bar{x}, y)} = F^4\psi + \frac{\hat{G}}{R}(\theta_{\bar{x}} + NC_{\bar{x}}) - D^{-1}F^2\psi \tag{2.15}$$

$$F^2\theta = P_1\delta(\psi_{\bar{X}}\theta_Y - \psi_Y\theta_{\bar{X}}) \tag{2.16}$$

$$F^2C = S_{C_E}\delta(\psi_{\bar{X}}C_Y - \psi_Y C_{\bar{X}}) - \left(\frac{S_C S_0}{N}\right)F^2\theta \tag{2.17}$$

where $\mathrm{F}^2 = \delta^2 \frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial y^2} \qquad \hat{G} = \delta G \approx O(1)$

The flow develops slowly with axial gradient of order δ and hence we take $\frac{\partial}{\partial \bar{x}} \approx O(\delta^{-1})$. We may note that the maintenance of slowly varying axial boundary temperature gives rise to the convection currents and in view of the compatibility at the zeroth order level the analysis is valid provided the thermal buoyancy parameter $G \approx O(\delta^{-1})$ implying $G \approx O(1)$

By using the perturbation technique, we write the solution as

$$\psi = \psi_0 + \delta\psi_1 + \delta^2\psi_2 + ...$$

$$\theta = \theta_0 + \delta\theta_1 + \delta^2\theta_2 + ...$$

$$C = C_0 + \delta C_1 + \delta^2 C_2 + ... \tag{3.1}$$

Let $\eta = \frac{y}{f(\bar{x})}$

Substitute equations (2.18) in equations (2.15) – (2.17) and separating the like powers of δ .

The zero order equations with boundary conditions

$$\frac{\partial^2\theta_0}{\partial\eta^2} = 0 \tag{3.2}$$

$$\frac{\partial^2 C_0}{\partial\eta^2} = -(\frac{S_C S_0}{N})(\frac{\partial^2\theta_0}{\partial\eta^2}) \tag{3.3}$$

$$\frac{\partial^4\psi_0}{\partial\eta^4} - (M_1^2 f^2)\left(\frac{\partial^2\psi_0}{\partial\eta^2}\right) = -\left(\frac{Gf^3}{R}\right)\left(\frac{\partial\theta_0}{\partial\bar{x}} + N\frac{\partial C_0}{\partial\bar{x}}\right) \tag{3.4}$$

$$\psi_0(+1) - \psi_0(-1) = -1 \quad \frac{\partial\psi_0}{\partial\eta} = 0 \quad \frac{\partial\psi_0}{\partial\bar{x}} = 0 \qquad \eta = \pm 1 \tag{3.5a}$$

$$\theta_0 = \gamma(\bar{x}) \qquad \eta = \pm 1 \tag{3.5b}$$

$$C = m \quad \eta = -1 \; C = 1 \quad \eta = +1 \tag{3.5c}$$

First order equations with boundary conditions

$$\frac{\partial^2\theta_1}{\partial\eta^2} = (P_1 f)\left(\frac{\partial\psi_0}{\partial\bar{x}}\frac{\partial\theta_0}{\partial\eta} - \frac{\partial\psi_0}{\partial\eta}\frac{\partial\theta_0}{\partial\bar{x}}\right) \tag{3.6}$$

$$\frac{\partial^2 C_1}{\partial\eta^2} = (S_C f)\left(\frac{\partial\psi_0}{\partial\bar{x}}\frac{\partial C_0}{\partial\eta} - \frac{\partial\psi_0}{\partial\eta}\frac{\partial C_0}{\partial\bar{x}}\right) - \left(\frac{S_C S_0}{N}\right)\frac{\partial^2\theta_1}{\partial\eta^2} \tag{3.7}$$

$$\frac{\partial^4\psi_1}{\partial\eta^4}-(M_1^2 f)\frac{\partial^2\psi_1}{\partial\eta^2}=-\left(\frac{\hat{G}f^3}{R}\right)\left(\frac{\partial\theta_1}{\partial\overline{x}}+N\frac{\partial C_1}{\partial\overline{x}}\right)+(Rf)\left(\frac{\partial\psi_0}{\partial\eta}\frac{\partial^3(\psi_0)}{\partial\eta^3}-\frac{\partial\psi_0}{\partial\overline{x}}\frac{\partial^3\psi_0}{\partial\eta^3}\right) \tag{3.8}$$

$$\psi_1(+1)-\psi_1(-1)=0\,,\frac{\partial\psi_1}{\partial\eta}=0 \qquad \frac{\partial\psi_1}{\partial\overline{x}}=0\,,\ \text{at}\quad \eta=\pm1 \tag{3.9a}$$

$$\theta_1=0\,,\quad C_1=0 \quad \text{at} \quad \eta=\pm1 \tag{3.9b}$$

Where $h_1^2=M_1^2 f^2,\quad M_1^2=D^{-1}$

4. SOLUTION OF THE PROBLEM

Solving equations (3.1) – (3.4) subjected to the conditions (3.5.a.b.c), we get,

$$\theta_0=\gamma(x)$$

$$C_0=\frac{a_1}{2}(\eta^2-1)+\left(\frac{1-m}{2}\right)\eta+\left(\frac{1+m}{2}\right)\eta$$

$$\psi_0=a_{12}Cosh[\beta_1\eta]+a_{13}Sinh[\beta_1\eta]+a_8\eta^4+a_9\eta^3+a_{10}\eta^2+a_{11}\eta$$

Similarly the solutions of the first order differential equations are

$$\begin{aligned}\theta_1=&-a_{37}(\eta^2-1)+a_{38}(\eta^3-\eta)+a_{39}(\eta^4-1)+a_{40}(\eta^5-\eta)+a_{41}(\eta^6-1)\\&+a_{42}(\eta^7-\eta)+a_{43}(\text{Sinh}[\beta_1\eta]-\text{Sinh}\,\beta_1)+(a_{45}\eta-a_{44})(\text{Cosh}[\beta_1\eta]\\&-\text{Cosh}\,\beta_1\eta)+(a_{46}+a_{48}\eta)(\text{Sinh}[\beta_1\eta]-\text{Sinh}\,\beta_1)+a_{47}(\eta^2\,\text{Cosh}[\beta_1\eta]-\text{Cosh}\,\beta_1)\end{aligned}$$

$$\begin{aligned}C_1=&b_{13}\eta^2+b_{14}\eta^3+b_{15}\eta^4+b_{16}\eta^6+(b_{18}+b_{20}\eta+b_{23}\eta^2)Sinh[\beta_1\eta]\\&+(b_{19}+b_{21}\eta+b_{22}\eta^2)Cosh[\beta_1\eta]+b_{25}\end{aligned}$$

$$\begin{aligned}\psi_1=&e_{49}+(e_{48}+e_{25})\eta+e_{26}\eta^2+e_3\eta^3+e_4\eta^4+e_5\eta^5+e_6\eta^6+e_7\eta^7+e_8\eta^8+e_9\eta^9\\&+e_{46}Cosh[\beta_1\eta]+e_{47}Sinh[\beta_1\eta]+e_{10}\eta Cosh[\beta_1\eta]+e_{11}\eta Sinh[\beta_1\eta]+e_{12}\eta^2Cosh[\beta_1\eta]\\&+e_{13}\eta^2Sinh[\beta_1\eta]+e_{14}\eta^3Cosh[\beta_1\eta]+e_{15}\eta^3Sinh[\beta_1\eta]+e_{16}\eta^4Cosh[\beta_1\eta]\\&+e_{17}\eta^4Sinh[\beta_1\eta]+e_{18}\eta^5Cosh[\beta_1\eta]+e_{19}Sinh[2\beta_1\eta]+e_{20}Cosh[2\beta_1\eta]+2e_{21}\beta_1\eta\end{aligned}$$

The rate of heat transfer (Nusselt number) on the plates have been calculated using the formula.

$$Nu=\frac{1}{f(\theta_m-\theta_w)}\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=\pm1} \qquad \theta_m=\frac{1}{2}\int_{-1}^{1}\theta\partial\eta \qquad C_m=\frac{1}{2}\int_{-1}^{1}C\partial\eta$$

5. DISCUSSION

In this paper we discussed the combined effect of radiation and convection on the flow of a viscous incompressible fluid through a porous medium in a horizontal wavy channel whose walls are maintained at non-uniform temperature. We take the Prandtl number P = 0.71. The walls are taken at $\eta=1+\beta e^{-x^2}$, where β >o represents the dilation and β <0 represents the constriction of the channel walls. In this analysis we confine to the case of β **>0.** The velocity, temperature and concentration distributions are presented in the figures 1-16 for different sets of governing parameters G, D^{-1}, N and Sc.

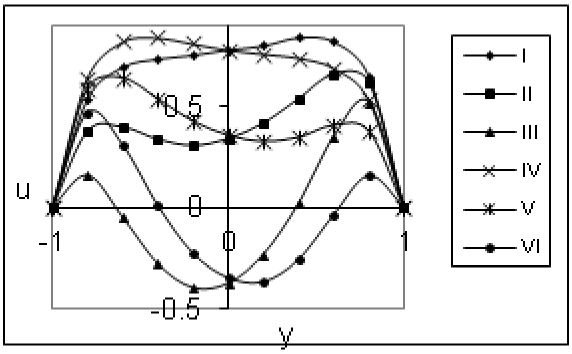

Fig.1: Variation of axial velocity (u) with G
$D^{-1}=5x10^2$,N=1,Sc=1.3,S_0=0.5,α=0.5

	I	II	III	IV	V	VI
G	10^3	$3x10^3$	$5x10^3$	-10^3	$-3x10^3$	$-5x10^3$

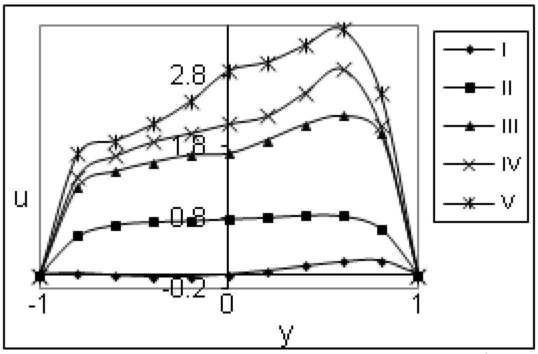

Fig.2: Variation of axial velocity (u) with D^{-1}
G=10^3,N=1,Sc=1.3,S_0=0.5,N1=4.0

	I	II	III	IV	V
D^{-1}	10^2	$2x10^2$	$3x10^2$	$5x10^2$	10^3

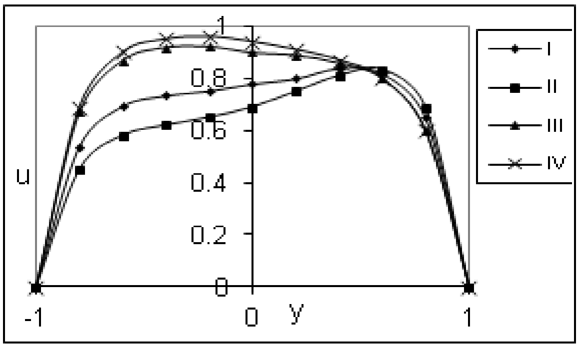

Fig.3: u with N, G=10^3,α=0.5,S_0=0.5,Sc=1.3

	I	II	III	IV
N	1	2	-0.5	-0.8

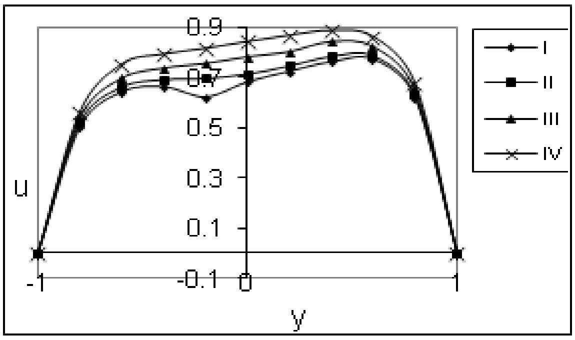

Fig.4: u with Sc ,G=10^3,N=1,α=2,S_0=0.5

	I	II	III	IV
Sc	0.24	0.6	1.3	2.01

The axial velocity u is shown in fig.1 for different G. $\boldsymbol{u} > 0$ is the actual flow and $\boldsymbol{u} < 0$ represents the reversal flow. It is found that the reversal flow appears in the lower half of the channel for G≥ 2×10^{3}. No such reversal flow appears for G < 0. The reversal flow enlarges with increase in G. Also the magnitude of u experiences an enhancement with increase in $|\boldsymbol{G}|$ (>< 0). The variation of u with D^{-1} reveals that the reversal flow which appears in the lower half for $D^{-1}= 10^3$ disappears for higher values of D^{-1}. Also lesser the permeability of the porous medium larger $|\boldsymbol{u}|$ in the entire flow region (fig.2). The variation of u with buoyancy ratio N shows that when the molecular buoyancy force dominates over the thermal buoyancy force $|\boldsymbol{u}|$ enhances when the buoyancy force act in the same direction and for the forces acting in the opposite directions it depreciates in the entire flow region (fig.3). The effect of Schmidt number S_c on u is shown in fig. 4. It is found that lesser the molecular diffusivity smaller $|\boldsymbol{u}|$ in the lower half and larger $|\boldsymbol{u}|$ in the upper half of the channel.

The secondary velocity v which arises due to the non-uniformity of the boundary and boundary temperature is shown in figures 5 - 8 for different values of G, D^{-1}, N and Sc. It is found that for all variations of the parameters the secondary velocity $\boldsymbol{v}$ is directed towards the boundary except for G<0, S_0=-1,$\boldsymbol{x} > \boldsymbol{\pi}/\boldsymbol{2}$, it is towards the mid region. An increase in $|\boldsymbol{G}| \leq \boldsymbol{2} \times \boldsymbol{10^3}$ enhances $\boldsymbol{v}$ and for higher$|\boldsymbol{G}| \geq \boldsymbol{3} \times \boldsymbol{10^3}$, depreciates $\boldsymbol{v}$ in the entire flow region (fig.5). From fig. 6 we notice that lesser the permeability of the porous medium smaller the secondary velocity in the flow region. When the molecular buoyancy force dominates over the thermal buoyancy force $\boldsymbol{v}$ depreciates when the buoyancy forces act in the same direction and for the forces acting the opposite directions $\boldsymbol{v}$ enhance in the flow region (fig.7). An increase in S_c enhances $\boldsymbol{v}$ in the entire region (fig8).

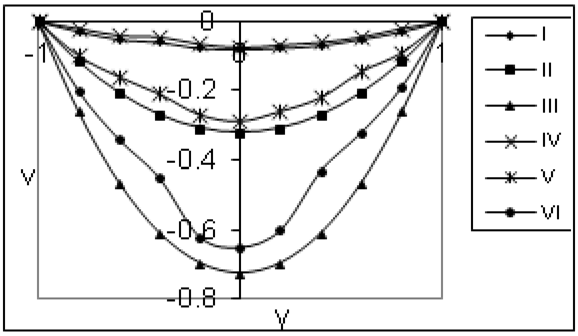

Fig.5: Variation Second Velocity V with G
$D^{-1}=5x10^2$,N=1,Sc=1.3,S_0=0.5

	I	II	III	IV	V	VI
G	10^3	3x103	$5x10^3$	-10^3	$-3x10^3$	$-5x10^3$

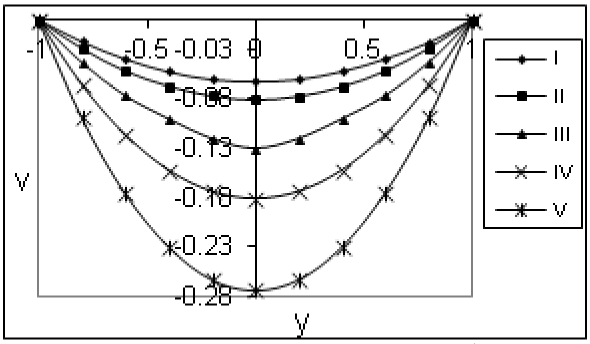

Fig.6: Variation Second Velocity V with D^{-1}
$G=10^3$,N=1,Sc=1.3,S_0=0.5

	I	II	III	IV	V
D^{-1}	10^2	$2x10^2$	$3x10^2$	$5x10^2$	10^3

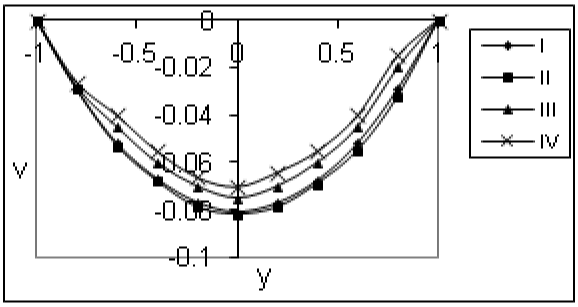

Fig.7: V with N $G=10^3$,α=0.5,S_0=0.5

	I	II	III	IV
N	1	2	-0.5	-0.8

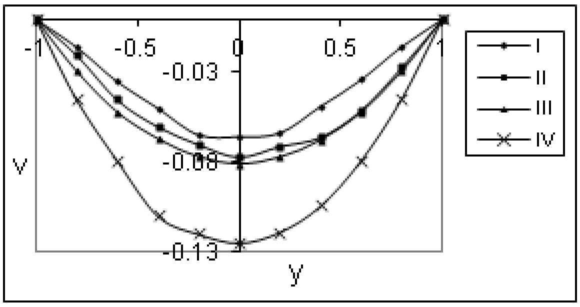

Fig.8: V with Sc $G=10^3$,N=1,α=2,S_0=0.5

	I	II	III	IV
Sc	0.24	0.6	1.3	2.01

The non-dimensional temperature $\boldsymbol{\theta}$ distribution is presented in figures 9- 12 for different variations. We follow the concentration that the non-dimensional temperature is positive or negative according as the actual temperature is greater or lesser than the equilibrium temperature. An increase in the Grashof number G > 0 enhances $\boldsymbol{\theta}$ and depreciates with G < 0 (figure 9). The variation of $\boldsymbol{\theta}$ with D^{-1} shows that lesser the permeability of the porous medium larger the actual temperature in the entire flow region (figure 10). The variation of $\boldsymbol{\theta}$ with buoyancy ratio N shows that the actual temperature enhances with N >0 and depreciates with increase in N < 0 (fig.11). An increase in S_c depreciates temperature $\boldsymbol{\theta}$(fig.12).

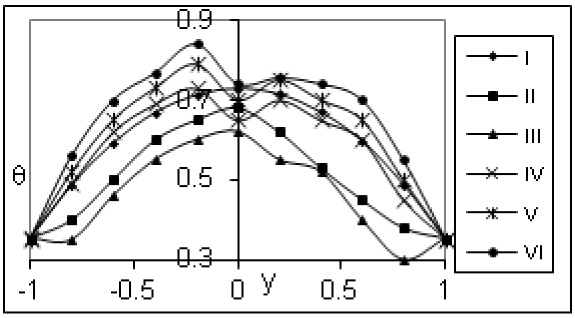

Fig.9: Variation of temperature (θ) with G
$D^{-1}=5x10^2$,N=1,Sc=1.3,S_0=0.5,α_1=0.5

	I	II	III	IV	V	VI
G	10^3	$3x10^3$	$5x10^3$	-10^3	$-3x10^3$	$-5x10^3$

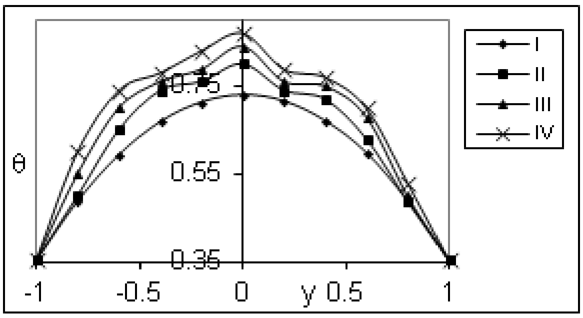

Fig.10: Variation of temperature (θ) with D^{-1}
$G=10^3$,N=1,Sc=1.3,S_0=0.5,N1=4

	I	II	III	IV	V
D^{-1}	10^2	$2x10^2$	$3x10^2$	$5x10^2$	10^3

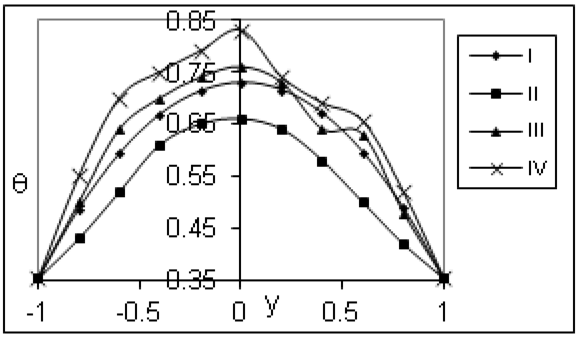

Fig.11: θ with N $G=10^3,\alpha=0.5, S_0=0.5, Sc=1.3$

	I	II	III	IV
N	1	2	-0.5	-0.8

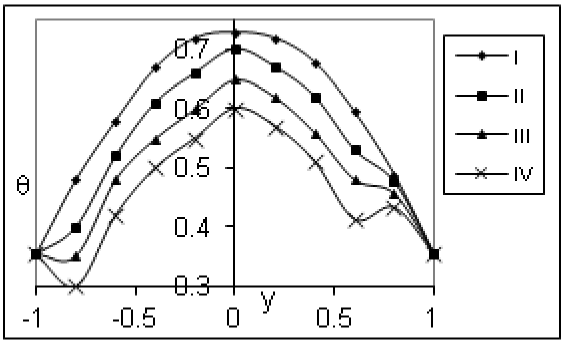

Fig.12: θ with Sc $G=10^3, N=1,\alpha=2, S_0=0.5$

	I	II	III	IV
Sc	0.24	0.6	1.3	2.01

The non-dimensional concentration (C) distribution is shown in figures 13-16 for different parameters G, D^{-1}, N and Sc. We follow the concentration that the non-dimensional concentration is positive or negative according as the actual concentration is higher or lesser than the equilibrium concentration. From figure 13 it is found that an increase in G > 0 reduces the actual concentration while it enhances with increase in G < 0. The profile of the concentration rises gradually from prescribed value 0 on $\boldsymbol{\eta = -1}$, attains a maximum at $\boldsymbol{\eta = -0.4}$ and then falls to its minimum at $\boldsymbol{\eta = 0.4}$ and again rises to attain its value 1 on $\boldsymbol{\eta = 1}$. It is found that an increase in G > 0 reduces the actual concentration in the lower half and enhances it in the upper half while an increase in G < 0 reduces the actual concentration in the upper half and enhances it in the lower half of the channel.

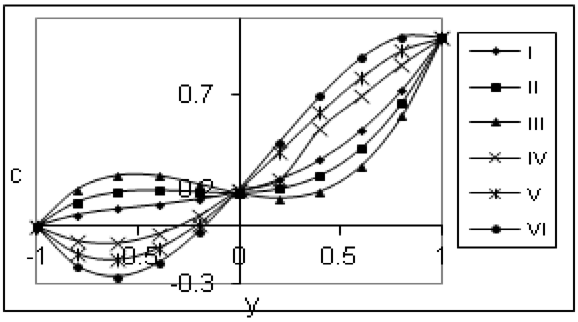

Fig.13 Variation of Concentration(C) with G
$D^{-1}=5x10^2, N=1, Sc=1.3, S_0=0.5,\alpha=0.5$

	I	II	III	IV	V	VI
G	10^3	$3x10^3$	$5x10^3$	-10^3	$-3x10^3$	$-5x10^3$

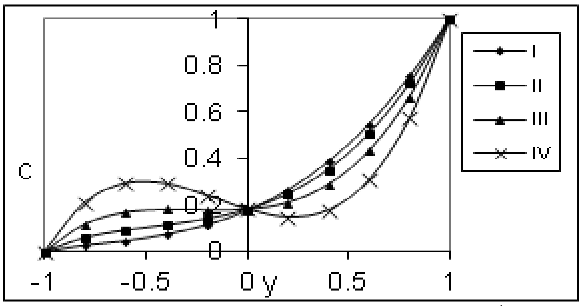

Fig. 14 Variation of Concentration(C) with D^{-1}
$G=10^3, N=1, Sc=1.3, S_0=0.5, N1=4.0$

	I	II	III	IV	V
D^{-1}	10^2	$2x10^2$	$3x10^2$	$5x10^2$	10^3

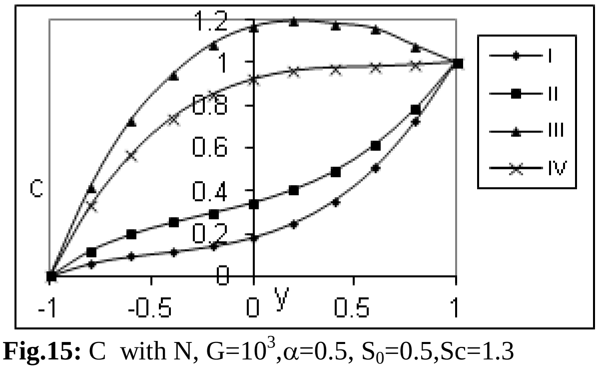

Fig.15: C with N, $G=10^3,\alpha=0.5, S_0=0.5, Sc=1.3$

	I	II	III	IV
N	1	2	-0.5	-0.8

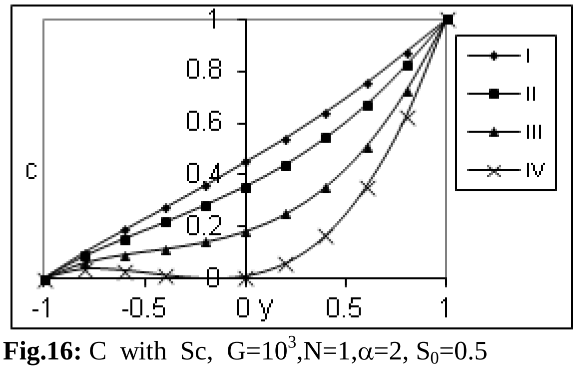

Fig.16: C with Sc, $G=10^3, N=1,\alpha=2, S_0=0.5$

	I	II	III	IV
Sc	0.24	0.6	1.3	2.01

The variation of C with D^{-1} reveals that lesser the permeability of the porous medium smaller the actual concentration in the lower half and higher the actual concentration in the upper half of the channel (fig.14). The behavior of C with buoyancy ratio N reveals that when the molecular buoyancy force dominates over the thermal buoyancy force the actual concentration enhances in the fluid region when the buoyancy forces act in the same direction and for forces acting in the opposite directions the actual concentration reduces in the entire fluid region (fig15). The variation of C with S_c exhibits that lesser the molecular diffusivity smaller the actual concentration in the lower half and larger the concentration in the upper half of the channel. For still lowering of the molecular diffusivity we notice depreciation in the actual concentration everywhere in the fluid region (fig.16).

The average Nusselt number (Nu) which measures the rate of heat transfer at $\boldsymbol{\eta} = \pm\mathbf{1}$ is shown in the tables 1– 4 for different values of the parameters. It is found that the rate of heat transfer enhances with increase in G > 0 and reduces with G < 0. Also lesser the permeability of the porous medium smaller $|\boldsymbol{Nu}|$ at $\boldsymbol{\eta} = \mathbf{1}$ and larger $|\boldsymbol{Nu}|$ at $\boldsymbol{\eta} = -\mathbf{1}$. The variation of Nu with non-uniform boundary temperature shows that an increase in the amplitude $\alpha_1 \leq 0.5$ and for higher $\alpha_1 \geq 0.7$, $|\boldsymbol{Nu}|$ experiences an enhancement at both the walls.An increase in the Reynolds number R reduces $|\boldsymbol{Nu}|$ for G > 0 and enhances for G < 0. When the molecular buoyancy force dominates over the thermal buoyancy force the rate of heat transfer at $\boldsymbol{\eta} = \mathbf{1}$ depreciates in the heating case and enhances in the cooling case, and enhances at $\boldsymbol{\eta} = -\mathbf{1}$ when the buoyancy forces act in the same direction, while for the forces acting in the opposite directions $|\boldsymbol{Nu}|$ at $\boldsymbol{\eta} = \mathbf{1}$ enhances for G > 0 and reduces for G < 0 and at $\boldsymbol{\eta} = -\mathbf{1}$ enhances for all G. The variation of Nu with S_c shoes that lesser the molecular diffusivity smaller $|\boldsymbol{Nu}|$ at $\boldsymbol{\eta} = \pm\mathbf{1}$ (Tables 3 and 4).

Table-1: Nu at η=1 α_1=0.5, β=0.5, N=1.0, S_c=1.3, S_0=0.5, R=35, $x=\pi/4$

G	I	II	III	IV	V
10^3	-1.45736	-0.65623	0.46386	-0.09475	1.27780
2×10^3	-1.60983	-0.80643	0.21548	-0.28367	0.91739
3×10^3	-1.76476	-0.94860	0.00030	-0.45450	0.62239
-10^3	-1.15955	-0.32880	0.09655	0.35024	2.30686
-2×10^3	-1.01411	-0.14991	0.10815	0.61476	3.07798
-3×10^3	-0.87090	-0.04039	0.01161	0.91482	4.15655
D^{-1}	10	20	30	20	20
M	2	2	2	3	5

Table-2: Nu at η= -1 α_1=0.5, β=0.5, N=1.0, S_c=1.3, S_0=0.5, R=35, $x=\pi/4$

G	I	II	III	IV	V
10^3	2.40403	3.55809	5.16686	4.36915	6.31606
2×10^3	2.82054	4.07607	5.56967	4.86408	6.43971
3×10^3	3.24377	4.56633	5.91864	5.31161	6.54091
-10^3	1.59050	2.42897	4.14077	3.20340	5.96303
-2×10^3	1.19319	1.81208	3.47324	2.51043	5.69849
-3×10^3	0.80199	1.15585	2.65674	1.72435	5.32847
D^{-1}	10	20	30	20	20
M	2	2	2	3	5

Table-3: Nu at η= 1 D^{-1}=20, M=2, α_1=0.5, β=0.5, S_0=0.5, $x=\pi/4$

G	I	II	III	IV	V	VI	VII	VIII
10^3	-1.4574	-0.5488	-0.8898	-0.9525	-1.2101	-1.0180	-0.3042	-0.5778
2×10^3	-1.6098	-0.5871	-1.5229	-1.8077	-1.9275	-1.5307	-0.1290	-0.6562
3×10^3	-1.7648	-0.6165	-2.7153	-4.0019	-2.6495	-2.0357	0.0307	-0.7323
-10^3	-1.1596	-0.4237	-0.2300	-0.2145	0.2110	0.0316	-0.7109	-0.4142
-2×10^3	-1.0141	-0.3106	-0.0364	-0.0218	0.9147	0.5688	-0.6486	-0.3288
-3×10^3	-0.8709	-0.1139	0.1103	0.1178	1.6140	1.1146	-0.5148	-0.2407
N	1.0	2.0	-0.5	-0.8	1.0	1.0	1.0	1.0
S_c	1.3	1.3	1.3	1.3	0.24	0.6	2.01	1.3
R	35	35	35	35	35	35	35	70

Table-4: Nu at η= -1 D^{-1}=20, M=2, α=2, α_1=0.5, β=0.5, S_0=0.5, $x=\pi/4$

G	I	II	III	IV	V	VI	VII	VIII
10^3	2.4040	3.5887	3.4912	3.5733	4.4804	4.1605	2.9719	3.2879
2×10^3	2.8205	4.0174	4.2675	4.3436	5.9604	5.2936	2.9374	3.5580
3×10^3	3.2437	4.3476	5.7296	6.5767	7.4500	6.4094	2.9060	3.8206
-10^3	1.5905	2.1853	2.6823	2.7222	1.5488	1.8413	3.0520	2.7239
-2×10^3	1.1931	1.4160	2.4449	2.5261	0.9969	1.6542	3.0988	2.4289
-3×10^3	0.8019	1.2902	2.2649	2.3839	0.6456	0.8517	3.1512	2.1252
N	1.0	2.0	-0.5	-0.8	1.0	1.0	1.0	1.0
S_c	1.3	1.3	1.3	1.3	0.24	0.6	2.01	1.3
R	35	35	35	35	35	35	35	70

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